Particle Velocimetry Data from COMSOL Model of Micro-channels

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Abstract: Particle velocimetry using image analysis is an effective non-intrusive method used for fluid velocity field estimation in micro-channels. However, the performance of velocity estimation algorithms is usually moderated by factors like particle density and the presence of strong velocity gradients. Consequently, there exists a need to run an experiment multiple times before the actual data collection process in order to determine the effectiveness of the imaging, qualitatively understand the nature of the velocity field and check the performance of an algorithm. We use COMSOL with MATLAB to generate velocity fields for micro-channel designs and then calculate changes in particle position for given imaging parameters. This is used to simulate images which can then be used to verify an algorithm's performance before it is used with real data. Since closed-form 2-d velocity equations are not easily available for most experimental micro-channel designs, this method helps simulate particle movement based on the underlying fluid dynamics equations. Also, it enables us to change experimental factors such as particle density and physical dimensions readily and reevaluate the algorithm performance, ultimately leading to a more efficient velocity field estimation with real data.
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Keywords: PIV, microfluidics, computational fluid dynamics, simulation, velocity fields.

1. Introduction

The work in this paper addresses the creation of realistic images to be used for velocity estimation in microfluidic channels. The flow channels of microfluidic devices have dimensions on the order of 1-100μm. A wide range of diagnostic techniques have been developed for micron resolution velocimetry and one of these methods is micro particle image velocimetry (μPIV). The measurement principle for μPIV is based on PIV for large scale applications where particle groups are matched in consecutive images and the unit mean particle displacement is used for the purpose of velocity estimation [3].

1.1 Particle image velocimetry

A standard experimental set-up for PIV is shown below. The flow is seeded with tracer particles for observing the fluid motion. A double pulsed LASER is used to produce light sheets at distinct time intervals.

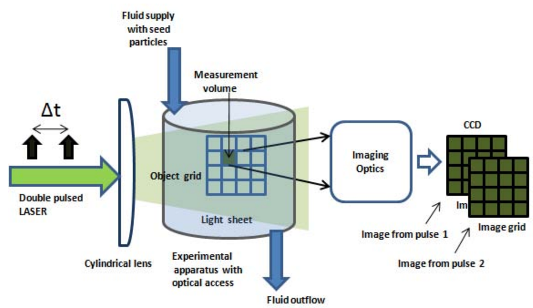

Figure 1 Block diagram for a PIV experimental procedure.

The light sheet is produced in the experimental setup which has optical access. The associated microscope optics magnify the measurement volume at the focal plane and a charge coupled device (CCD) based image acquisition technique obtains and stores the images at the consecutive pulses. In the case of μPIV, a similar set up exists but in micron resolution and with volume illumination as opposed to a light sheet.

During image acquisition the light originating from tracer particles in the focal plane of the microscope objective is collected by the optics. The position of the tracer particles is thus captured as "dots" or tiny "blobs" at one time instant t_1. A second digital image records the position of the tracers at time t_2, after an interval $\Delta t = t_2 - t_1$. During fluid motion, there would exist a difference in the positions of tracer particles for the two consecutive recordings and the tracer particles with velocity v would have shifted by a certain displacement Δs such that

$$v = \frac{\Delta s}{\Delta t} \quad (1)$$

The figure below shows two consecutive image frames obtained from a PIV experiment. The position of the cross-correlation peak gives the mean displacement of the region of interest (ROI). In actual practice, each consecutive image set is subdivided into interrogation regions which are then cross-correlated to find the mean velocity vector for that interrogation region.

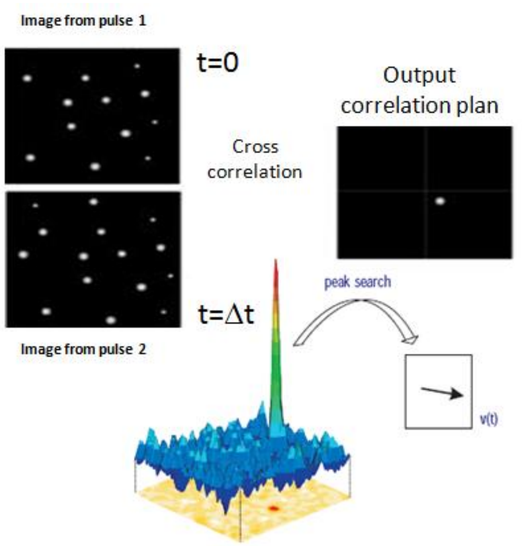

Figure 2. Displacement and velocity vector from cross-correlation of consecutive images.

