A Moisture Transfer Model for Drying of Grain

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Abstract: A kernel of grain is modeled as an isothermal sphere, with descriptive differential equations scaled or rendered into non-dimensional form, where moisture transfer is governed by internal diffusivity, not by surface conditions. The time dependent problem is solved with COMSOL and the average non-dimensional moisture content and its time-rate of change are obtained; by regression, these numerical data yield a new kernel-average moisture model that is extended to volumetric mass transfer for a bed or column of kernels.

One-dimensional, time-dependent equations are derived for a "drying column" with axially flowing air, and with a stationary and flowing bed of kernels; these coupled equations are solved with the Diffusion and Convection tool in COMSOL. Void fractions and superficial velocities are specified, and rates of moisture transfer from the kernels to the air are determined. The results show the reduction in kernel moisture with time and along the length of the column, and the increase of the air humidity ratio. It is found that a considerable volume of air-flow is required for effective drying.

Keywords: drying, mass transfer, diffusion, grain kernels, heat transfer.

1. Introduction

One of the important problems in food production is the drying of grain and seed kernels. Grains and seeds are generally harvested with a fairly high moisture content, yet high-quality grains for storage and export require a reduced moisture content [1]. The transfer of moisture in grains is a two-step process: first there is the mass-transfer diffusion of moisture within the kernels, and then the convection of water vapor in the air, or drying medium.

Grain kernels have complex composite structures, and are ellipsoidal in shape [3]; nevertheless, in this paper the common homogeneous spherical model is used to determine the moisture transfer within a kernel [2]. The descriptive differential equations are scaled, or rendered into non-dimensional form, where moisture transfer is governed by internal diffusivity, not by surface conditions. The time dependent problem is solved with COMSOL and the average non-dimensional moisture content and its time-rate of change are obtained; by regression, these numerical data yield a new kernel-average moisture model.

The single kernel model is extended to volumetric mass transfer in a bed or moving column of kernels having specified constant void fraction, or porosity. The resulting mass transfer per unit volume is a source term in the Diffusion and Convection tool in COMSOL, with equations solved as one-dimensional, time-dependent. Added to these models are the general heat transfer equations for multi-physics computations.

The results indicate that moisture transfer in both a fixed bed, or bin, and in a moving column of grain can be effective. However, the effect of heating the incoming air is minimal; the reason is that the air takes on the grain temperature instead of the reverse, due to the high heat-capacity of the grain.

2. Single-Kernel Moisture Transfer

Grain kernels are non-homogeneous and ellipsoidal in shape [3]; however, it has been shown that the diffusivities in homogeneous ellipsoids and spheres are comparable [4]. Here the kernel is modeled as a sphere, and the moisture concentration equation is given by

$$\frac{\partial M_k}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_k \frac{\partial M_k}{\partial r}\right) \tag{1a}$$

where subscript k denotes the kernel interior. Usually, this is subject to the convective type boundary condition,

$$-D_k \frac{\partial M_k}{\partial r}\bigg|_R = h_m\left(M_k\big|_R - M_{eq}\right) \tag{1b}$$

where M_k is the moisture concentration within the kernel, and M_{eq} its equilibrium value. Here the moisture diffusivity is in the form of an Arrhenius-type relation, $D_k = A_k\exp(-B_k/T_a)$, where A_k and B_k are constants, and T_a is the ambient air temperature surrounding the kernel.

For soft wheat it is reported that A_k = 71.4 m^2/h = 0.020 m^2/s, and B_k = 6155 K [2]; for Canadian red spring wheat, A_k = 0.010 m^2/s is reported [3]. Hence, for moderate temperatures $D_k \approx 10^{-11}$ m^2/s (as also shown in [4]), whereas the thermal diffusivity is $\alpha_k \approx 10^{-8}$ m^2/s; thus, with α_k three orders larger than D_k, heat diffusion is rapid and the local air temperature can be used throughout the kernel.

