Created in COMSOL Multiphysics 6.3

Uncertainty Quantification Study for a Microstrip Patch Antenna

Introduction

In this tutorial, an uncertainty quantification analysis is performed on the Microstrip Patch Antenna model from the RF Module Application Library to explore how variations in input parameters, such as material properties or geometric variations, impact the antenna's performance in terms of the S-parameter.

The analysis begins with a screening study to identify which parameters most significantly influence the key performance metrics. That is followed by sensitivity analysis to evaluate the relative effects of these parameters and their interactions. Next, uncertainties are propagated through the model to examine their influence on the distribution of the output characteristics. Finally, a reliability analysis is conducted to determine the probability of achieving the specific performance criteria.

This comprehensive approach provides a detailed understanding of how parameter uncertainties affect the overall performance and reliability of the microstrip patch antenna.

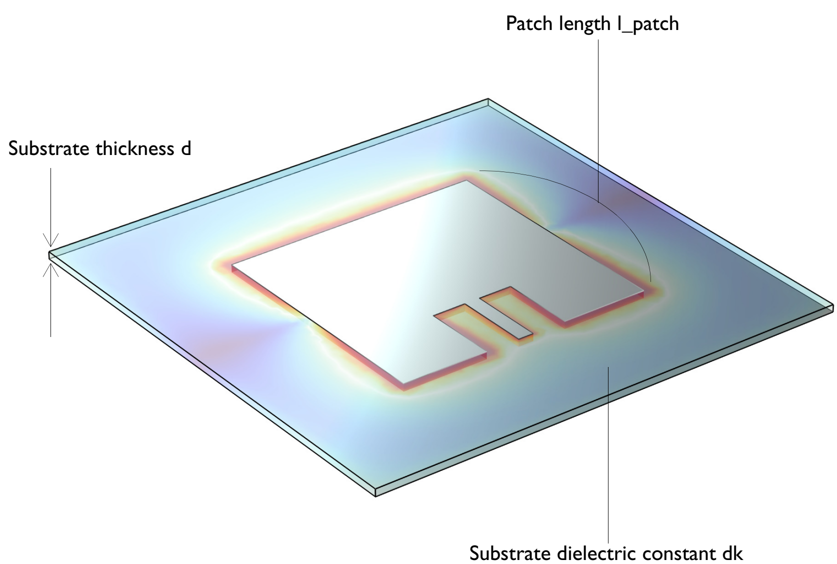


Figure 1: Microstrip patch antenna with evaluated parameters.

Note: In addition to the RF Module, this example requires the Uncertainty Quantification Module.

Model Definition

The frequency responses of narrow-band or high-Q devices are sensitive to variations in design parameters as well as mesh settings during the simulation. The underlying assumption here is that the current mesh is sufficiently well-structured and not a primary source of unreliable results. Instead, only design parameters such as geometry size and material properties are considered influential to the computation.

After loading the *Microstrip Patch Antenna* model from the Application Library, additional studies on uncertainty quantification are conducted sequentially based on Study 1, Frequency Domain:

- Screening, MOAT
- Sensitivity analysis
- Uncertainty propagation
- Reliability analysis

SCREENING, MOAT

The screening study identifies which input parameters most significantly influence each Quantity of Interest (QoI). It utilizes the Morris One-At-a-Time (MOAT) technique to assess sensitivity, and provides two key metrics:

- MOAT Mean: Measures the average effect of each input parameter on the QoI.
- MOAT Standard Deviation: Indicates the variability or uncertainty in the effect of each input parameter, highlighting interactions with other parameters or nonlinear effects.

The following table details the range of variation for each parameter used in the screening study.

TABLE 1: EVALUATED PARAMETERS

PARAMETER	DESCRIPTION	INITIAL VALUE (MEAN)	STANDARD DEVIATION (σ)	RANGE
d	Substrate thickness	60 mil	0.035*d	d ± 2σ
l_patch	Patch length	52 mm	0.005*l_patch	l_patch ± 2σ
dk	Dielectric constant	3.38	0.005*dk	dk ± 3σ

The unit 'mil' used for the substrate thickness refers to the unit milliinch, as shown in Table 1, is a commonly used standard for printed circuit boards (PCBs) in the United States. Typical substrate thicknesses are 10 mil, 20 mil, and 60 mil, which correspond to 0.254 mm, 0.508 mm, and 1.524 mm, respectively.

- The substrate thickness can vary within about ±7% based on a manufacturer's data sheet. If the standard deviation (σ) for the thickness is 3.5% of the nominal thickness, then the range specified by d ± 2σ will cover approximately 95.45% of the variations.
- The tolerance for l_patch is initially based on a scenario using a milling machine with a loosely anchored circuit board. This results in an undesired tolerance of 2σ, 0.520 mm, where σ is 0.005*l_patch. After computing the probability of conditions for this scenario, another reliability analysis is performed assuming non-high-precision PCB fabrication, with an etching tolerance of 0.127 mm (0.005 inches). In this case, σ is set to 0.00125* l_patch, making 2σ approximately equal to 0.127 mm.
- According to a manufacturer's data sheet, the dielectric constant (relative permittivity) dk is given as 3.38 ± 0.05 For strict range coverage such as 99.73%, the dielectric constant varies within dk ± 3σ, where σ is 0.005*dk, which equals 0.0169. Therefore, the range corresponds to 3.38 ± 0.0507, that is close to the example value suggested by manufacturers.