1.2 Acquiring particle images from an experimental setup.

The acquisition of high quality particle images from an experimental setup is dependent on multiple factors [1]. One of the most important factors is tracer particle density. Since the method of PIV is based on correlation, uniform tracer particle density would help better estimate the flow field. At very low concentration of tracer particles, the correlation match found is often erroneous giving rise to spurious velocity vectors. Such a condition arises as the underlying velocity field is coarsely sampled by tracer particles. In some cases even though the tracer particle concentration is sufficient, the particle population becomes sparse at certain locations due to sharp velocity gradients. The size of particles is also important. While smaller particles follow the flow better, they are more affected by Brownian motion. Larger particles are less affected by Brownian motion but follow the flow less faithfully. Lastly, high speed imaging needs to be done with small exposure times failing which, velocity blurs are obtained instead of distinct particles.

1.3 Using COMSOL to generate particle images.

We study the use of COMSOL for generating realistic particle image sequences for an experimental microfluidic setup. In COMSOL, we can design a device and incorporate the physics in an ideal experimental condition environment. By simulation, we can then obtain the spatial distribution of velocity within the device. The velocity vectors can then be used in another mathematical software design environment (e.g. MATLAB) to estimate particle displacements at regular or irregular time intervals. Using this information we then simulate particle image frames which now conform to the device physics and can be used to test µPIV analysis algorithms.

2. Governing Equations

In this study, we use COMSOL to obtain particle images to estimate the flow of a non-viscous, incompressible fluid around a cylinder that is transverse to the direction of the flow. Apart from this, we assume that far from the cylinder, the flow is unidirectional and uniform while writing the governing equations. The graphic for this study is as below.

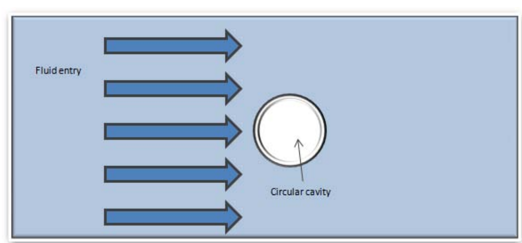

Figure 3. Fluid motion around circular cylinder

Let the cylinder be of radius R, our goal is to find the velocity V. The velocity far away from the cylinder is

$$V = V_c \hat{i} + 0\hat{j} \quad (2)$$

where V_c is a constant. At the cylinder boundary we have

$$V.\hat{n} = 0 \quad (3)$$

„n" is a vector normal to the cylinder surface. The flow is assumed to be without vortices (irrotational) and so there also exists a velocity potential ϕ

$$\nabla \times V = 0$$
$$V = \nabla \phi \quad (4)$$

The flow must also satisfy Laplace"s equation since it is incompressible i.e.

$$\nabla^2 \phi = 0 \quad (5)$$

In polar coordinates this equation becomes

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (6)$$

The solution for this with the appropriate boundary conditions is

$$\phi(r,\theta) = V_c\left(r + \frac{R^2}{r}\right)\cos\theta \quad (7)$$

Now from (4) the components of V in polar coordinates are obtained as

$$V_r = \frac{\partial \phi}{\partial r} = V_c\left(1 - \frac{R^2}{r^2}\right)\cos\theta \quad (8)$$

and

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -V_c\left(1 + \frac{R^2}{r^2}\right)\sin\theta \quad (9)$$

4. Methods and simulations

4.1 Device design and simulation in COMSOL

The ability to design a device in a simulation environment and obtain the velocity field within the device is key towards the generation of realistic particle images. Simulation in COMSOL was thus the first step in our study. In the COMSOL environment, we used water as the fluid and a simple device was designed. Laminar, incompressible flow was selected for the physics. Both the inlet and outlet velocities were maintained at 1 μm/s.

