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Failure Stress Analysis of Fiber Reinforced of Composite Laminates under Uniaxial/Biaxial Loading

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Abstract: The main objective of the present work is to perform stress analysis on composite laminates under uniaxial/biaxial loading to serve as a preliminary data for test verification. A detailed calculation based on the Classical Lamination Theory was performed for a laminate with stacking sequence $[90/45/-45/0]_s$. The material used in the analysis was carbon epoxy (AS4/3501-6). We started by applying a pure uniaxial followed by a biaxial load. The finite element software COMSOL was used to model the geometry, material, loading conditions, and to perform the stress analysis to validate the output results of the analysis. Experimental results available in the literature were also used to validate the results of the analysis. Computer software was also developed that enables the user to study general composite layup including hybrid composites under uniaxial/biaxial loading conditions. Moreover, several carpet plots were also conducted to show the interacting behavior of two independent variables such as Young's modulus, Poisson's ratio and Shear modulus.

Keywords: Composite Materials, Biaxial load.

1. Introduction

Composite material systems are increasingly used in almost every industrial branch. Several applications could be found in the automotive, aeronautical, and sports sectors, additionally, there are numerous and ever increasing industrial uses, including wind turbines, storage tanks, high-speed and precision machinery, gas turbine engines, and medical diagnostic equipment. Their outstanding mechanical performance added to their lowweight and other unique and tailorable physical properties make composites the material system of first choice for many applications. The properties of a composite material depend on the properties of the constituents, their geometry, and the distribution of the phase. The constituent parts of the structural components manufactured from these composite material systems are usually subjected to complex loading that leads to multi-axialstress and strain fields at critical surface locations which leads for the need of a reliable design procedure validated rigorously by multi-axial experimental data in order to ensure satisfactory performance over the predefined service period and to avoid the use of high safety factors to cover the high level of uncertainty. The development of the field of composite materials in structural applications relies on the ability to model and simulate the behavior of these materials successfully. Numerous theories have been proposed to predict the response of composite materials, but in order to apply these analysis tools for structural design, experimental validation under a variety of complex loading conditions is mandatory. As composite materials generate complex biaxial and multi-axial stress states [1], even for simple uniaxial loading, the current practice of using solely uniaxial analysis is not suitable and, consequently, analysis closer to reality is of paramount importance. The composite testing community has successfully developed the ability to characterize the uniaxial response of fibre-reinforced composite materials. Several standardized test methods have been developed to evaluate the in-plane shear, the axial and transverse tensile, and the axial and transverse compressive response [2].

2. Macromechanical Analysis of Lamina

The main difference between a lamina and a laminate, where a lamina is a thin layer of a composite material that is generally of a thickness on the order of 0.005 in. (0.125 mm), and a laminate is constructed by stacking a number of such lamina in the direction of the lamina thickness. Mechanical structures made of these laminates are subjected to various loads, such as bending and twisting. The design and analysis of such laminated structures demands knowledge of the stresses and strains in the laminate. Also, design tools, such as failure theories, and stiffness models need the values of these laminate stresses and strains. Understanding the mechanical analysis of a lamina precedes understanding that of a laminate. A lamina is unlike an isotropic homogeneous material. For example, if the lamina is made of isotropic homogeneous fibers and an isotropic homogeneous matrix, the stiffness of the lamina varies from point to point depending on whether the point is in the fiber, the matrix, or the fiber–matrix interface. Accounting for these variations will make any kind of mechanical modeling of the lamina very complicated. For this reason, the macromechanical analysis of a lamina is based on average properties and considering the lamina to be homogeneous.