Using ±2σ balances high reliability with practical constraints. While ±3σ provides a more stringent coverage, ±2σ is known to be generally sufficient and practical for many applications, including manufacturing, quality control, and statistical analysis. In this example, both ±2σ and ±3σ are used to illustrate a range of options for parameter variations.

SENSITIVITY ANALYSIS

The sensitivity analysis study evaluates the proportion of impact that each input parameter has on the QoI, providing a deeper understanding of how parameters influence the output. It calculates two types of Sobol indices:

- First-Order Sobol Index: Reflects the direct contribution of each parameter to the variability of the QoI when varying that parameter alone.
- Total Sobol Index: Accounts for the total contribution of each parameter, including interactions with other parameters, to the overall variability of the QoI.

UNCERTAINTY PROPAGATION

The uncertainty propagation examines how uncertainties in input parameters propagate through the model to affect the QoI. It computes the statistical variation of the QoI based on input uncertainties. The output includes a kernel density estimation (KDE) plot, which provides a smoothed estimate of the probability density function for the QoI, illustrating how its distribution is shaped by the uncertainties in the input parameters.

RELIABILITY ANALYSIS

The reliability analysis determines the probability that a specific condition related to the QoI will be satisfied. It evaluates the likelihood of meeting a predefined criterion based on the QoI. For example, it calculates the probability that the S-parameter is below −10 dB, helping to assess the reliability of meeting performance specifications. The S-parameter S_{11} in dB indicates the level of reflection or impedance mismatch at the antenna input port. The threshold of −10 dB is conventionally used, and when it is lower than −10 dB, the reflection due to the impedance mismatch is considered acceptable.

The reliability test is repeated with a smaller tolerance, assuming a scenario where the circuit is fabricated via an etching process. This results in fewer deviations from the specified design parameters compared to the case of a milling machine is used and the circuit board is poorly anchored.

STUDY SUMMARY

The following table provides a quick overview of all study steps used for uncertainty quantification in this model..

TABLE 2: STUDY STEP SUMMARY

STUDY	PURPOSE	NOTABLE RESULTS	NOTE
Study 1	Reference study to create the following Screening.	S11dB value	S_{11} dB shows how much reflection or impedance mismatch at the excited port.
Study 3, UQ Screening	• To identify which input parameters are influential • Reference uncertainty quantification study step to create the following Sensitivity Analysis	MOAT, comp.emw.s11dB plot	MOAT plot indicates which parameters significantly influences the quantity of interest (QoI), S_{11} dB.

TABLE 2: STUDY STEP SUMMARY

STUDY	PURPOSE	NOTABLE RESULTS	NOTE
Study 4, UQ Sensitivity Analysis	• To analyze the sensitivity of the QoI to the selected parameter • Reference uncertainty quantification study step to create the following Uncertainty Propagation	• Sobol Index, comp1.emw.S11dB • Polynomial Chaos Expansion, surrogate model for sensitivity analysis	Sobol plot shows the sensitivity of S_{11} dB to a single parameter variations, and the overall sensitivity contribution including interactions with other parameters.
Study 5, UQ Uncertainty Propagation	• To compute the probability density function of the QoI • Reference uncertainty quantification study step to create the following Reliability Analyses	• Kernel Density Estimation, comp1.emw.S11dB plot	KDE plot estimates the probability density function of S_{11} dB
Study 6, UQ Reliability Analysis 1	To compute the probability under the given conditions (milling with loose anchoring combined with dielectric constant variations).	• Probability for Conditions value • Gaussian Process, surrogate model for reliability analysis	Chance that S_{11} dB will fall below -10 dB.
Study 7, UQ Reliability Analysis 2	To compute the probability under the given conditions (non-high-precision PCB etching tolerance combined with dielectric constant variations).	• Probability for Conditions value • Gaussian Process, surrogate model for reliability analysis	Chance that S_{11} dB will fall below -10 dB.

Since the surrogate models were based on the data from the wider range of the design parameter l_patch variations (l_patch ± 2*0.005*l_patch), the reliability analysis with a narrow range l_patch variations (l_patch ± 2*0.00125*l_patch), that is within the worst-case range, remains valid.

Results and Discussion

The screening results, as shown in Figure 2, indicate that the length of the patch (l_patch) and the dielectric constant of the substrate (dk) are influential on QoI whereas the thickness of the substrate (d) is relatively less significant.