The surface mass transfer coefficient, h_m, is usually obtained from the Sherwood number, $Sh = h_m d_k/D_a$, where D_a is the moisture diffusivity in air [similar to the Nusselt number for surface heat transfer, $Nu = (h/\rho C)d_k/\alpha$]. For example, for Sh ≈ 5, and $D_a \approx 0.26\text{x}10^{-4}$ m^2/s, a kernel with d_k = 4 mm yields $h_m \approx$ 0.13 m/s.

For the solution of (1a,b) the scaled variables are introduced, $m = (M - M_{eq})/(M_0 - M_{eq})$, $\xi = r/R$, and $\tau = t/t_k$, where $t_k = d_k^2/D_k \approx 3.7\text{x}10^5$ s; then, (1a) and (1b) appear as

$$\frac{\partial m}{\partial \tau} = \frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2 \frac{\partial m}{\partial \xi}\right) \qquad (2a)$$

$$-\left.\frac{\partial m}{\partial \xi}\right|_1 = \frac{Bi_m}{2} m\big|_1 \qquad (2b)$$

where $Bi_m = h_m d_k/D_k \approx 10^7$ is the mass transfer Biot number; subject to initial condition, $m = 1$. However, so large a mass Biot number means that the internal resistance to moisture transfer is vastly greater than that at the surface. In other words, the surface concentration will take on the equilibrium value, M_{eq}, almost instantaneously, with large gradients occurring within the kernel. Therefore, for all practicality, the boundary condition (2b) may be replaced by

$$m|_1 = 0 \qquad (2c)$$

These equations are easily solved by the diffusion module in COMSOL, as shown in Fig. 1(a) for time = t/t_k = 0.112; there is a smooth change in concentration from the center to the surface:

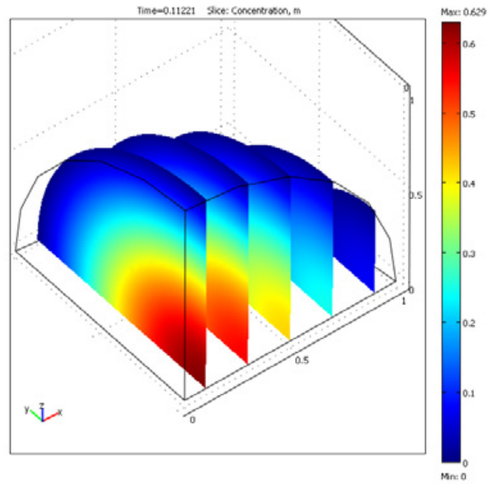

Figure 1(a). Kernel Scaled Moisture Concentration.

If the volumetric average concentration, and its (negative) time derivative, are computed in COMSOL, then their variations with τ appear as in Fig. 1(b):

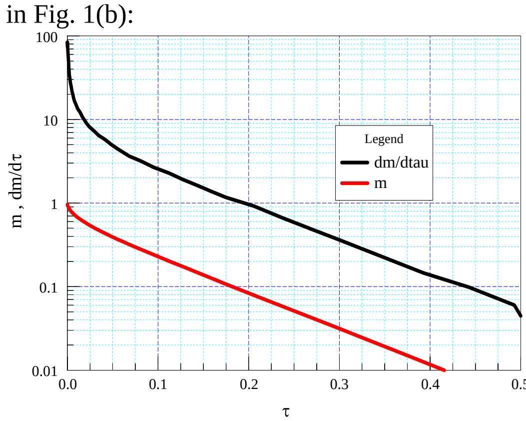

Figure 1(b). Kernel Average Concentration.