2.1 Stress-Strain Relations

The state of stress for general anisotropic material can be represented by nine stress components, σ_{ij} (where i, j=1, 2, 3) acting on the sides of an element cube. Similarly, the state of deformation is represented by nine strain components, ε_{ij}. Applying symmetry of the stress and strain tensors

$$\sigma_{ij} = \sigma_{ji} \tag{1}$$

$$\varepsilon_{ij} = \varepsilon_{ji} \tag{2}$$

Thus the stress-strain relation for anisotropic body is given as follows for a three-dimensional body in a 1–2–3 orthogonal Cartesian coordinate system

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad \left(\text{where i, j} = 1, 2, 3, \ldots, 6\right) \tag{3}$$

Where the 6 x 6 [C] matrix is called the stiffness matrix which contains 36 constants. For Orthotropic material which have three mutually perpendicular planes of material symmetry. The stress-strain relation have the same form as anisotropic material, however, the number of independent elastic constants are reduced to nine, because the stiffness and the compliance terms are interrelated. An orthotropic material is called a transversely isotropic when one of its principal planes is a plane of isotropy, that is at every point there is a plane on which the mechanical properties are the same in all directions. The stress-strain relations for transversely isotropic materials are simplified for a two-three planes of isotropy; thus, the stress-strain relations for a transversely isotropic material are reduced to five. In most structural applications, composites materials are used in the form of thin laminates loaded in the plane of the laminate. Thus, composite laminae and laminates can be considered to be under a condition of plane stress, with all stress components in the out–of-plane direction being zero. An isotropic material is characterized by an infinite number of planes of material symmetry through a point. We summarize the number of independent elastic constants for various types of materials as follows

•Anisotropic: 36
• Orthotropic: 9
• Transversely isotropic: 5
• Isotropic: 2

2.2 Stress-Strain Relations for a Thin Lamina (Two-Dimensional)

A thin, unidirectional lamina is assumed to be under a state of plane stress and the equations that govern the stress and strain are shown in Eq. (4) where Q_{ij} is the reduced stiffness coefficients. For a unidirectional lamina, these engineering elastic constants are E_1, E_2, υ_{12} and G_{12}.

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{4}$$

3. Macromechanical Analysis of Laminates

It is apparent that the overall behavior of a multidirectional laminate is a function of the properties and stacking sequence of the individual layers. The so-called classical lamination theory predicts the behavior of the laminate within the framework of several assumptions and restrictions. Figure 1 shows two cross sections before and after loading, we can observe the deformation that has occurred after loading. Assume u_0, v_0, and w_0 to be the initial displacements in the x, y, and z directions, respectively, at the midplane and u, v, and w are the displacements at any point in the x, y, and z directions, respectively. At any point other than the midplane, the two displacements in the x–y plane will depend on the axial location of the point and the slope of the laminate midplane with the x and y directions.

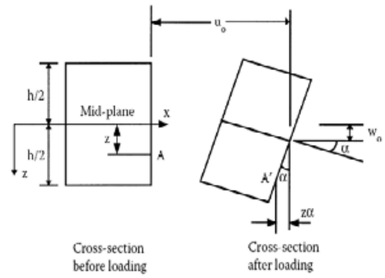

Figure 1. Relationship between displacements through the thickness of a plate to midplane displacements and curvatures

If the strains are known at any point along the thickness of the laminate, the stress–strain in each lamina can be defined as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{5}$$

The reduced transformed stiffness matrix, [Q], corresponds to that of the ply located at the point along the thickness of the laminate. For a certain linear strain variation through the thickness, which can result from axial and flexural loading, the variation of the modulus E_x from layer to layer causes discontinues stress variation. Because of the discontinuous variation of stresses from layer to layer, it is more convenient to deal with the integrated effect of these stresses on the laminate. Thus, we seek expressions relating forces and moments to laminate deformation. More detailed discussion could be found in [3].

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} \left[\left(\overline{Q}_{ij} \right) \right]_k \left(h_k - h_{k-1} \right) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} \left[\left(\overline{Q}_{ij} \right) \right]_k \left(h_k^2 - h_{k-1}^2 \right) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} \left[\left(\overline{Q}_{ij} \right) \right]_k \left(h_k^3 - h_{k-1}^3 \right) \end{aligned} \tag{6}$$

The [A], [B], and [D] matrices defined in Eq. (6) are called the extensional, coupling, and bending stiffness matrices, respectively. The extensional stiffness matrix [A] relates the resultant in-plane forces to the in-plane strains, and the bending stiffness matrix [D] relates the resultant bending moments to the plate curvatures. The coupling stiffness matrix [B] couples the force and moment terms to the midplane strains and midplane curvatures.