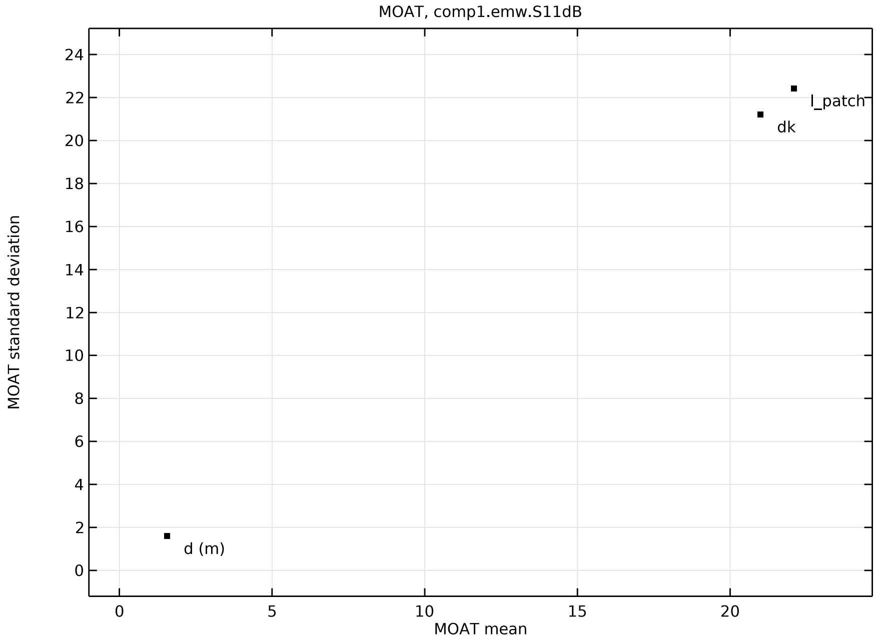


Figure 2: Screening results with three parameters: the length of the patch (l_patch), the dielectric constant of the substrate (dk), and the thickness of the substrate (d).

Figure 3 presents the results of the two Sobol indices. The first-order index describes the sensitivity of the QoI when a single parameter varies independently. The total index reflect the overall sensitivity contribution of a parameter, including its interactions with other parameters. For all parameters, the total Sobol index is greater than the first-order index, indicating that interactions between parameters are not negligible. This suggests that the combined effects of parameters play a significant role in the sensitivity of the QoI. Additionally, the results show that the length of the patch has a greater sensitivity on the S-parameter compared to the variation in the dielectric constant.

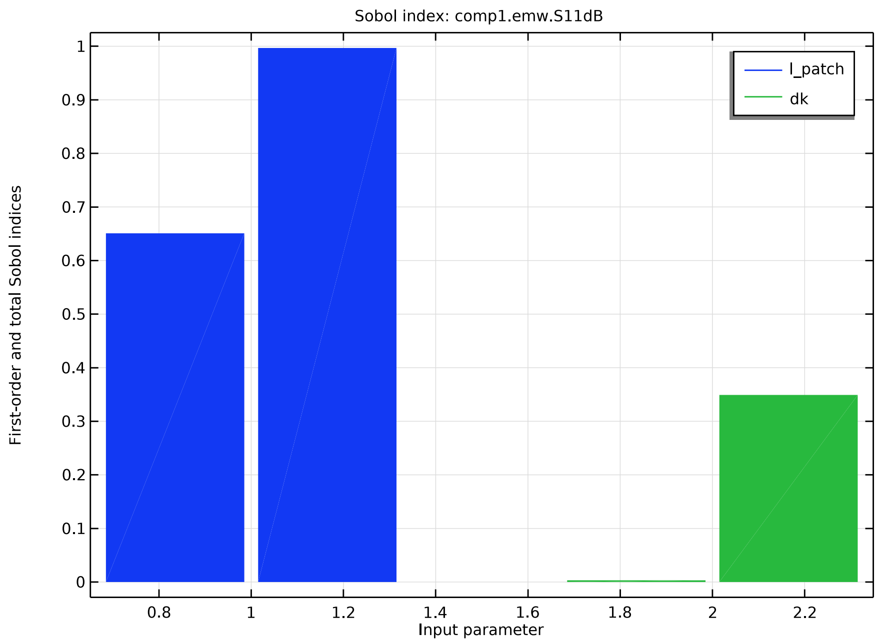


Figure 3: Sobol indices for sensitivity analysis, showing the first-order and total indices for evaluating parameter sensitivity and interactions.

Figure 4 displays the kernel density estimation (KED) of the QoI. This KDE plot provides a smoothed estimate of the probability density function of the QoI, reflecting how its values are distributed based on the input parameter uncertainties. By using KDE, the distribution of the QoI is visualized in a continuous manner, allowing for a clearer under standing of it probabilistic behavior and variability. This method helps in identifying the likelihood of different outcomes and understanding the range and distribution of the QoI given the inherent uncertainties in the model parameters.

