Design and Stress Analysis of a General Aviation Aircraft Wing

Ghassan M. Atmeh[*1], Zeaid Hasan[2] and Feras Darwish[3]

[1, 3]Jordan University of Science and Technology, Irbid, Jordan, [2]Texas A&M University, College Station, Texas

* Corresponding Author: atmehg@yahoo.com

Abstract: In the present study, a general aviation airplane is designed and analyzed. The design process starts with a sketch of how the airplane is envisioned. Weight is estimated based on the sketch and a chosen design mission profile. A more refined method is conducted based on calculated performance parameters to achieve a more accurate weight estimate which is used to acquire the external geometry of the airplane. A three-dimensional layout of the airplane is created using RDS software based on conic lofting, then placed in a simulation environment in Matlab which proved the designs adherence to the design goals. In addition, static stress analysis is also performed for wing design purposes. Using the finite element software package COMSOL, the calculated aerodynamic loads are applied to the wing to check the wing reliability. It is shown that the designed wing could be a good candidate for similar general aviation airplane implementation.

Keywords: Aircraft Design, Structural Analysis

1. Introduction

The design of an aircraft is a prolonged process that has mainly three phases; the first is the conceptual design phase, and it is the phase employed in this paper. This phase deals with the layout of the aircraft and what major characteristics it must have in order to achieve its design goals, if the conceptual design is to be a success no major changes should be implemented on it in future phases. Hence, the conceptual design engulfs the major characteristic of the aircraft while delivering a layout of its major components. The second and third phases of the design process are the preliminary and detail design phases; these phases deal with the analysis of the aircraft components in all major aspects of aerospace such as structures, dynamics, control and others. The conceptual design approach utilized in this study is based on reference [1]. The method is highly statistical based on data of other aircraft of the same category. First, a set of design goals is set to provide a baseline for the design.

Second, a sketch of the airplane is created along with a design mission profile. By using a simple cruise mission profile consisting of five segments which are takeoff, climb, cruise, loiter and landing. A crude method of weight estimation based on the sketch and mission profile is conducted to achieve an estimate of the weight of the airplane. In order to achieve a more precise estimate of the weight the wing loading and thrust-to-weight ratio are estimated. These along with the mission profile provided a more accurate estimate of the airplane weight. The weight, which is statistically integrated into major parts of the airplane, is used to generate a layout of the aircraft exterior geometry. Conic lofting is used to generate a three-dimensional layout of the airplane using the RDS software [1]. The three-dimensional model is used to create a simulation of the airplanes takeoff, the simulation shows the ability of the design to achieve its goals. Structural analysis of the airplane wing is conducted to define the geometry of the wing spars and skin. The wing section is designed to have two spars, one at a quarter of the cord position and one at the three quarters of the cord. Angled spar flanges are used along with stringers to stiffen the skin. It is assumed that the stringers and spar flanges only carry the directs stresses while the skin and spar webs carry shear stresses, this enables the idealization of the stringers and flanges areas into a concentrated area named booms to carry the direct stresses along with the thin skin to carry the shear stresses. Proper stress analysis is used to calculate the area of the booms and the thickness of the skin. The finite element software COMSOL is utilized to verify the results of the structural analysis by employing the finite element method to calculate the stresses in the wing due to the aerodynamic loading.

2. The Conceptual Design

2.1 Design goals

The conceptual design is commenced by defining a set of design goals based on gathered data for several airplanes of the same category.

The goals are as follows: Single engine airplane with a maximum velocity greater than 250 ft/s, stall velocity greater than 60 ft/s, takeoff ground roll less than 1000ft, landing distance less than 1400 ft, range of 550 nm, service ceiling of 13000 ft and a maximum rate of climb of 1200 ft/min.

2.2 Mission profile and initial sketch

A simple mission profile consisting of five segments was used. The takeoff, climb, cruise, loiter and landing segments are shown in figure 1. Figure 2 shows the initial sketch of the aircraft.
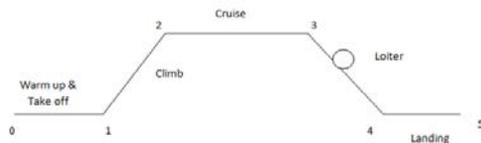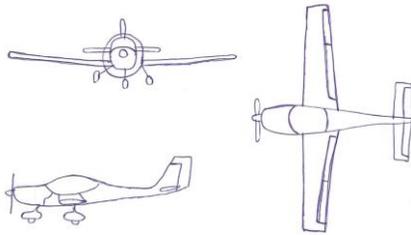