4. Failure Theories

When dealing with composite materials, micromechanical failure theories have been proposed by extending and adapting isotropic failure theories to account for the anisotropy in stiffness and strength of the composite. Lamina failure theories can be classified in the following three groups: Limit or noninteractive theories, in which specific failure modes are predicted by comparing individual lamina stress or strains with corresponding strengths or ultimate strains, for example Maximum Stress and Maximum Strain theories where no interaction among different stress components on failure is considered. Interactive theories (the Tsai-Hill and the Tsai-Wu theories) in which all stress components are included in one expression (failure criterion). Overall failure is predicted without reference to particular failure modes. Partially interactive or failure mode based theory (the Hashin-Rotem) where separate criteria are given for fiber and interfiber failures, more detailed description of these theories can be found in [4].

5. Analysis

A detailed calculation for a specific case of a composite layup is performed in order to get the failure stress by considering two types of failure criteria's which are the First Ply Failure (FPF) criteria in which the laminate is considered failed when the first layer (or group of layers) fails, and the iterative Ultimate Laminate Failure (ULF) criteria where there is no generally accepted definition of what constitutes such failure but It is generally accepted that a laminate is considered failed when the maximum load level is reached. The flow diagrams for each failure criteria are shown in Figure 2 and Figure 3 respectively. The calculation was performed using four different theories that were previously mentioned and the results obtained are compared with each other. The geometry and loading condition for the biaxial loading is shown in Figure 4, the factor n is the ratio of the stress applied in the transverse direction to that in the axial direction. A summary of each case is shown in Eq. (7), as can be observed, when the value of n = 0, a pure axial load (uniaxial) is applied which is a special case of the biaxial loading.

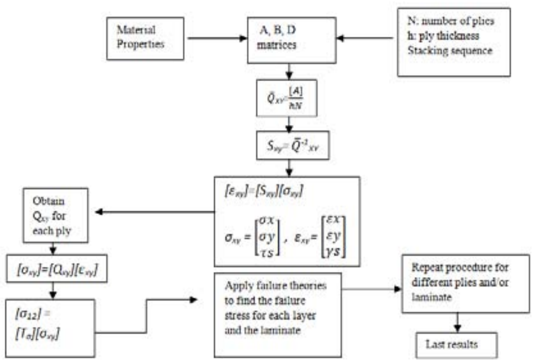

Figure 2. Flow chart for first ply failure criteria

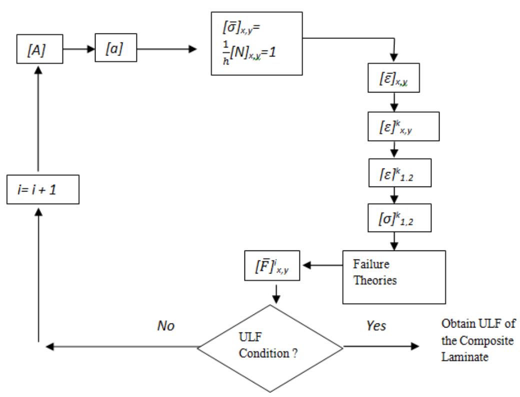

Figure 3. Flow chart for ultimate laminate failure criteria

6. Results

Based on the Classical lamination plate theory, several loading cases were considered with different values of n applied to a symmetric laminate having the following stacking sequence $[90/45/\text{-}45/0]_s$; the ply thickness is 0.005 inch. The material used is carbon/epoxy (AS4/3501-6). The minimum load based on the FPF and ULF criteria's are summarized in Tables. (1-6).