The reliability analysis aims to determine the probability that S_{11} is below –10 dB. It produces a table titled Probability for Conditions in the Reliability Analysis group, which indicates a value of approximately 0.55. This suggests that, under the given conditions, there is about a 55% chance that S_{11} will be less than –10 dB.

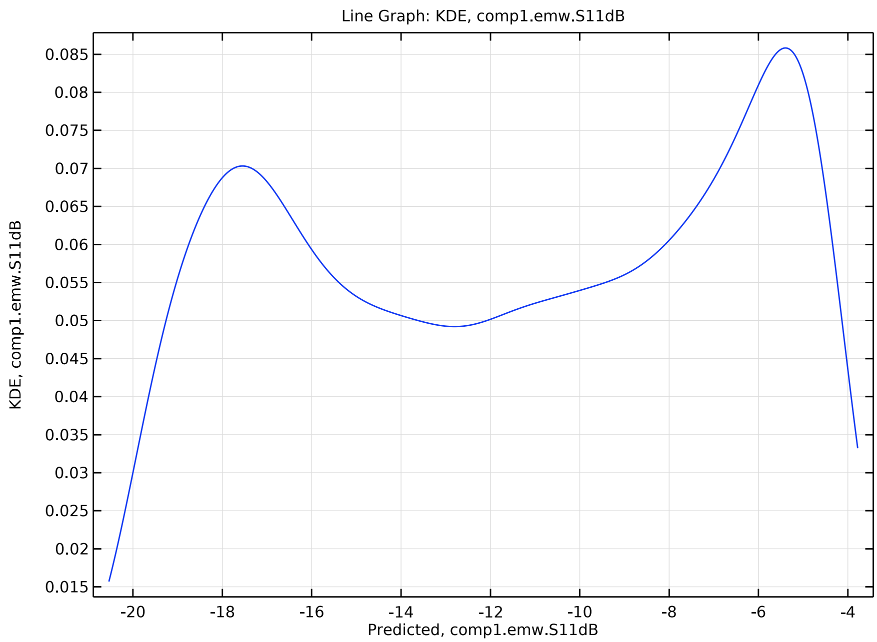


Figure 4: Kernel density estimation (KED) illustrating the probability density function of the QoI.

Figure 5 visually represents how variations in the two input parameters influence the S_{11} value, highlighting the interaction between these parameters and their combined effect on the output. The response surface plot provides insights into the dependency and sensitivity of S_{11} with respect to the input parameters, offering a comprehensive view of how changes in the input impact the antenna's performance.

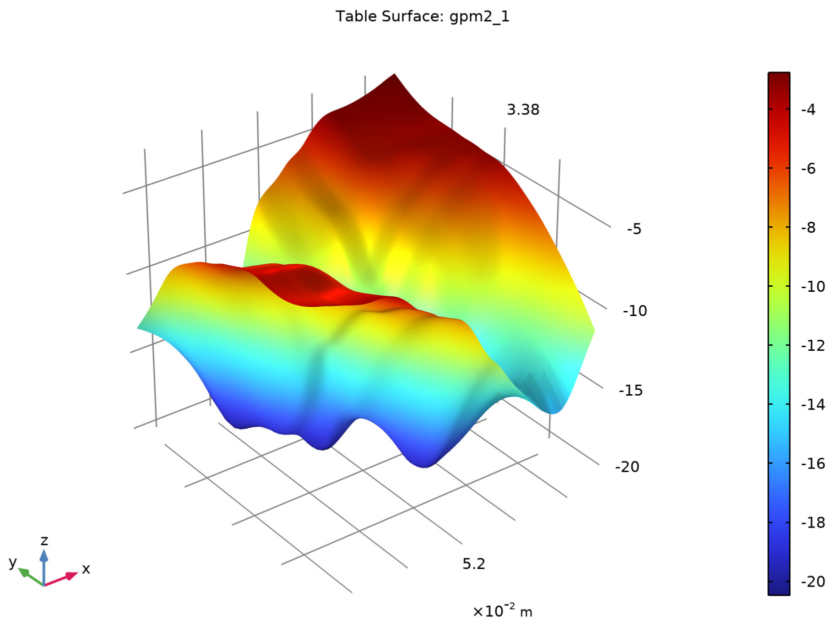


Figure 5: Response surface illustrating the relationship between S_{11}, and two input parameters.

The repeated reliability analysis yields a probability of approximately 0.86 in the Probability for Condition under the Reliability Analysis 1 group. This indicates that, under the updated conditions, there is about a 86% chance that S_{11} will be less than –10 dB.

DISCUSSION ON UNCERTAINTY LEVELS

The uncertainty quantification study for the microstrip patch antenna underscores the significant impact of parameter variations, such as material properties, substrate thickness, and fabrication tolerances, on antenna performance. The computed probability distribution reveals these inherent uncertainties, reflecting the challenges of reproducing narrow-band devices accurately.

In practice, achieving consistent performance in narrow-band prototypes is difficult due to deviations in design parameters. These deviations can lead to issues like a resonant frequency shift of 5–10 MHz and impedance mismatch at the intended operating frequency.

