Modelling Flow through Fractures in Porous Media

Ekkehard Holzbecher[*,1], LiWah Wong[1] and Marie-Sophie Litz[2]
[1]Georg-August Universität Göttingen (GERMANY), [2]Freie Universität Berlin (GERMANY)
*Corresponding author: Georg-August Universität Göttingen, Goldschmidtstr. 3, 37077 Göttingen, eholzbe@gwdg.de

Abstract: There are various alternative options concerning modeling fluid flow within fractures in porous media. We give a general overview, with remarks concerning the modeling using COMSOL Multiphysics. Moreover we define and study two test cases for intercomparison. Finally for one of the testcases some results of an extensive sensitivity study is presented.

Keywords: porous media, fracture, Darcy's Law, free fluids, streamfunction

1. Introduction

There are various natural circumstances in which fluid flux through a porous medium is dominated by fracture flow. Geological layers can be fractured, as well as porous materials used in technical devices, for example in membranes, filters etc.

For the modeler there are various options to treat fractures in porous media. First of all is the question of scale. Three cases can be distinguished. On the smallest scale processes between one or few fractures and the surrounding low permeable matrix are studied in detail. On the larger scale a fracture network is considered with a multitude of fractures, partially intersecting. On the next larger scale individual fractures are not taken into account any more. Instead the mixed system, consisting of fractures and matrix, is taken as a homogenous medium, similar to the description of porous media by Darcy's Law. Diodato (1994) suggests a classification into 'explicit discrete fracture formulation', 'discrete fracture networks' and two continuum formulations. Here we deal with the smallest scale, the micro-scale only.

For the numerical modelling we examine various different approaches. For the fracture we choose different geometries and different descriptions of the underlying physics. In addition we use the potential and streamfunction formulation in 2D, for which we also implement a formulation, in which the fracture is modelled as a 1D interior boundary. In that case a jump condition has to be used for the streamfunction.

We present the most important findings that we obtained so far from the intercomparison of the different approaches, in which we also include a comparison with an analytical solution. We also show the results of a parameter sensitivity study, varying fracture width, the ratio between high and low hydraulic conductivities and fracture angle in relation to the regional flow field. Moreover we studied constellations in which the fracture does not have a constant aperture, but is represented by a very slender ellipse (typical half-axes ratio is 1:400).

The ongoing studies are performed in the framework of the GeBo project, funded by Lower Saxony and BakerHughes, concerning the utilization of geothermal energy of deep geological strata. It is intended to consider the distributions of temperature and mass concentration in future studies, in addition to flow modelling.

2. Testcase Set-ups

Using COMSOL Multiphysics we examine two constellations of a fracture in a porous medium. In one set-up the fracture is located in a free in principle unbounded flow field within the porous medium. In the other set-up the flow field is sandwiched between two impermeable strata.

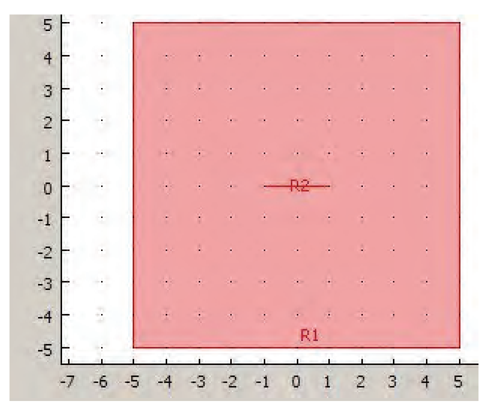

Figure 1. Fracture (R2) in a free flow field (set-up 1)

Set-up 1 has a horizontally placed fracture in a diagonal flow field. The same constellation was studied by Sato (2003) and by Churchill & Brown (1984). The geometry and extension of the model region is depicted in Figure 1.