Figure 1. Mission profile

Figure 2. Initial sketch

2.3 Weight estimation from initial sketch

The airplane gross takeoff weight can be divided into crew, payload, fuel and empty weight. The crew and payload weights are known from the design goals. The fuel and empty weight however are unknown and are dependent on the takeoff gross, hence, the weight estimation is an iterative process based on an initial guess of the gross takeoff weight. Equation (1.1) is used to estimate the gross takeoff weight

$$W_o = \frac{W_{crew} + W_{payload}}{1 - (W_f/W_o) - (W_e/W_o)} \qquad (1.1)$$

The empty weight fraction can be expressed based on statistical data in [1]. The fuel weight fraction however is calculated based on the mission profile segments where each segment consumes a fraction of fuel and the total fuel weight fraction is the multiple of these fractions. This crude estimation method gives a takeoff gross weight of 1670 lb for the airplane.

2.4 Weight estimation based on performance parameters

In order to obtain a more refined estimation of the airplane weight several performance parameters must be estimated such as the maximum lift-to-drag ratio, the maximum lift coefficient, the wing loading and the thrust-to-weight ratio. The latter two are interconnected, this causes the need to estimate one of them, call it X, and use that estimate to calculate the other, call it Y . The estimate X is recalculated based on the value Y. The maximum lift-to-drag ratio is estimated by assuming CD_o and K to have values of 0.03 and 0.05 respectively. This yields a maximum lift-to-drag ratio of 12.9. The maximum lift coefficient is highly dependent on the airfoil sections of the wing. At this early design stage two airfoils are used which are the NACA 23018 and NACA 23012 at the root and at the tip respectively. The value of these airfoils maximum lift coefficients is available via lift-curves, taking the average of these values and introducing a factor to account for the three-dimensional effect gives a maximum lift coefficient of 2.34. When estimating the horsepower-to-weight ratio and the wing loading, the maximum horsepower-to-weight ratio and minimum wing loading are selected, this is to ensure they satisfy all the mission segments. The hp/W is estimated at different conditions and used to estimate the wing loading. The hp/W is re-estimated using the wing loading were necessary. The following final results were found (hp/W of 0.07 and a W/S of 16 lb/ft^2). Now that the required performance parameters are available the gross-takeoff weight is more accurately estimated to be 2100 lb.

2.5 Geometry sizing

The geometry of the airplane can now be unveiled using the weight estimated in the previous section. The fuselage length is

estimated statistically to be 25.4 ft. The fuselage fineness ratio, which is the ratio of the length divided by the maximum diameter, is chosen to be 5.5 rendering a maximum diameter of 4.6 ft. By finding the values for the weight and wing loading of the airplane, the plan form area of the wing (S) can be calculated to be 131.25 ft^2. The span can be estimated from an aspect ratio of 8 to be 32.4 ft. The taper ratio which is the ratio between the tip and the root cords is 0.5, this gives a root and tip cord values of 5.4 ft and 2.7 ft respectively. The mean aerodynamic cord is positioned at 7.2 ft from the root with a length of 4.2 ft. Figure 3 shows an outline of the wings reference area.
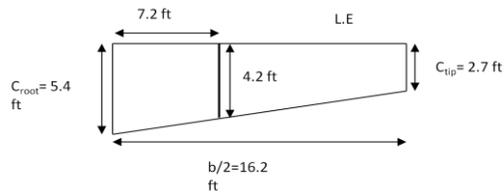

Figure 3. Wing outline

The vertical and horizontal tails of the airplane are seized in a similar manner however; depending on the tail volume coefficients the plan form areas of both are found to be 11 ft^2 and 25.3 ft^2 respectively. In addition, the vertical tail is found to have a span of 4.06 ft and a tip and root cord of 1.8 ft 3.61 ft respectively. The horizontal tail also has a span of 10 ft and tip and root cord of 1.7 ft and 3.1 ft respectively.