The reliability analysis provides probabilities for scenarios near the specified threshold, where S_{11} is below –10 dB. This threshold defines the range the reflection due to the impedance mismatch is considered acceptable.

The analysis offers the following insights combined with variations in the dielectric constant:

- Reliability analysis for milling with loose anchoring: the computed probability is approximately 55%, indicating that if the metal part of the board is poorly cut, leading to deviations in the patch size from the intended design parameter l_patch, there is a 55% chance that S_{11} will be below –10 dB.
- Reliability Analysis for non-high-precision PCB etching: the estimated probability is about 86%, suggesting that under non-high-precision etching conditions, there is an 86% chance of acceptable impedance mismatch with S_{11} below –10 dB.

Application Library path: `RF_Module/Antenna_Arrays/microstrip_patch_antenna_uq`

Modeling Instructions

APPLICATION LIBRARIES

1. From the **File** menu, choose **Application Libraries**.
2. In the **Application Libraries** window, select **RF Module** > **Antenna Arrays** > **microstrip_patch_antenna_inset** in the tree.
3. Click **Open**.

GLOBAL DEFINITIONS

Parameters 1

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, locate the **Parameters** section.
3. In the table, enter the following settings:

Name	Expression	Value	Description
dk	3.38	3.38	Dielectric constant

COMPONENT 1 (COMP1)

In the **Model Builder** window, expand the **Component 1 (comp1)** node.

MATERIALS

Substrate (mat2)

1. In the **Model Builder** window, expand the **Component 1 (comp1) > Materials** node, then click **Substrate (mat2)**.
2. In the **Settings** window for **Material**, locate the **Material Contents** section.
3. In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	dk	1	Basic

STUDY 1

In the **Model Builder** window, right-click **Study 1** and choose **Uncertainty Quantification > Add Uncertainty Quantification Study Using Study Reference**.

STUDY 3, UQ SCREENING

In the **Settings** window for **Study**, type `Study 3, UQ Screening` in the **Label** text field.

Uncertainty Quantification

1. In the **Model Builder** window, under **Study 3, UQ Screening** click **Uncertainty Quantification**.
2. In the **Settings** window for **Uncertainty Quantification**, locate the **Quantities of Interest** section.
3. Click **Add**.
4. In the table, enter the following settings:

Expression	**Individual solution selection**
`comp1.emw.S11dB`	From "Solution selection"

5. Locate the **Input Parameters** section. Click **Add**.

6 In the table, enter the following settings:

Parameter	Source type	Parameter description
d (Substrate thickness)	Analytic	Normal

7 From the **Distribution** list, choose **Normal(μ,σ)**.

8 In the **Mean** text field, type `d`.

9 In the **Standard deviation** text field, type `0.035*d`.

10 From the **CDF-Lower** list, choose **Manual**.

11 In the **Lower bound** text field, type `d-2*0.035*d`.

12 From the **CDF-Upper** list, choose **Manual**.

13 In the **Upper bound** text field, type `d+2*0.035*d`.

14 Click + **Add**.

15 In the table, enter the following settings:

Parameter	Source type	Parameter description
l_patch (Patch length)	Analytic	Normal

16 From the **Distribution** list, choose **Normal(μ,σ)**.

17 In the **Mean** text field, type `l_patch`.

18 In the **Standard deviation** text field, type `0.005*l_patch`.

19 From the **CDF-Lower** list, choose **Manual**.

20 In the **Lower bound** text field, type `l_patch-2*0.005*l_patch`.

21 From the **CDF-Upper** list, choose **Manual**.

22 In the **Upper bound** text field, type `l_patch+2*0.005*l_patch`.

23 Click + **Add**.

24 In the table, enter the following settings:

Parameter	Source type	Parameter description
dk (Dielectric constant)	Analytic	Normal