In the 2D model region R1 we require the differential equation

$$\nabla K_{low} \nabla \varphi = 0 \tag{1}$$

for the hydraulic potential φ to be fulfilled. In the 1D or 2D region R2 we require:

$$\nabla K_{high} \nabla \varphi = 0 \tag{2}$$

Conditions at the outer boundaries of R1 are of Dirichlet type throughout:

$$\begin{aligned} \varphi(x,5) &= 0.5 - .05*(x+5) \\ \varphi(x,-5) &= 1 - .05*(x+5) \\ \varphi(5,y) &= 0.5 - .05*(y+5) \\ \varphi(-5,y) &= 1 - .05*(y+5) \end{aligned} \tag{3}$$

With these we specify diagonal inflow with an angle of 45° from the lower left part. If a 1D approach for the fracture is used, the boundary condition on the two end points of interval R2 is:

$$\frac{\partial \varphi}{\partial x}(\pm 1, 0) = 0 \tag{4}$$

If we model R2 as 1D, this is a combined 1D/2D problem! A formulation for combinations of 1D, 2D and 3D subdomains in general is given by Sembera *et al*. (2007), who also propose a finite element model approach for such situations. Similarly Angot *et al*. (2009) compare 'global Darcy models' with 'asymptotic' approaches. Very recently Grillo *et al*. (to appear) deal with lower-dimensional approaches for density-driven flow in fractured porous media.

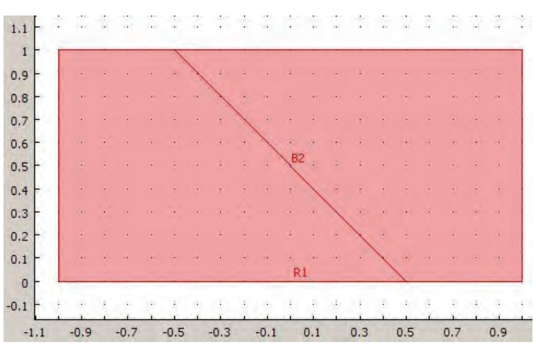

Figure 2. Fracture (B2) in a bounded flow field (set-up 2)

Set-up 2 has a diagonally placed fracture in a flow field that is bounded two sides. The geometry and extension of the model region is depicted in Figure 2.

Lower and upper horizontal lines represent no-flow boundaries. The flow is induced by a pressure, resp. head gradient between the left and right boundary, inducing flow from left to right. Thus there are boundary conditions of Dirichlet- and of Neumann type. In Figure 2 the slope of R2 –1, corresponding with an angle of 45°. Slope and angle, respectively, were changed in a parameter variation study.

For the parameter variations we used $K_{low} = 1/K_{high}$ and obtain for the ratio between the conductivities:

$$K_{ratio} = \frac{K_{high}}{K_{low}} = K_{high}^2 \tag{5}$$

Note that we use normalized values for length and velocity in both models. A real world problem with reference length L and velocity v_0 we can transfer to the normalized system by the transformations $x \to x/L$ and $y \to y/L$ and normalized velocity $v \to v/v_0$. For the given choice of conductivities we obtain a formula for the reference velocity as a function of fracture conductivity K_f and matrix conductivity K_m of the real system

$$v_0 = \sqrt{K_m K_f} \quad \text{or} \quad v_0 = K_{ratio}\sqrt{K_m} \tag{6}$$

3. Analytical Solutions

For the described geometry Churchill & Brown (1984) gave an analytical solution in form of a complex potential $\Phi = \varphi + i\Psi$. They are concerned with a relatively impermeable disturbance in a flow field, which is tilted by the angle α in relation to the coordinate axis:

$$\Phi(z) = \Phi_o(z\cos(\alpha) - \mathrm{i}\sqrt{z^2 - a^2}\sin(\alpha)) \tag{7}$$

where a denotes half of the length between the two end positions of the horizontal barrier (see also: Sato, 2003). z=x+iy denotes the complex variable in the plane.