2.6 Design layout

The main objective of the conceptual design is to provide a drawing of the airplanes smooth, producible and aerodynamically accepted external features. The process of lofting is one that defines the external geometry of an airplane. Conic lofting is a lofting technique which incorporates a family of second degree curves such as a circle, ellipse, parabola and hyperbola. In this study, the conic lofting technique is used to generate a drawing of the designed airplane using the RDS software [1]. Figure 4 shows an isometric and three-view layout of the airplane developed in RDS.
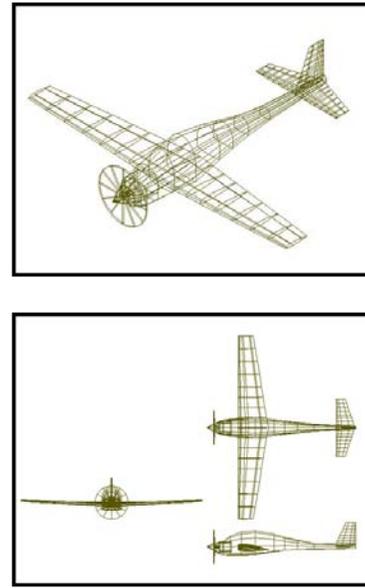

Figure 4. Isometric and three view layout

3. Takeoff Simulation

The knowledge of the external geometry of the airplane enables us to create a three-dimensional model using the V-realm Builder and place it in a simulation environment, as shown in figure 5, by incorporating Matlab's simulink. The intention of the simulation is to show the designs ability to satisfy some design goals such as the takeoff distance, maximum speed and rate of climb. The simulation is divided into three main parts; the takeoff, climb and cruise segments of the mission profile. The simulation is a three-degree-of-freedom one, including translation in the X and Z direction and pitching (rotation) along the Y direction. Three equations are solved to generate the simulation

$$\dot{u} = \frac{F_x}{m} - Qw - g\sin(\theta)$$

$$\dot{w} = \frac{F_z}{m} + Qu + g\cos(\theta)$$

(1.2)

$$\dot{Q} = \frac{M}{I_{xx}}$$

(1.3)
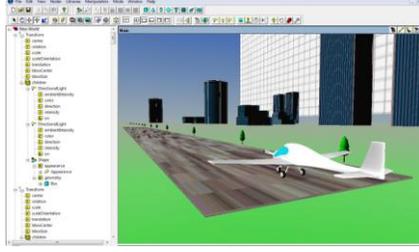
Figure 5. Simulation environment and three-dimensional model

4. Wing Stress Analysis and Design

4.1 Aerodynamic loads and pitching moment

The aerodynamic loads that mainly affect aircraft wings are the lift and drag forces. The lifting line theory is applied in this study to achieve the span wise loading on the wing; it assumes that the presence of a vortex generating a circulation causes the load, it also assumes that the wing is segmented into several span wise locations were a two-dimensional flow exists. After calculating the loads at the span wise locations separately, they are brought together in order to form the span wise load distribution. For this preliminary design phase, the wing is assumed to have only one airfoil shape along the span which is the NACA 2412. A code written in Mallab is developed in order to generate the span wise load distribution. First the unknown vortex strength distribution (circulation, Γ) is approximated by the following Fourier series expansion

$$\Gamma(\theta) = 2bV \sum_{n=1}^{N} A_n \sin(n\theta) \qquad (1.4)$$

Where

$$\theta = \cos^{-1}\left(-\frac{2y}{b}\right) \qquad (1.5)$$

Were V is the velocity, b is the span and y is the location along the span. In order to determine the Fourier coefficients A_n, the following lifting line equation is solved

$$A_n \sin(n\theta(y)) + \sum_{n=1}^{N} nA_n \frac{\sin(n\theta(y))}{\sin(\theta(y))} = \alpha_G(y) - \alpha_o(y) \ (1.6)$$

Where, α_G is the geometric angle of attack and α_o is the zero lift angle of attack. Now that the Fourier coefficients are known, the lift coefficient for each span wise location is calculated using the following equation

$$c_l = \frac{\left(4b \times A_n \sin\left((2M-1) \times \theta\right)\right)}{c} \qquad (1.7)$$

Where c is the cord of the airfoil at a specific span wise location calculated using Eq. (1.8)

$$c = c_r\left(1 - 2\frac{\lambda - 1}{b}y\right) = \frac{c_t}{\lambda}\left(1 - 2\frac{\lambda - 1}{b}y\right)(1.8)$$
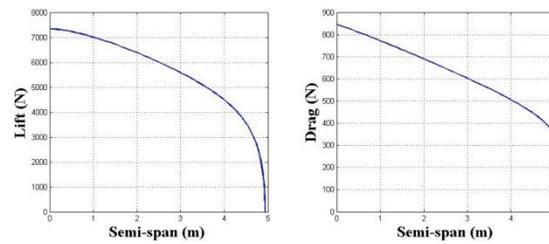
The drag coefficient is calculated from the drag polar equation Eq. (1.9)

$$c_d = c_{d0} + kc_l^2 \qquad (1.9)$$

The lift and drag forces are calculated at each span wise position by plugging their coefficients into Eq. (1.10). The lift and drag distribution along the span are shown in Figure 6.