25 From the **Distribution** list, choose **Normal(μ,σ)**.

26 In the **Mean** text field, type `dk`.

27 In the **Standard deviation** text field, type `0.005*dk`.

28 From the **CDF-Lower** list, choose **Manual**.

29 In the **Lower bound** text field, type `dk-3*0.005*dk`.

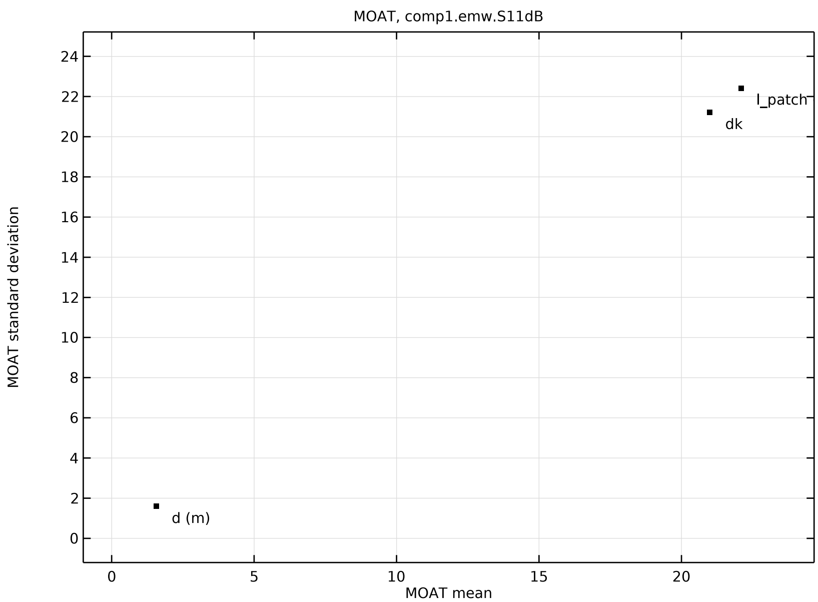
30 From the **CDF-Upper** list, choose **Manual**.

31 In the **Upper bound** text field, type `dk+3*0.005*dk`.

32 In the **Study** toolbar, click **Compute**.

RESULTS

MOAT, comp1.emw.S11dB



The screening results indicate that the length of the patch `l_patch` and the dielectric constant of the substrate `dk` are influential on the quantity of interest. In contrast, the thickness of the substrate `d` appears to be less significant. A high value of the Morris One-At-a-Time (MOAT) mean (horizontal axis) indicates that the parameter significantly influences the quantity of interest. Conversely, a high MOAT standard deviation (vertical axis) suggests that the parameter is influential, possibly interacting with other parameters or having a nonlinear effect. The next step is to conduct a sensitivity analysis. Use the screening results to decide which parameters to include in this analysis. Since sensitivity analysis is more computationally demanding than screening, it is advisable to select a subset of the parameters identified in the screening study. In this example, we exclude the thickness of the substrate `d`. There is no need to redefine all the uncertainty quantification parameters; instead, you can create a new **Uncertainty Quantification** study for the sensitivity analysis by reusing information from the screening study.

STUDY 3, UQ SCREENING

Uncertainty Quantification

Right-click **Uncertainty Quantification** and choose **Add New Uncertainty Quantification Study For** > **Sensitivity Analysis**.

STUDY 4, UQ SENSITIVITY ANALYSIS

1. In the **Model Builder** window, click **Study 4**.
2. In the **Settings** window for **Study**, type `Study 4, UQ Sensitivity Analysis` in the **Label** text field.

1. In the **Model Builder** window, under **Study 4, UQ Sensitivity Analysis** click **Uncertainty Quantification**.
2. In the **Settings** window for **Uncertainty Quantification**, locate the **Input Parameters** section.
3. In the table, click to select the cell at row number 1 and column number 1.
4. Click **Delete**.

   This excludes the thickness of the substrate `d` for the sensitivity analysis.

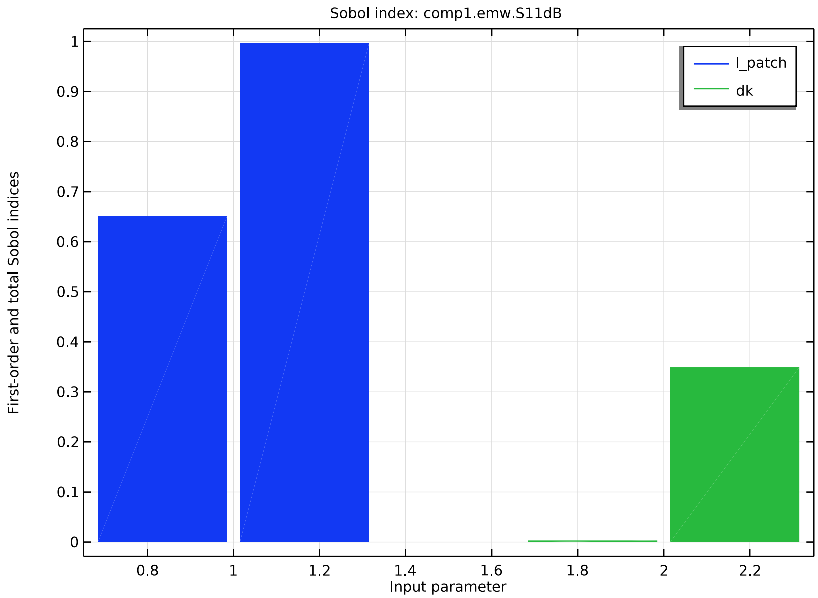
Uncertainty Quantification 2

In the **Study** toolbar, click **Compute**.

RESULTS

Sobol Index, comp1.emw.S11dB

1. In the **Model Builder** window, under **Results** > **Uncertainty Quantification Graph 1** click **Sobol Index, comp1.emw.S11dB**.



The sensitivity analysis utilizes the Sobol method, also known as variance-based sensitivity analysis. The results include a set of Sobol indices, presented in both a Sobol table and a Sobol plot. There are two types of Sobol indices: the first-order index and the total index.