The solution can be adjusted for the case, where a relatively permeable object disturbs the 1D flow field. Then the real part and the imaginary part of the complex potential exchange their roles. Moreover the sign of the angle has to be changed, and for the complex potential solution one obtains the formula

$$\overline{\Phi}(z) = -\mathrm{i}\Phi_o(z\cos(\alpha) - \mathrm{i}\sqrt{z^2 - a^2}\sin(\alpha)) \quad (8)$$

Formula (8) can be re-written as

$$\overline{\Phi}(z) = \Phi_o(\sqrt{z^2 - a^2}\sin(\alpha) - \mathrm{i}z\cos(\alpha)) \quad (9)$$

Note that this solution does not have a hydraulic conductivity or permeability as parameter. Thus it can not be an exact solution for set-up 1.

At the fracture this analytical solution shows a jump for the streamfunction Ψ, but not for the real potential φ. In general the streamfunction fulfils a jump condition along the fracture:

$$\Psi^+ - \Psi^- = \frac{K_{high}d}{K_{low}}\frac{\partial\varphi}{\partial s} \quad (10)$$

(Sato 2003) where d denotes the aperture of the fracture.

4. Use of COMSOL Multiphysics

The models were set-up using COMSOL Multiphysics 3.5a. There are various options that have been applied. We used the diffusion mode (di) for the solution of equations (1) and (2), with diffusivities K_{low} and K_{high} respectively. We also used alternative formulations utilizing options from the COMSOL Earth Science Module. For the matrix we took Darcy-mode (esdl), where instead of equation (1) the differential equation

$$\nabla\frac{k_{low}}{\eta}\nabla p = 0 \quad (11)$$

with permeability k_{low} and fluid viscosity η for the hydraulic pressure p is valid. Then for the fracture we studied the following options*

- Darcy-mode (esdl)
- Navier-Stokes-mode (chns)
- Brinkman-mode (chns)

If there is free flow within the fracture, i.e. if it is similar to a tube, the Navier-Stokes equations are the appropriate mode for modeling the most likely laminar flow within the free space. If the fracture itself is filled, or partially filled, with porous material of smaller size, it is surely more appropriate to use Darcy's Law within the fracture as well. An intermediate way to cope with the problem is to use the Brinkman equations, in which free fluid flow and porous media flow are combined:

$$\frac{\eta}{k}\mathbf{v} + \nabla p - \nabla\frac{\eta}{\theta}\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right) = 0 \qquad \nabla\cdot\mathbf{v} = 0 \quad (12)$$

where θ denotes porosity. The advantage is, that depending on the local variable size, the dominance of either one or the other regime is automatically taken into account.

Another alternative is the direct use of the streamfunction Ψ, which fulfills the potential equation

$$\nabla K_{low}^{-1}\nabla\Psi = 0 \quad (13)$$

in the low permeable region and

$$\nabla K_{high}^{-1}\nabla\Psi = 0 \quad (14)$$

in the fracture. For the solution of differential equations (13) and (14) we utilized the diffusion mode (di) also. Note that the streamline model is coupled with the potential solution, from which total flow is calculated.

Moreover we explored the option to use a lower dimensional representation of the fracture, i.e. here to combine a 1D fracture model with a 2D geometry for the low permeable porous medium. This procedure was introduced using the potential and the streamfunction formulations, mentioned above.

The potential values, calculated for the fracture in the 1D fracture geometry, are used as boundary conditions in the 2D geometry. For that purpose we use the COMSOL option to enter 'extrusion coupling variables'. In case of the streamfunction formulation we implemented condition (10) at the fracture, also using extrusion coupling. Note that in this model approach there is an inherent coupling between the potential and the streamfunction, which is easily to be implemented by COMSOL Multiphysics.

Meshing was always made with a drastic refinement near to the fracture. In figure 3 we show as an example a mesh for the elliptic fracture approach in set-up 2.