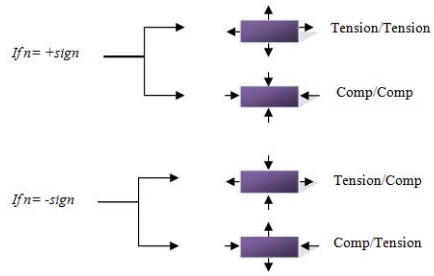

Figure 4. Loading possibilities applied to the composite laminate

$$n = \begin{Bmatrix} 0 \rightarrow Uniaxial\_Load \\ 1 \rightarrow Biaxial\_Load \\ 0 \prec n \prec 1 \rightarrow Biaxial\_Load \\ n \succ 1 \rightarrow Biaxial\_Load \end{Bmatrix} \qquad (7)$$

Table 1: Failure load based on the FPF & ULF criteria's for several different failure theories; n = 0, Tension

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	50.5/90°	50.5/90°	49.9/90°	46.6/90°
ULF (ksi)/ Ply	115/0°	114/0°	112/0°	108/0°

Table 2: Failure load based on the FPF & ULF criteria's for several different failure theories; n = 0, Compression

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	71.7/45°	71.7/45°	68/45°	85/45°
ULF (ksi)/Ply	180/90°	180/90°	167/90°	153/90°

Table 3: Failure load based on the FPF & ULF criteria's for several different failure theories; n = 1, Tension/ Tension

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	51/90°	51/90°	51/90°	51/90°
ULF (ksi)/Ply	51/0°	51/0°	51/0°	51/0°

Table 4: Failure load based on the FPF & ULF criteria's for several different failure theories; *n* = 1, Compression/ Compression

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	136/0°	136/0°	136/0°	136/0°
ULF (ksi)/Ply	136/90°	136/90°	136/90°	136/90°

Table 5: Failure load based on the FPF & ULF criteria's for several different failure theories; *n* = -1, Tension/Compression

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	35.8/45°	35.8/45°	35.8/45°	35.9/45°
ULF (ksi)/Ply	85/0°	85/0°	74/0°	67/0°

Table 6: Failure load based on the FPF & ULF failure criteria's for several different failure theories; n= -1, Compression/Tension

	Max Stress	Max Strain	Tsai-Hill	Tsai-Wu
FPF (ksi) Ply	35.8/45°	35.8/45°	35.8/45°	35.9/45°
ULF (ksi)/Ply	85/90°	85/90°	74/90°	67/90°

Table 1 and Table 2 show the results of applying a pure uniaxial load in tension and compression on the proposed composite laminate respectively. As shown in the tables, the minimum stress occurred in the 90° ply for the tension case as expected since it has the least resistance against the load unlike the compression case where the minimum load occurred in the 45°/-45° plies. Table 3 and Table 4 show the results of applying a biaxial load in tension/tension case and compression /compression on the proposed composite laminate respectively. As shown in the tables, the stress is almost equally distributed throughout all the plies of the laminate since the laminate is symmetric and balanced while the composite laminate will fail in the 90°/0° plies at the same time due to the effect of the biaxial load. Table 5 and Table 6 show the results of applying a pure biaxial load but this time the load fraction *n* has a negative value giving the possibility of applying tension/compression or compression /tension to the proposed composite laminate respectively. As shown in the tables, the minimum stress occurs in the 45°/-45° ply for the tension/completion case since these are the least resistive plies regarding the given loading condition while the compression/tension case gives a reverse behavior keeping the 45°/-45 plies the least resistive hence the first to fail.