For the mesh, the triangulation method was set to Delaunay and the maximum element size was reduced to incorporate more elements. The mesh so obtained is shown below.

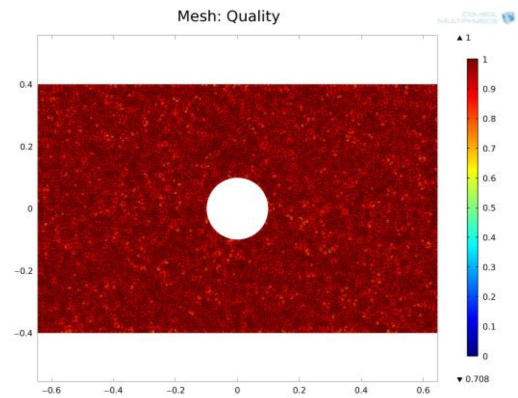

Figure 4. Triangular mesh quality

A stationary study was conducted and the solution time was approximately 12 seconds for about 50,000 degrees of freedom. Results were then plotted for the velocity field and the for streamlines that would be obtained for ideal tracers in the velocity field.

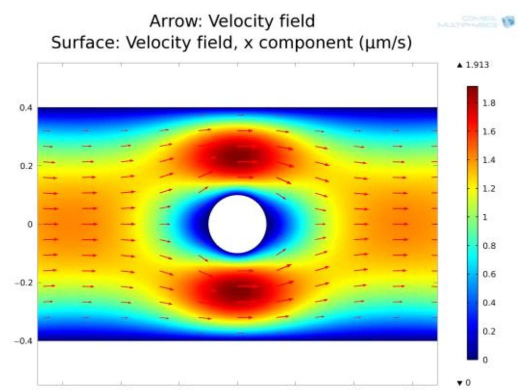

Figure 5. Velocity field x-component

The figures 5 and 6 show the horizontal and vertical component fields respectively. Note that for the selected conditions the magnitude of the y-component is smaller and so the net magnitude surface plot in figure 7 is quite similar to the x-component magnitude plot. Streamlines are shown in figure 8, the streamline positioning was start-point controlled and the „tube" line style was selected for the plot.

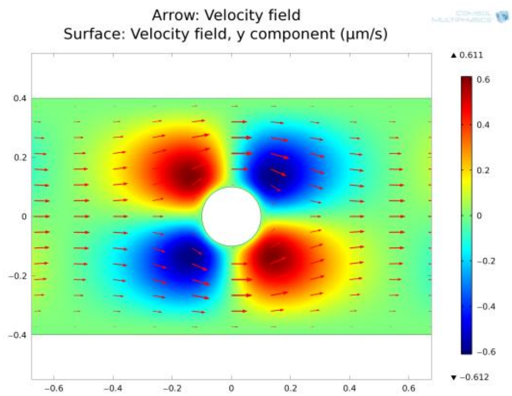

Figure 6. Velocity field y-component

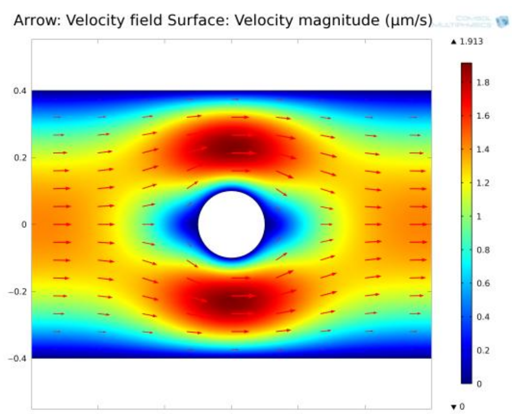

Figure 7. Total velocity magnitude.