* In COMSOL Multiphysics 4.0 there are different modes: Darcy-mode (dl), laminar flow (sfp), Brinkman- mode (br), fracture flow (esff), free and porous media (fp)

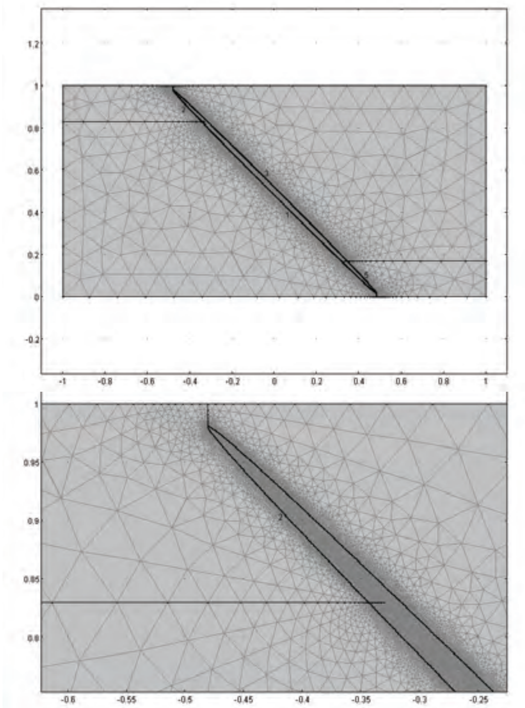

Figure 3. Finite element mesh of set-up 2 with elliptic fracture; top: complete model region, bottom: detail

For intercomparison we also modeled the two set-ups by models with complete 2D geometry in which the fracture is resolved by a very fine 2D mesh.

Concerning convergence rates for the two set-ups see Bradji *et al.* (2010).

5. Results

5.1 Free Fluid and Porous Medium Solution

As mentioned above for the fracture the porous medium or the free fluid approach can be used. We used the Brinkman-approach to vy between the two alternatives. For an expected parameter range we did not find significant differences between the two approaches, i.e. the Darcy-term in the Brinkman approach seems to be dominant, as long as velocities are within the usual range.

5.2 Analytical and Numerical Solution

A graphical representation of the results for set-up 1 is given in Figure 4. The numerical model is here based on a lower dimensional, i.e. 1D representation, and the diffusion mode, as described above, was used to calculate the potential. K_{low} was chosen as 0.1. The potential distribution is visualized by the colormap, representing flow from red to blue. Flowpaths and velocity field, both depicted in white, are based on the velocity derived from the solution for φ. Streamlines, in magenta are plotted on basis of the analytical solution for the streamfunction, given by equation (9).

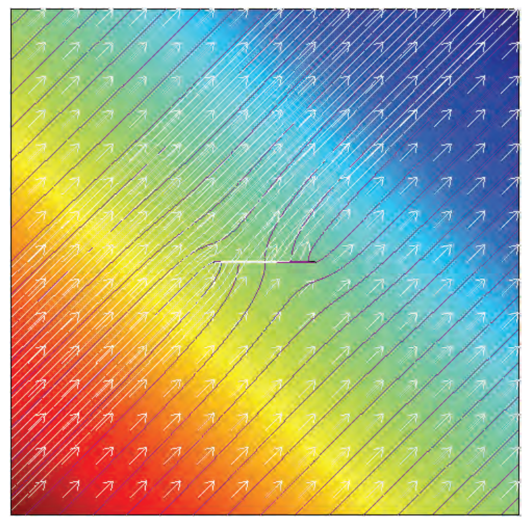

Figure 4. Potential colorplot, streamlines (magenta), flowpaths (white) and velocity field (white arrows) for set-up 1 (see text)

Near to the outer bounds of the model region streamlines from the analytical solution and flowpaths from the numerical solutions are obviously identical, while they do not coincide exactly at the fracture, although they qualitatively show the same flow pattern. As mentioned above the analytical solution does not consider any permeability and thus can not describe the numerical solution.

5.3 Streamfunction and Potential Solution

In Figure 5 we show numerical results for set-up 2. The solutions were obtained for 1D and 2D representations of the fracture with a constant aperture of 0.001 (in the 2D case) and K_{low}=0.1 and we used the diffusion mode for potential and streamfunction.