$$L = \frac{1}{2}\rho_\infty V_\infty^2 cc_l$$
$$D = \frac{1}{2}\rho_\infty V_\infty^2 cc_d \qquad (1.10)$$

Figure 6. Lift and drag distribution

The pitching moment is calculated based on the pitching moment coefficient of the airfoil section using Eq. (1.11)

$$M_z = \frac{1}{2}\rho_\infty V_\infty^2 Scc_m \qquad (1.11)$$

4.2 Shear force distribution and bending moment

The presence of the lift and drag forces create two shear forces, S_y and S_x along with two bending moments, Mx and My. The span wise distribution of the shear forces and the moments are shown in figure 7 and figure 8 respectively.
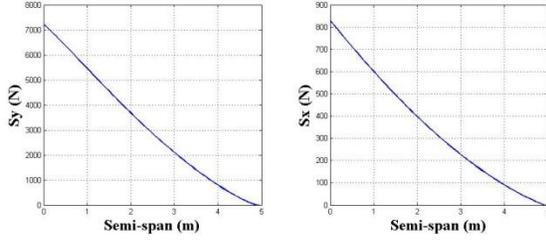

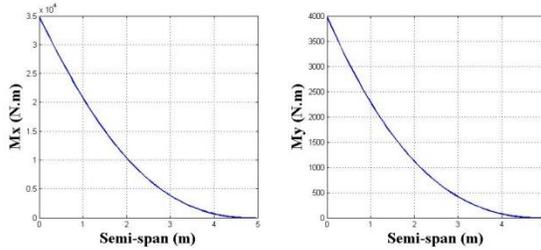
Figure 7. Shear forces distribution

Figure 8. Moments distribution

4.3 Wing stress analysis

Two of the Idealized booms are placed at the quarter of the cord each having an area of A, and another two at three quarters of the cord each having an area of 0.8A. The reason for a higher area at the quarter cord is due to the fact that the aerodynamic center is at that location requiring a larger area at that position. The following analysis calculates the area of these booms based on the bending stress and finds the thickness of the skin according to the shear flow in it

4.3.1 Boom area calculation

To achieve the boom areas, direct stresses are calculated since the spar-flanges and stringers mostly carry the bending stresses. First the centeroid location is calculated in order to calculate the second moments of area of the booms. Using Eq. (1.12-1.14), the areas of the booms, which is imbedded in the second moment of area, are calculated by substituting the yield strength of the annealed steel 4140 alloy, with a factor of safety of 1.25, into Eq. (1.12). The area is found to be A = 0.0026 m^2

$$\sigma_z = \frac{\overline{M_y}}{I_y} x + \frac{\overline{M_x}}{I_x} y \qquad (1.12)$$

$$\frac{\overline{M_x}}{I_x} = \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} \qquad (1.13)$$

$$\frac{\overline{M_y}}{I_y} = \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} \qquad (1.14)$$

4.3.2 Skin thickness calculation

Calculating the skin thickness is done by utilizing the shear flow in it. The shear center is calculated by substituting a value of zero for S_x and S_y in Eq. (1.15) and Eq. (1.16). The shear forces are assumed to act at the shear center

$$\frac{\overline{S_y}}{I_x} = \frac{S_y I_y - S_x I_{xy}}{I_x I_y - I_{xy}} \qquad (1.15)$$

$$\frac{\overline{S_x}}{I_y} = \frac{S_x I_x - S_y I_{xy}}{I_x I_y - I_{xy}} \qquad (1.16)$$

The shear flow is then calculated based on Eq. (1.17)

$$q_s = q_b + q_{s,0} \qquad (1.17)$$

In a closed section structure the ability to choose a point with a known value of the shear flow is difficult, hence, a "cut" is made to create an open section and the value of the basic or "open" section shear flow (q_b) is calculated by using Eq. (1.18). Having two spars the wing has a three-cell structure requiring three cuts.

$$q_b = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \int_0^s txds -$$
$$\left(\frac{S_y I_{yy} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \int_0^s tyds \qquad (1.18)$$

The value of $q_{s,0}$, which compensates for the cut in the closed structure, is calculated using Eq.(1.19). Were G is the modulus of rigidity, A is the area if the each cell and $d\theta/dz$ is the rate of twist.