- First-order index: Indicates the sensitivity of the quantity of interest when varying a single parameter alone.
- Total index: Reflects the overall sensitivity contribution of a parameter, including interactions with other parameters.

In this case, the first-order and total indices are not equal, within the computed accuracy, for all parameters, suggesting significant interaction between parameters. The Sobol plot reveals that the length of the patch is the most sensitive parameter, which aligns with the findings from the screening analysis.

STUDY 4, UQ SENSITIVITY ANALYSIS

Uncertainty Quantification

In the **Model Builder** window, under **Study 4, UQ Sensitivity Analysis** right-click **Uncertainty Quantification** and choose **Add New Uncertainty Quantification Study For** > **Uncertainty Propagation**.

STUDY 5, UQ UNCERTAINTY PROPAGATION

1 In the **Model Builder** window, click **Study 5**.
2 In the **Settings** window for **Study**, type `Study 5, UQ Uncertainty Propagation` in the **Label** text field.

1 In the **Model Builder** window, under **Study 5, UQ Uncertainty Propagation** click **Uncertainty Quantification**.
2 In the **Settings** window for **Uncertainty Quantification**, locate the **Uncertainty Quantification Settings** section.
3 From the **Compute action** list, choose **Analyze only**.

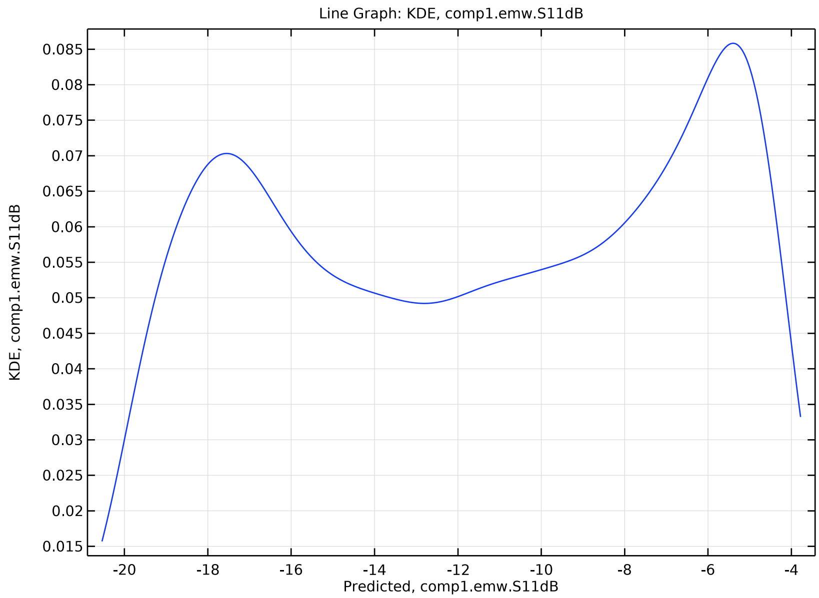
Uncertainty Quantification 3

In the **Study** toolbar, click = **Compute**.

RESULTS

Kernel Density Estimation, comp1.emw.S11dB

1 In the **Model Builder** window, under **Results** > **Uncertainty Quantification Graph 2** click **Kernel Density Estimation, comp1.emw.S11dB**.



The uncertainty propagation study uses a method called kernel density estimation (KDE). KDE can be thought of as a smoothed histogram that estimates the probability density function of the quantity of interest, based on the input parameters and their distributions.

STUDY 5, UQ UNCERTAINTY PROPAGATION

Uncertainty Quantification

In the **Model Builder** window, under **Study 5, UQ Uncertainty Propagation** right-click **Uncertainty Quantification** and choose **Add New Uncertainty Quantification Study For** > **Reliability Analysis**.

STUDY 6, UQ RELIABILITY ANALYSIS FOR MILLING WITH LOOSE ANCHORING

1 In the **Model Builder** window, click **Study 6**.

2 In the **Settings** window for **Study**, type `Study 6, UQ Reliability Analysis for Milling with Loose Anchoring` in the **Label** text field.

1 In the **Model Builder** window, under **Study 6, UQ Reliability Analysis for Milling with Loose Anchoring** click **Uncertainty Quantification**.

2 In the **Settings** window for **Uncertainty Quantification**, locate the **Uncertainty Quantification Settings** section.

3 Find the **Surrogate model settings** subsection. In the **Relative tolerance** text field, type `0.01`.

4 Locate the **Quantities of Interest** section. In the table, enter the following settings:

Function name	Expression	Individual solution selection	True if	Threshold
S11dB	comp1.emw.S11dB	From "Solution selection"	Smaller than threshold	-10

5 Locate the **Surrogate-Based Response Surface** section. In the table, enter the following settings:

Parameter	Point generation method	Distribution resolution	Parameter value list
l_patch	Distribution	100	
dk	Distribution	100	

Uncertainty Quantification 4

In the **Study** toolbar, click **Compute**.