The potential distribution is visualized by the color plot. The streamlines are obtained as contours of the streamfunction. The starting points for the streamlines was set on the left side, from where they pass through the low permeable matrix to reach the fracture. It is clear to see that they remain within the fracture for a longer distance, until they move further through the right part of the matrix towards the outlet. The passage in the fracture coincides only roughly with the jump of the streamfunction in the 1D model. The origin of these deviances is not yet completely clear. We suppose that the interface condition

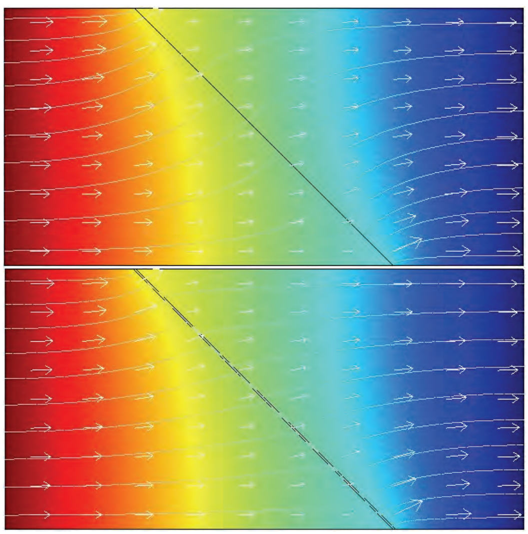

Figure 5. Potential colorplot and streamlines for set-up 2 (K_{low}=0.1, K_{ratio}=100); top: 1D fracture representation, bottom: 2D fracture representation

Bradji *et al.* (2010) show a similar figure obtained from the same model, only for a lower value of K_{low} and thus a higher value for K_{ratio} (Figure 5). Comparison of the flow fields shows that the passage through the fracture increases with K_{ratio}, as it can be expected.

It is worth noting that in the 1D fracture model the jump in streamlines can be visualized only by the streamfunction approach. Using Particle or streamline tracing based on the velocity field does not deliver any passages through the fracture. The reason is that the end position of the flowpath or streamline in the left matrix sub-domain is taken as starting position for a further tracing in the right sub-domain. The streamfunction approach turns out to be extremely useful for the visualization of the streamline pattern.

6. Parameter Variation

For set-up 2 we performed an extensive parameter variation study. Variations concerned the aspect ratio, the fracture width, resp. the half-length ratio in case of elliptic fractures, anisotropy in fracture and matrix, fracture angle with respect to the closed boundaries, as well as the connection with open boundaries.

As an example we show in Figure 6 the resulting flow pattern in a matrix-fracture-system with layers. The fracture is represented by an ellipse, which connects two permeable layers on both sides. The mesh for that model was already shown in Figure 3. Streamlines and velocity field show how the flow is focused in the more permeable parts of the model region. The result was obtained using Darcy's Law mode and pressure as dependent variable.

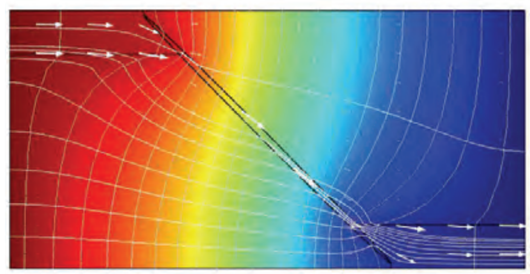

Figure 6. Potential colorplot, streamlines and velocity field for set-up 2 with elliptic fracture and two permeable layers

In all variations we focused our interest on the change of flow in comparison to the flow through a pure matrix of low conductivity. The fracture increases the flux and the model shows in which scale we have the increase to expect. One example is depicted in Figure 7, in which we show results for the 2D modeling of a fracture constant aperture in a homogeneous porous medium.

On the *x*-axis we assign dimensionless K_{ratio}, varying over several orders of magnitude from 1 (no fracture) to 10^6. As expected the shown curves are monotonically increasing, as the effect clearly becomes more pronounced when the conductivities of fracture and matrix deviate more. However for small and high ratios the flux increase is not affected. For low K_{ratio} the difference is simply too small, to be effective. For high K_{ratio} the path of flow through the impermeable zone becomes significant. The matrix has to be passed, and the flux through the fracture can not be increased.