Except at the very beginning, both curves are of exponential-decay type, and at long times nearly proportional by the factor 12. Therefore, a model that reflects this moisture transfer is given by

$$-\frac{d\overline{m}}{d\tau} = \beta\,\overline{m} \quad ; \quad \beta = \frac{12\tau + 0.11}{\tau + 0.0011} \qquad (3a,b)$$

Where β is the proportionality factor. A comparison of this approximate model with the numerical data is shown in Fig. 1(c):

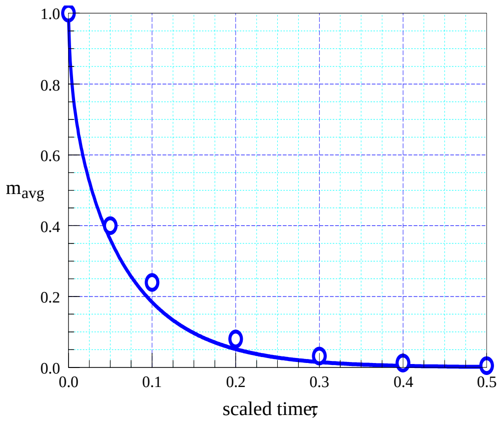

Figure 1(c). Kernel Average Moisture Transfer.

If we now take the moisture transfer from one kernel [kg/s] as

$$-V_{1k}\rho_k \frac{d\bar{M}}{dt} = h_k S_{1k}\rho_k(\bar{M} - M_{eq}) \qquad (3c)$$

where V_{1k} and S_{1k} are the volume and surface area of the kernel. Then, with $S_{1k}/V_{1k} = 6/d_k$, and comparison of (3c) with (3a), we find that the effective transfer coefficient is

$$h_k = \beta d_k/6t_k = 2/3\, \beta\, D_k/d_k \qquad (3d)$$

That is, the average moisture transfer rate depends mostly on the internal diffusivity, with a very small magnitude of $h_k \approx 10^{-8}$ m/s, and not on the external Sherwood number.

The Equilibrium Moisture Content in (3c) (M_{eq}= EMC) depends on the air vapor pressure, and may be evaluated using the Chung-Henderson correlation [2]:

$$M_{eq} = E_{eq} - F_{eq}\ln[-(T_C+C_{eq})\ln(p_v/p_{vs})] \qquad (3e)$$

where the relative humidity is $\varphi = p_v/p_{vs}$, where E_{eq}, F_{eq}, and C_{eq} are constants, and T_C is the kernel temperature in degrees Celsius. This S-shaped function is shown in Fig. 2(a) for hard wheat, and for kernel temperatures of 25 (red), 50 (green) and 75 (blue) degrees C:

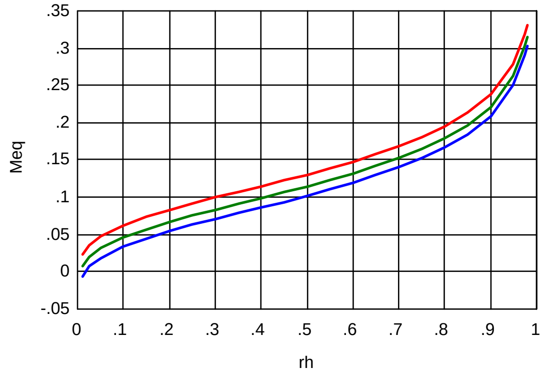

Figure 2(a). EMC Curves.

The M_{eq} is the limit to which the grain can be dried. Thus, to dry to an optimum 15% moisture requires the air relative humidity to be somewhere below 80%. In Fig. 2(b), this limiting *rh* is shown as a function of the temperature:

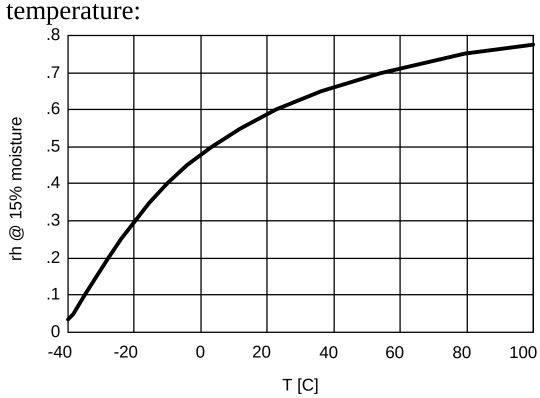

Figure 2(b). Minimum Relative Humidity.

The vapor pressure is related to the humidity ratio, or specific humidity, as $\omega = 0.622p_v/(p-p_v) \approx 0.622p_v/p$ and the saturated vapor pressure, p_{vs}, is related to the temperature; thus we have the functional relationship: $M_{eq} = M_{eq}(\omega, T)$.