RESULTS

In the **Model Builder** window, expand the **Results** > **Tables** node.

Probability for Conditions

The reliability analysis performs importance sampling to refine the full model results near the specified threshold for our quantity of interest. Specifically, we are interested in the probability that S_{11} is lower than -10 dB. The reliability analysis provides a table named **Probability for Conditions** under the **Reliability Analysis** group, which shows a value of approximately 0.55. This indicates that, under the given conditions, there is about a 55% chance that S_{11} will fall below -10 dB.

Study 6, UQ Reliability Analysis for Milling with Loose Anchoring

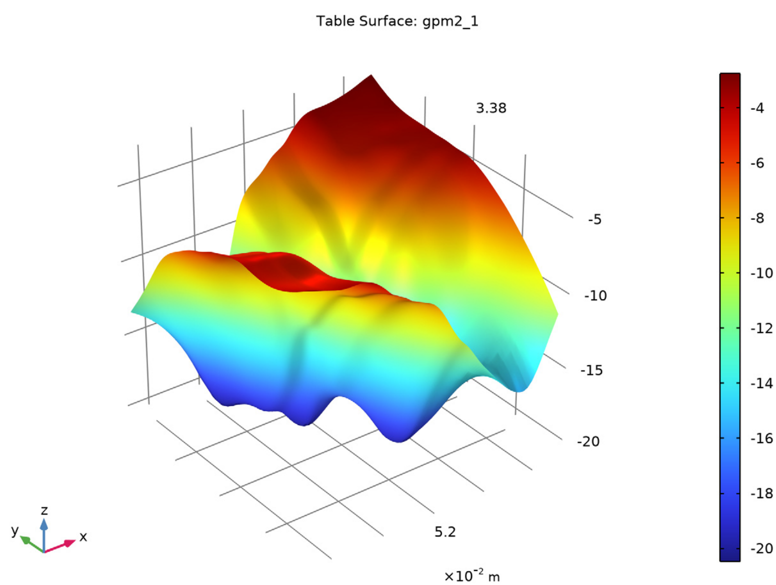
Uncertainty Quantification

1 In the **Model Builder** window, expand the **Results** > **Tables** > **Reliability Analysis** node, then click **Study 6, UQ Reliability Analysis for Milling with Loose Anchoring** > **Uncertainty Quantification**.
2 In the **Settings** window for **Uncertainty Quantification**, locate the **Surrogate-Based Response Surface** section.
3 Click **Response Surface** in the upper-right corner of the section.

Results

Response surface

1 In the **Model Builder** window, under **Results** click **Response surface**.



This is a response surface illustrating the relationship between two input parameters, `dk` and `l_patch`, and the S-parameter output.

Repeat the reliability analysis using a different range and standard deviation value for **l_patch**, representing a non-high-precision PCB etching scenario.

STUDY 5, UQ UNCERTAINTY PROPAGATION

Uncertainty Quantification

In the **Model Builder** window, under **Study 5, UQ Uncertainty Propagation** right-click **Uncertainty Quantification** and choose **Add New Uncertainty Quantification Study For** > **Reliability Analysis**.

STUDY 7, UQ RELIABILITY ANALYSIS FOR NON-HIGH-PRECISION PCB ETCHING

1 In the **Model Builder** window, click **Study 7**.

2 In the **Settings** window for **Study**, type `Study 7, UQ Reliability Analysis for Non-High-Precision PCB Etching` in the **Label** text field.

1 In the **Model Builder** window, under **Study 7, UQ Reliability Analysis for Non-High-Precision PCB Etching** click **Uncertainty Quantification**.

2 In the **Settings** window for **Uncertainty Quantification**, locate the **Uncertainty Quantification Settings** section.

3 Find the **Surrogate model settings** subsection. In the **Relative tolerance** text field, type `0.01`.

4 Locate the **Quantities of Interest** section. In the table, enter the following settings:

Function name	Expression	Individual solution selection	True if	Threshold
S11	comp1.emw.S11dB	From "Solution selection"	Smaller than threshold	-10

5 Locate the **Input Parameters** section. In the table, click to select the cell at row number 1 and column number 3.

6 In the **Standard deviation** text field, type `0.00125*l_patch`.

7 In the **Lower bound** text field, type `l_patch-2*0.00125*l_patch`.

8 In the **Upper bound** text field, type `l_patch+2*0.00125*l_patch`.

9 Locate the **Surrogate-Based Response Surface** section. In the table, enter the following settings:

Parameter	Point generation method	Distribution resolution	Parameter value list
l_patch	Distribution	100	
dk	Distribution	100	

Uncertainty Quantification 5

1 In the **Study** toolbar, click **Compute**.

Check **Probability for Conditions** under the **Reliability Analysis 1** group, which shows a value of approximately 0.86. This implies that, under the given conditions—representing a non-high-precision PCB etching tolerance combined with variations in the dielectric constant—there is about a 86% chance that S_{11} will fall below -10 dB.