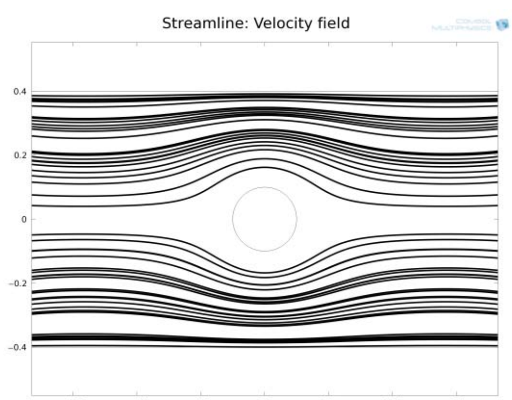

Figure 8. Streamline plot for velocity field obtained in COMSOL

Velocity fields obtained in this step are used in the next step to generate particle images under different conditions of imaging.

4.2 Importing velocity field vectors from COMSOL

We use the MATLAB-COMSOL interface to obtain the velocity field magnitudes of the horizontal and vertical components and their corresponding grid point coordinates. Since these are obtained as matrices they can be directly used in the MATLAB environment. In some cases where the COMSOL generated velocity is coarsely sampled, we interpolate to get a finer velocity resolution. The interpolation can be done in MATLAB or from within the COMSOL-MATLAB environment.

4.3 Generating particle images

The flow chart below summarizes the generation of particle images. The user inputs are shown on the left side and they control the imaging and particle density.

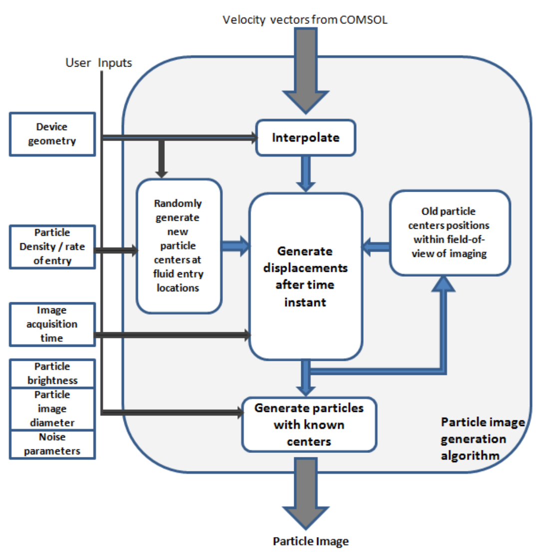

Figure 9. Block diagram for particle image generation procedure.

The individual particle displacements are based on equation 1 where Δt is also an user input. Setting Δt to be very low makes each particle closely follow the fluid velocity but requires more frames to get from inlet to outlet. At each iteration particle positions are verified to be within the actual permissible fluid path. For our current study particles moving outside the fluid path were excluded from the image. After valid displacements are generated for particle centers, each particle is created as a 2-d Gaussian profile with the spread dependant on the particle image diameter. This is a standard procedure for particle image generation for Monte Carlo methods [2].

5. Experimental Results

Image frames generated from the algorithm are shown below. The first frame shows a time instant when particles are entering the device. For this case there are no pre-existing particles within the imaging field-of-view, but we can also set the particle entry rate to zero and have random distribution of particles at the start.

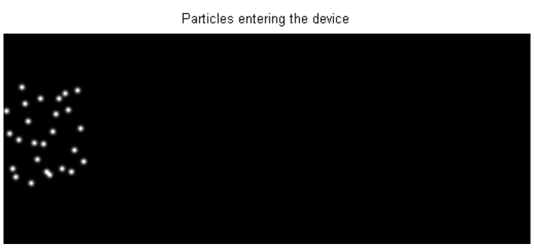

Figure 10. Particles entering from the left into the device.

Next, we show three frames for low, moderate and high tracer particle entry rate or seeding density.

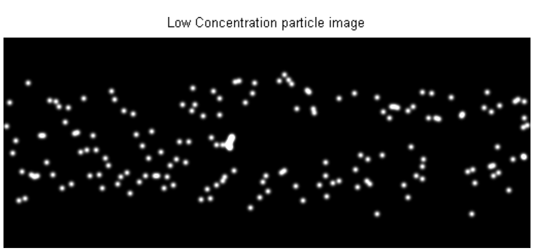

Figure 11. A low concentration particle image

It can be seen that the spatial extent is poorly sampled by tracers for the figure 11. Since tracers are normally injected in solution medium, we can estimate from simulations what concentration of particles would be good for the real experiment.