3. One-Dimensional Transport Equations

3.1 Moisture Mass Transfer

Let M be the kernel-average moisture concentration, then for N_k kernels in total volume V_T, the moisture transfer to the air is $Q_m = N_k S_{1k} h_k \rho_k (M - M_{eq})$, where S_{1k} is the surface area of 1 kernel. The solid-phase volume is $V_s = N_k V_{1k} = (1-\varepsilon)V_T$; thus, $N_k = (1-\varepsilon)V_T/V_{1k}$ and $Q_m = (1-\varepsilon)V_T(S_{1k}/V_{1k})h_k\rho_k(M - M_{eq})$; for a sphere, $S_{1k}/V_{1k} = 6/d_k$, and the moisture transfer per unit volume is

$$q_{Vm} = \frac{Q_m}{V_T} = (1-\varepsilon)\rho_k \frac{6h_k}{d_k}(M - M_{eq}) \quad (4a)$$

If the kernels are taken to enter control volume, $V_T = Adx$, with superficial velocity, U_s, then the rate of change of moisture in the solid phase is [5]:

$$(1-\varepsilon)\rho_k \left(\frac{\partial M}{\partial t} + U_s \frac{\partial M}{\partial x} \right) = -q_{Vm} \quad (4b)$$

or,
$$\frac{\partial M}{\partial t} + U_s \frac{\partial M}{\partial x} = -6\frac{h_k}{d_k}(M - M_{eq}) \quad (4c)$$
$$= -4\beta \frac{D_k}{d_k^2}(M - M_{eq})$$

In the air, the amount of moisture is represented as the humidity ratio, ω = mass of water-vapor divided by the mass of air; thus, the moisture leaving the solid, q_{Vm}, is gained by the air with velocity, U_a:

$$\varepsilon\rho_a \left(\frac{\partial \omega}{\partial t} + U_a \frac{\partial \omega}{\partial x} \right) = q_{Vm} \quad (5)$$

3.2 Heat Transfer

There is volumetric heat transfer from the air to the solid phase that is similar to the moisture transfer, as follows:

$$q_V = \frac{Q}{V_T} = (1-\varepsilon)\frac{6h_a}{d_k}(T_a - T_s) \quad (6a)$$

but in this case, the Biot number is $O(1)$, and h_a is obtained from a Nusselt number correlation.

The enthalpy *increase* in the air flowing with axial superficial velocity U_a is the sum of convective heat transfer from the kernels and the heat content in the moisture transfer:

$$\varepsilon\rho_a \left(\frac{\partial C_a^{'} T_a}{\partial t} + U_a \frac{\partial C_a^{'} T_a}{\partial x} \right) = -q_V + u_m q_{Vm} \quad (6b)$$

where the moisture internal energy is $u_m - u_{m0} = C_m(T_s - T_{m0})$; thus

$$\varepsilon\rho_a \frac{D_a C_a^{'} T_a}{Dt} = \frac{6(1-\varepsilon)}{d_k} \times \quad (6c)$$
$$\left\{ h_a(T_s - T_a) + u_m \rho_s h_k (M - M_{eq}) \right\}$$

where the effective specific heat is $C'_a = C_a + \omega C_m$, and ω is the humidity ratio.

For the solid phase, there is energy added directly by heat exchange, q_V, that goes into heating the kernels, minus that carried away by the moisture exchange, q_{Vm}, and minus the heat that goes into evaporating the water in the kernels; this results in the balance

$$(1-\varepsilon)\rho_s \left(\frac{\partial C_s^{'} T_s}{\partial t} + U_s \frac{\partial C_s^{'} T_s}{\partial x} \right) = q_V - u_m q_{Vm} \quad (7a)$$
$$- (1-\varepsilon)\rho_s h_{fg} \frac{\partial M}{\partial t}$$

or
$$\rho_s \left\{ \frac{D_s C_s^{'} T_s}{Dt} + h_{fg}\frac{\partial M}{\partial t} \right\} = \frac{6}{d_k} \times \quad (7b)$$
$$\left\{ h_a(T_a - T_s) - u_m \rho_s h_k (M - M_{eq}) \right\}$$

where the effective specific heat is $C'_s = C_s + C_m M$.

