Special Calculations

In this section:

ⓠ | Lumped Ports with Voltage Input

S-Parameter Calculations

For high-frequency problems, voltage is not a well-defined entity, and it is necessary to define the scattering parameters (S-parameter) in terms of the electric field. To convert an electric field pattern on a port to a scalar complex number corresponding to the voltage in transmission line theory an eigenmode expansion of the electromagnetic fields on the ports needs to be performed. Assume that an eigenmode analysis has been performed on the ports 1, 2, 3, … and that the electric field patterns $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, … of the fundamental modes on these ports are known. Further, assume that the fields are normalized with respect to the integral of the power flow across each port cross section, respectively. This normalization is frequency dependent unless TEM modes are being dealt with. The port excitation is applied using the fundamental eigenmode, the mode with subscript 1. The computed electric field $\mathbf{E}_c$ on the port consists of the excitation plus the reflected field. That is, on the port boundary where there is an incident wave, the computed field can be expanded in terms of the mode fields as

$$\mathbf{E}_c = \mathbf{E}_1 + \sum_{i = 1} S_{i1}\mathbf{E}_i \, ,$$

whereas on all other port boundaries, the computed field is given by

$$\mathbf{E}_c = \sum_{i = 1} S_{i1}\mathbf{E}_i$$

The S-parameter for the mode with index k is then given by multiplying with the conjugate of the mode field for mode k and integrating over the port boundary. Since the mode fields for the different modes are orthogonal, the following relations are obtained for the S-parameters

$$S_{11} = \frac{\displaystyle\int_{\text{port } 1} ((\mathbf{E}_c - \mathbf{E}_1) \cdot \mathbf{E}_1^*) dA_1}{\displaystyle\int_{\text{port } 1} (\mathbf{E}_1 \cdot \mathbf{E}_1^*) dA_1}$$

$$S_{21} = \frac{\displaystyle\int_{\text{port } 2} (\mathbf{E}_c \cdot \mathbf{E}_2^*) dA_2}{\displaystyle\int_{\text{port } 2} (\mathbf{E}_2 \cdot \mathbf{E}_2^*) dA_2}$$

$$S_{31} = \frac{\displaystyle\int_{\text{port } 3} (\mathbf{E}_c \cdot \mathbf{E}_3^*) dA_3}{\displaystyle\int_{\text{port } 3} (\mathbf{E}_3 \cdot \mathbf{E}_3^*) dA_3}$$

and so on. To get S_{22} and S_{12}, excite port number 2 in the same way.

POWER FLOW NORMALIZATION

The fields $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, and so on, should be normalized such that they represent the same power flow through the respective ports. The power flow is given by the time-average Poynting vector,

$$\mathbf{S}_{\text{av}} = \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*)$$

The amount of power flowing out of a port is given by the normal component of the Poynting vector,

$$\mathbf{n} \cdot \mathbf{S}_{\text{av}} = \mathbf{n} \cdot \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*)$$

Below the *cutoff frequency* the power flow is zero, which implies that it is not possible to normalize the field with respect to the power flow below the cutoff frequency. But in this region the S-parameters are trivial and do not need to be calculated.

In the following subsections the power flow is expressed directly in terms of the electric field for TE, TM, and TEM waves.