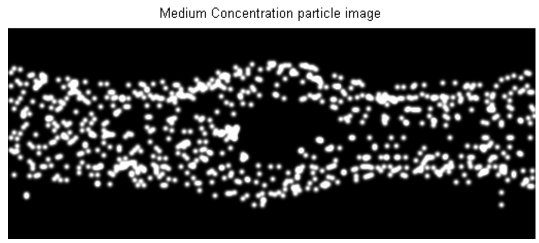

Figure 12. A medium concentration particle image. (5 times the low concentration)

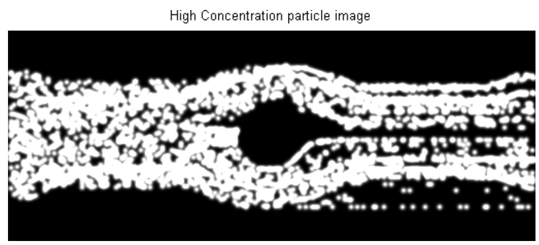

Figure 13. A high concentration particle image (10 times the low concentration)

For steady flow streamlines and pathlines traced by particles should appear similar. The pathlines obtained from an image stack are shown in figure 14 and they appear very similar to the streamlines obtained in figure 8.

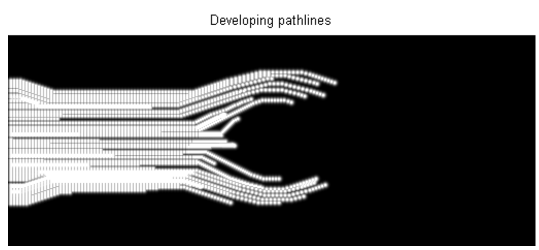

Figure 14. Developing pathlines from particle movement. (superposition after 100 frames)

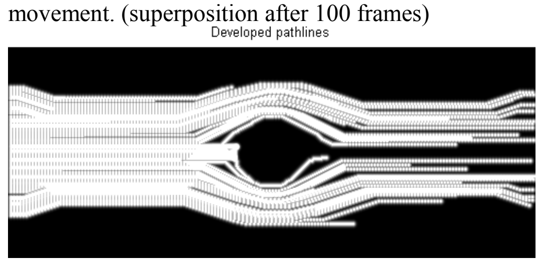

Figure 15. Superposition of particle pathlines after 200 frames.

6. Conclusions

In this work we show how COMSOL can be used for generating image sequences to be used for PIV based work. The target here is to harness the power of computational fluid dynamics to generate particle image sequences for particle velocity fields under idealized experimental conditions and for arbitrary user designed flow circuits. What we achieve by doing this is a good idea of required experimental conditions for efficient image acquisition. The experimental condtions may be particle concentration for the seed particles or their image size which is controlled by the optics. The experimental conditions can even be indirect, like voltage for an electrophoresis experiment which can appear as scale factors for the magnitude of the velocity field.

The proposed method for particle image generation would also help in designing better algorithms for μPIV. For most velocity field estimation algorithms based on imaging there is always a requirement for gold standard images with known velocities and experimental parameters. The images generated by our method would always have a known reference for comparison of algorithm performance while the devices and velocity fields can be as desired by the user.

7. Future Work

The work described in this paper proposes the particle image generation method using COMSOL and shows the results obtained for a well known fluid dynamics problem under ideal conditions. Our current directions are towards dynamically changing physical dimensions and positions of obstacles and obtaining images for such scenarios. Moreover the effect of particle size needs to be taken into account for predicting the motion of particles.

8. References

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3. Santiago, J.G., Wereley, S.T., Meinhart, C.D., Beebe, DJ and Adrian, RJ, A particle image velocimetry system for microfluidics, *Experiments in Fluids*, **25**, 316--319 (1998).