4. Results

4.1 Stationary Bed or Bin

Consider a 5m tall bin of any cross-sectional area. For the case of the grain being stationary with 50% void fraction, initially at 25% moisture and at 5^{O}C; the inlet air is 50^{O}C, 40% rh, and with a superficial velocity of 5 cm/s (which for a 3.6m [5 ft] diameter bin corresponds to about 1/3 hp fan power). Then the moisture over 50 hrs at 5 hr intervals varies along the 5m elevation as shown in Fig. 3(a):

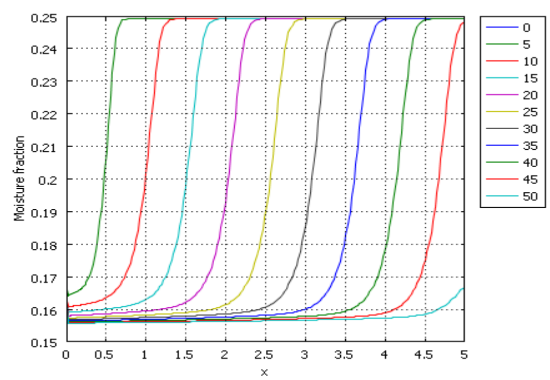

Figure 3(a). Fixed-bed Moisture Function.

The moisture is rapidly reduced near the air inlet, and then throughout the column in a traveling-wave type behavior.

This is also seen in Fig. 3(b) for t = 20 hrs, where the moisture M (red), the equilibrium moisture M_{eq} (green), and the air humidity ratio ω (blue), vary along the 5m height as follows:

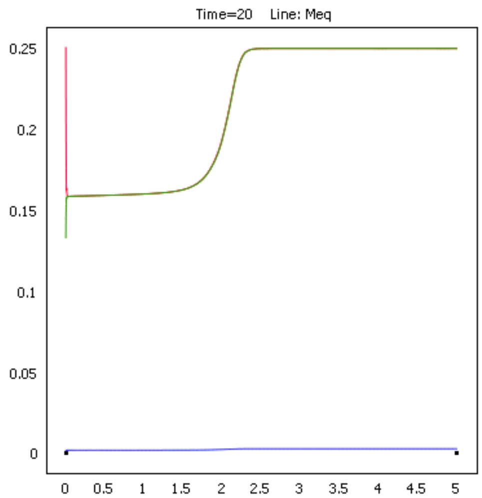

Figure 3(b). Fixed-bed Moisture Function.

As seen, the kernel-average moisture drops quickly from 25% to about 15% at the inlet, and the equilibrium (or kernel surface) moisture increases from 13%.

At 50 hrs the grain is uniformly dry to about 16%, which may be near optimum grain quality [1]. The traveling wave pattern in the equilibrium (surface) moisture is shown as an extrusion plot in Fig. 3(c):

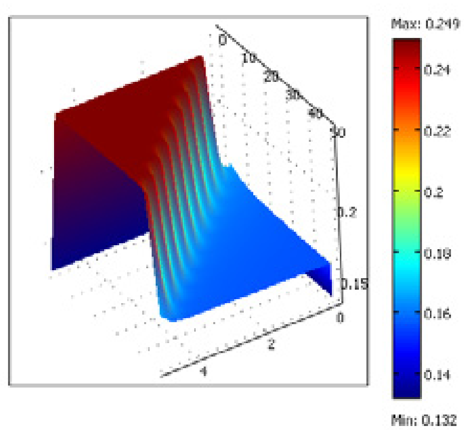

Figure 3(c). Moisture Traveling Wave.

This example is typical of a grain storage bin on a farm, with dry ambient air supplied by a blower. The heat content in the 50 C air is much less than that of the 5 C grain in the bed or bin. Therefore the air temperature quickly takes on that of the grain, as expected, and as shown in Fig. 3(d) for times 0.01s to 0.5s.