$$\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{Gt}ds \qquad (1.19)$$

The three-cell structure requires three cuts, this produces three equations. By setting the rate of twist to zero and by solving them simultaneously the value of $q_{s,0}$ for each cell is found. To find the shear center, the summation of moment due to the shear forces about the shear center is equated to that produced from the shear flow about the shear center. After evaluating the position of the shear center, the aero-loads are applied at the aerodynamic center causing a rate of twist. The rate of twist for cells one and two form an equation and the rate of twist for cell two and three form another, both equations have three variables q_1, q_2 and q_3 therefore, a third equation is required found from the relationship between the applied torque from the lift and drag forces along with pitching moment on the shear center and the shear flow given in Eq. (1.20)

$$T = \sum_{i=1}^{3} 2A_i q_i \qquad (1.20)$$

After calculating the shear flow, the thickness is computed from Eq. (1.21)

$$q = t \times \tau \qquad (1.21)$$

The thickness is found to be t = 0.5 mm

5. Finite Element Implementation

In order to analyze the designed wing and study its reliability, the commercial finite element software COMSOL is used in order to implement and model the wing under the calculated aerodynamic loads. The geometry of the wing is constructed using the CAD software Pro-E and imported into COMSOL. The aerodynamic loads are defined and the whole model is meshed using free tetrahedral elements with a total of 11000 elements. The finite element mesh is shown in Figure 9. In order to verify that the proposed design is safe and reliable, elastic failure theories such as the Max

Stress Theory is applied in order to check that the stress in the span wise direction given in Eq. (1.12) does not exceed the yield strength of the material used in the wing structure which is aluminum for the spar-webs, wing skin and ribs while steel is used for the flanges. To simplify the model, the FE model for the wing is constructed of two steel spars with rectangular cross sections and three aluminum ribs including the skin as shown in figure 9. After applying the load it was found that the maximum span-wise stress $\sigma_z = 318$ MPa. By comparing it to the yield strength of annealed steel 4140 (361 MPa) we can conclude that the designed wing can sustain the loads according to the elastic failure theories. A maximum deflection of 6.2 cm in the out of plane direction was inflicted on the wing due to the load as shown in figure 10.
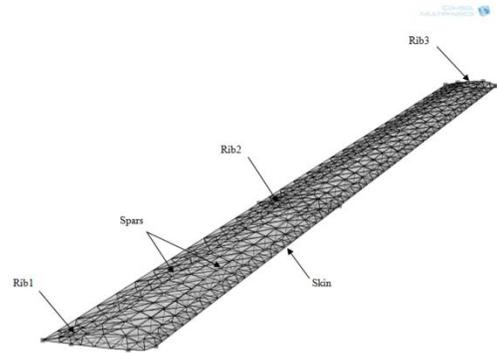

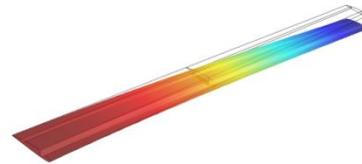
Figure 9. Finite Element Mesh

Figure 10. Wing Deflection

7. Conclusions

Designing an airplane requires time and effort and like any other designing process it usually starts with general layout followed by detailed analysis of each component. This study starts by introducing a conceptual design of a general aviation airplane, then transcends to designing the wing. The wing is the part that provides an essential asset for heavier than air flight, the lift. Sadly this lift is not free; the price of drag is a cumbersome one that accompanies lift. These two main forces in flight and the pitching moment affect an aircraft wing during its life-time; therefore it must be designed accordingly. The conceptual design provides a general shape of the wing. The paper then moves on to a more comprehensive design describing the parts of the wing and the characteristics they require to perform under the calculated loads that the wing is designed to undergo. A structural analysis was performed to calculate the geometry of the wing components based on the stresses they are expected to carry. The wing is then modeled using Pro-E and imported into COMSOL Multiphysics to perform a finite element analysis showing the wing as a respectable candidate for an airplane similar to the one introduced at the beginning of paper.

8. Acknowledgements

9. References

1. Raymer, D.P., "Aircraft Design: A Conceptual Approach", *American Institute of Aeronautics and Astronautics*, Washington D.C, 1992

2. Al-Mawahra F., Zaza, O.,"Structural Analysis of an Aircraft Wing," Thesis Submitted to the Faculty of the Jordan University of Science and Technology.

3. Anderson, Jr., J.D., "Aircraft Performance and Design", *McGraw-Hill*, 1999

4. Megson, T.H.G., "Aircraft structures for engineering students", *Elsevier Ltd*, 2007

5. Houghton, E.L., Carpenter, P.W., "Aerodynamics for Engineering Students", *Elsevier*, 2003