The effect (flux increase) becomes also more pronounced with the slope of the fracture in the system, i.e. the angle between fracture and closed boundaries. The slope of –10 represents an almost vertical line, and results in an angle of 45° and has almost no effect on the flux. With increasing slope the two tips of the fracture move towards the open boundaries. The effect on the flux increases, as for the fluid the chance of bypassing the impermeable part becomes bigger. We see an increase of flux by factors of 4 or 5, when the ends of the fracture come closer to the outer bounds. With a slope of almost –0.5 the fracture connects the two open faces of the model region, and flow increases infinitely with the permeability of the fracture, as clearly in the logarithmic scale.

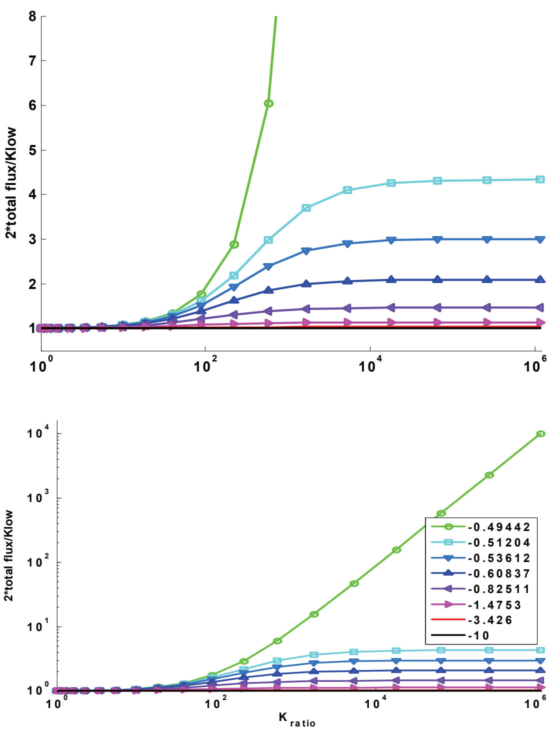

Figure 7. Increased flux in dependence of K_{ratio} (x-axis) and slope (legend); top: linear scale on y-axis, bottom: logarithmic scale

7. Summary and Outlook

Concerning the modeling of flow between a low permeable matrix and a fracture we outlined and examined several options. Only few of our results could be presented in the paper.

We presented a comparison between analytical and numerical solution, showing that the analytical solution is not a valid asymptotic for the numerical solutions.

For 1D fracture representations the streamfunction provides the only way to visualize the streamline pattern through the fracture. However deviations concerning the streamline patterns of 1D and 2D models are not yet resolved.

The results are of general interest concerning the upscaling of the mentioned methods for modeling fracture networks. In order to keep the number of DOFs in a practicable size, it is intended to use a lower-dimensional approach, here to represent the fractures as 1D geometries in a 2D geometry.

The results here show how the streamfunction can be used to obtain streamline patterns for the entire model region. However, aside from the just mentioned deviations, the approach has the disadvantage that the matrix has to be divided in subdomains also for the formulation of the jump conditions. Moreover, a generalization concerning 1D or 2D fractures in 3D is not at hand.

Altogether COMSOL Multiphysics is a very appropriate tool for the here presented tasks, which becomes even more advantageous if transport processes have to be considered in addition. As a demonstration in Figure 8 we show a snapshot from heat transport through the system of set-up 2. Cold water is entering from the left, gradually replacing the hot water that is initially present in the entire system. Unsteady transport seems to be a good way to visualize flow.

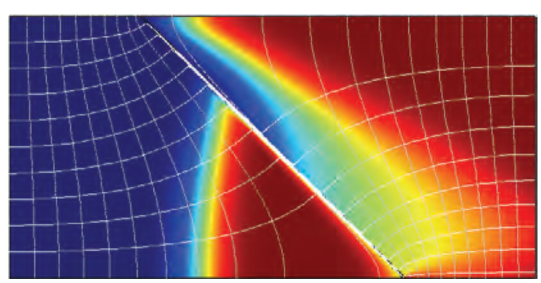

Figure 8. Snapshot from a simulation of heat transport through set-up 2

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