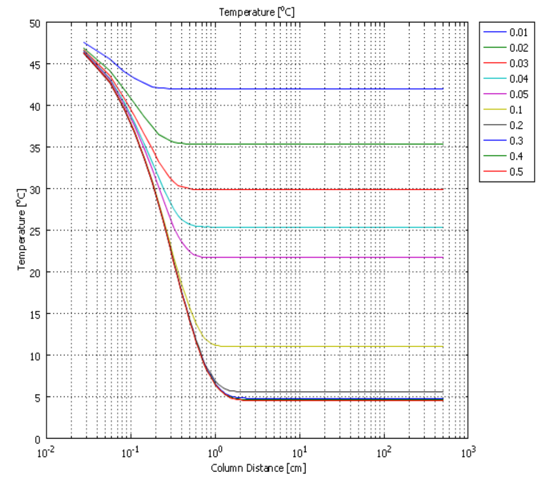

Figure 3(d). Variation of Air Temperature.

Here it is seen that the air temperature has already attained that of the grain within 2 cm of the inlet, and in about ½ second. The conclusion is that heating slow-moving inlet air in a stationary bin has little or no effect.

4.2 Moving Column of Grain

Instead of a fixed bed of grain, the kernels could be made to flow with velocity U_s in a smaller duct that is fed from the bin. Then the axis-symmetric viscous flow of kernels in the bin would appear as in Fig. 4:

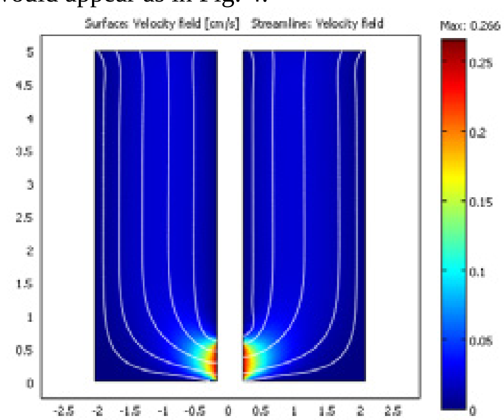

Figure 4. Typical Grain Flow-field in Bin.

We consider the case of a 1 ft [0.3m] diameter duct where U_s = 0.05m/s, U_a = 5m/s and the void fraction is ε = 85% (which would need about 2/3 hp air pumping power). For an inlet absolute humidity ratio of 0.005, and the kernels 20 C, the kernel moisture along the duct appears as in Fig. 5(a), where the air inlet temperatures are 25, 35, 50, and 80 C, as shown.

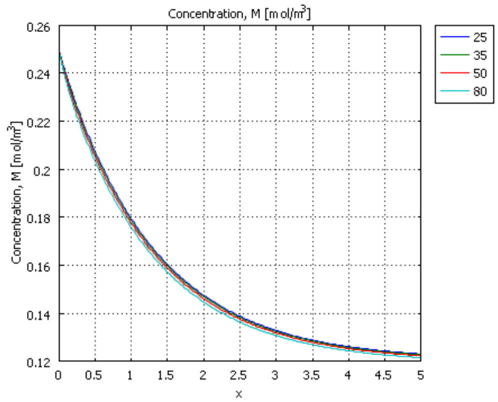

Figure 5(a). Moving Grain-Column Moisture.

It is seen that the kernel-moisture at the outlet of the duct can be reduced to below 15% and that the inlet air temperature has minimal effect. This is due to the temperature of the air quickly attaining that of the grain, as seen in the semi-log plot of Fig. 5(b):

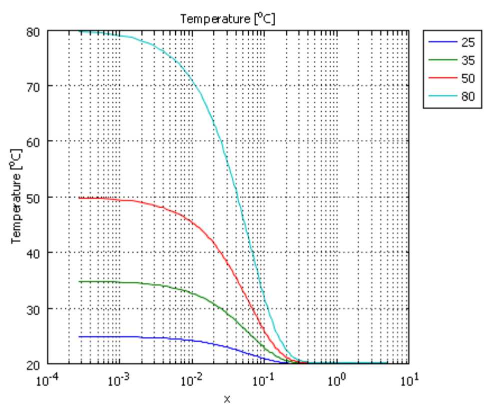

Figure 5(b). Air Temperatures.