7. Finite Element Verification

In order to verify the results obtained from the previous analysis the is used to implement a certain case study. A composite laminate made of Carbon/Epoxy (AS4/3501-6); the model consists of 8 plies with fiber orientation $[90/45/\text{-}45/0]_s$. The model implemented was used to examine the stress of each ply resulting from a tension stress followed by a compression stress in order to verify the results that were obtained from the previous analysis. Eight rectangular plies were used in the analysis. The element used are free tetrahedral with a total of 12000 elements in the model. We will apply the failure load that we have obtained from our previous analysis to one end of the laminate, while the other end constrained. It was observed that the principle stress for the uniaxial case σ_2 = 30 ksi in the 90° ply exceeded the transverse tensile strength of the material which means that it failed as shown in Table 7 while all the principle stresses in the rest of the plies have not exceed the corresponding strength. Table 8 shows the result of applying a compression stress; it can be observed that the shear stress in the +-45° plies exceeded the corresponding shear strength leading them to fail under the given load while the other plies not; these results aggre with the results previously obtained, hence, according to the result of the FE model, we found that the results that we obtained from the analysis are reliable and correct. In addition, Table 9 shows a comparison of the ULF values obtained from experimental results found in [5] with those obtained from our previous analysis and both show good agreement.

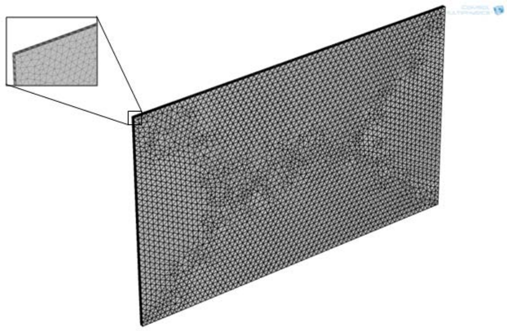

Figure 5: Finite element mesh

Table 7: Failure stress summery of the FEA model for n=0, tension

Ply	σ_1 (ksi)	σ_2 (ksi)	τ_6 (ksi)	F_{1t} (ksi)	F_{2t} (ksi)	F_6 (ksi)
90°	8.6	**30.17**	0.3	330	**8.3**	11
45°	47.8	4.5	-7.9	330	8.3	11
-45°	47.5	4.4	7.9	330	8.3	11
0°	120	3.5	0.27	330	8.3	11

Table 8: Failure stress summery of the FEA model for n=0, compression

Ply	σ_1 (ksi)	σ_2 (ksi)	τ_6 (ksi)	F_{1c} (ksi)	F_{2c} (ksi)	F_6 (ksi)
90˚	-12.4	28.4	-0.3	250	33	11
45˚	-67	-6.8	**12.4**	250	33	**11**
-45˚	-67	-6.8	**-12.4**	250	33	**11**
0˚	-189	-0.04	-0.3	250	33	11

Table 9: Comparison between experimental & analytical results

Laminate	ULF - Experimental (ksi)	ULF - Previous Analysis (ksi)
$[60/-60/0]_s$ ***n* = 0**	106	115

8. Carpet Plots

Carpet plot is one that illustrates the interacting behavior of two independent variables, which among other things facilitates interpolation in both variables at once, mainly used as a design tool. A designation of a certain layup is $[0_m/90_n/\pm 45_p]_s$, where *m*, *n*, *p* denote the number of 0°,90°,+-45° plies, respectively. The in-plane engineering constants of a symmetric laminate depend only on the proportion of the various plies in the entire laminate and not on the exact stacking sequence. Thus, in-plane engineering constants are a function of the fractional values α, β, γ, where

$$\alpha=2m/N \quad \beta=2n/N \quad \gamma=4p/N$$

N: total number of plies

As we mentioned a carpet plot is a parametric family of curves with one of the fractions α, β, γ as a variable and the other two as parameters, keeping in mind that $\alpha+\beta+\gamma = 1$. Such plots for Young's modulus, shear modulus, and Poisson's ration are shown in Fig. 8, 9, 10 respectively for Carbon/Epoxy material (AS4/3501-6).