The humidity ratio, ω, increases as moisture is transferred to the air; it is only marginally affected by the air temperature, as shown in Fig. 5(b):

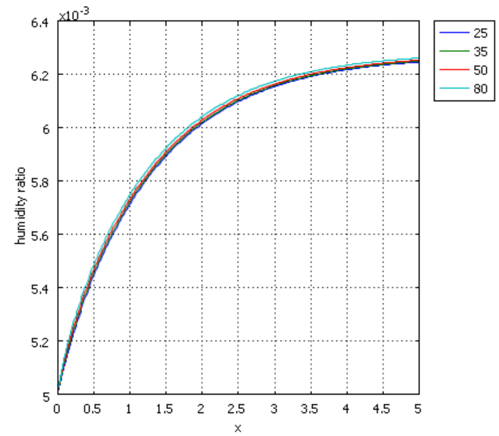

Figure 5(b). Air Humidity Ratio.

Similarly, the relative humidity also increases, as in Fig. 5(c):

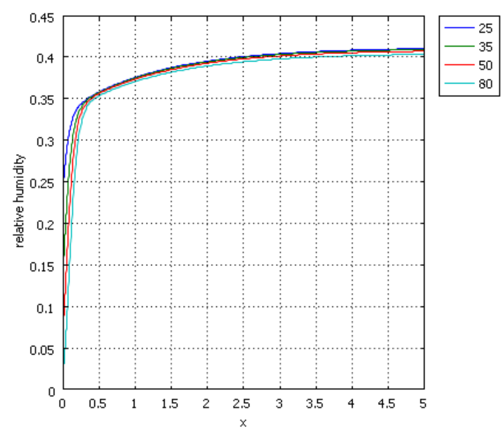

Figure 5(c). Air Relative Humidity.

It is estimated that the air fan power for this column would be about 2/3 hp, and that the time for the grain in a bin to move through the column, as illustrated in fig. 4, would be about 13 hours.

5. Conclusions

A new single-kernel model was constructed, and solved in non-dimensional form with COMSOL; the generated numerical data were correlated into a moisture transfer model that was applied to a column of kernels with specified porosity.

Solutions of the one-dimensional column show the development of moisture transfer, which can be useful in the design of dryers.

6. References

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7. Appendix

<u>Nomenclature</u>

C	=	specific heat [J/kgK]
d_k	=	effective particle (kernel) diameter [m]
D_k	=	moisture diffusion coefficient in kernels [m^2/s]
h_a	=	unit surface heat transfer coefficient [W/m^2K]
h_{fg}	=	latent heat of vaporization [J/kg]
h_k	=	effective mass transfer coefficient [m/s]
h_m	=	unit surface mass transfer coefficient [m/s]
M	=	kernel or solid-phase moisture [fraction or %]
Q	=	heat transfer [W]
Q_m	=	mass transfer [kg/s]
q_V	=	volumetric heat transfer (Q/V_T) [W/m^3]
q_{Vm}	=	volumetric mass transfer (Q_m/V_T) [$kg/s.m^3$]
S_{1k}	=	surface area of one kernel (d_k^2 sphere)
Sh	=	Sherwood number ($h_m d_k/D_m$)
t_k	=	kernel moisture time constant ($d_k^2/4D_k$)
U_a	=	superficial air velocity [m/s]
u_m	=	moisture internal energy ($u_{m0} + C_m\{T_s\text{-}T_{m0}\}$) [J/kg]
U_s	=	superficial solid velocity [m/s]
V_{1k}	=	volume of one kernel ($d_k^3/6$ sphere)
V_T	=	total volume [m^3]
ρ	=	density
ε	=	porosity, void fraction
ω	=	humidity ratio, specific humidity