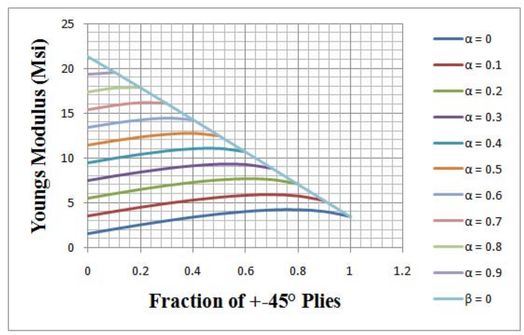

Figure 6. Carpet Plot for Young's modulus of $[90/45/-45/0]_s$ carbon epoxy laminates (AS4/3501-6)

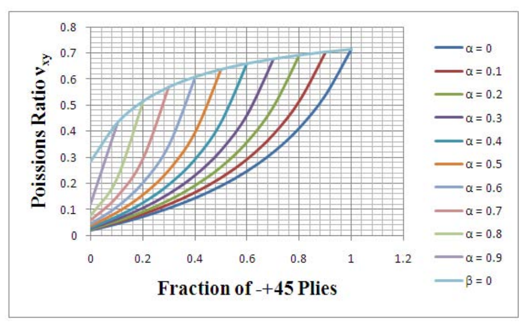

Figure 7. Carpet Plot for Poisson's ratio of $[90/45/-45/0]_s$ carbon epoxy laminates (AS4/3501-6)

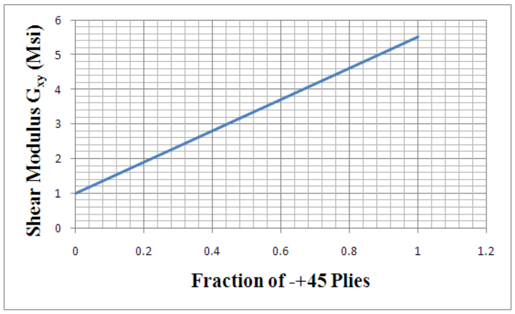

Figure 9. Carpet Plot for Shear modulus of $[90/45/-45/0]_s$ carbon epoxy laminates (AS4/3501-6)

9. Computer Software

The Composite Analyzer is an engineering program that analyzes laminated composite plates under uniaxial and biaxial loading according to classical laminated plate theory. Familiarity with such analysis is assumed. Input consists of ply material properties, material strengths, ply fiber orientation and stacking sequence, ply thickness, type of composite (Hybrid: Composite Laminates containing plies of two or more different types of material) and unit system(English/SI). Output consists of apparent laminate material properties, ply stiffness and compliance matrices, laminate "ABD" matrices, lamina failure load and mode based on Maximum Stress, Maximum Strain, Tsai-Hill, and Tsai-Wu failure theories, failure load and mode for the hole laminate based on first ply failure (FPF) also based on ultimate laminate failure (ULF) and Carpet plots for orthotropic laminates (An orthotropic laminate requires that the number of +θ angle plies equals the number of –θ angle plies [90 /+-θ/0]). The main interface page of the program is shown in Fig. 11.

10. Conclusion

This paper presented a general approach in analyzing composite laminates under uniaxial and biaxial loading using two different failure criteria's. It was observed that the failure occurred in the 90º ply when applying a uniaxial load while the failure occurred in the 45º / -45º plies in the biaxial case which obtains the least resistance in that case. Verification using the FEA software COMSOL was performed in addition to experimental verification. Computer software was also conducted that enables the user to analyze a broad variety of composite layups. Also, carpet plots for young's modulus, poisson's ratio and shear modulus were introduced.

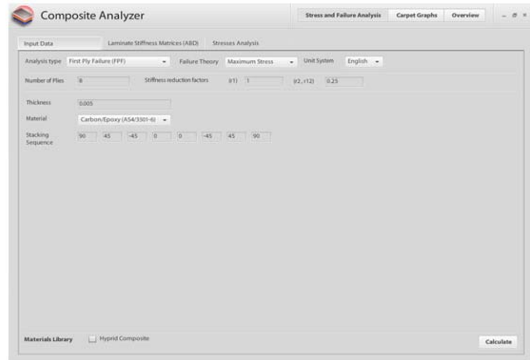

Figure 8. Main Interface of the program

11. References

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