Advancements in Acoustical Topology Optimization

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Abstract
Acoustical topology optimization has reached a point where it is applicable to industrial product development cases. With topology optimization, solutions can be found that would be impossible with traditional engineering methods, even if standard simulations are included in the approach. The technology has been refined here, and new methodologies are demonstrated via examples to showcase how for example 'aesthetics' constraints, which are not present in the underlying geometry or physics, can be imposed for the topology optimization domain using various mapping techniques and mathematical operations.
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Introduction
Topology optimization is one type of geometry optimization, and in contrast to parameter or shape optimization, topology optimization arguably offers the freest form of geometry modification, allowing for highly creative and non-intuitive solutions. One downside of this, however, is that while a geometry might result from the optimization that meets all targets well, it might not be possible to deduce its actual inner workings.

Topology optimization is still a relatively new technique with most of the defining work done within the last 50 years, with the focus being on structural mechanics applications. Acoustical topology optimization has had even fewer years to mature, and has for now remained mainly an academic exercise, whereas structural topology optimization has begun to creep into the industry.

The design of products in the audio industry is to a large extent done via a combination of prior knowledge and trial-and-error. Such an approach will sometimes be seen described as an 'optimization' of said products, but we will here focus on the formal mathematical approach of topology optimization with the underlying physics being acoustics.

In recent years, COMSOL has evolved its optimization module to a point where it is now possible to move topology optimization from academia into the industry. Acculution is now designing audio components for clients purely via acoustical topology optimization, and this is seemingly a first in the audio industry. When using the technique in the industry, the approaches described in the academic literature will typically not suffice, and we will touch upon some of the issues associated with industrial cases.

It will be demonstrated how acoustic topology optimization can result in designs that trial-and-error approaches could never achieve, as the solution space is too vast. The designs that emerge from topology optimization can meet targets across both a wide range of frequencies as well as across spatial angles, such as for controlling acoustic radiation patterns and associated pressure levels.

In some industrial cases, the physics setup related to the topology optimization may not have the desired symmetry axes that are sought in the final optimized design, so it is also demonstrated how auxiliary design constraints, such as extra mirror planes or repeating, possibly scaled, patterns can be implemented in the optimization setup without them being explicitly included in the physics setup.

Acoustical Topology Optimization Theory
For acoustical topology optimization some domains of the total geometry will be handled as standard acoustics with material data applied directly, whereas other domains will have their material values dictated by a design variable field defined for the latter domains only. For all domains the Helmholtz equation assuming harmonic time dependency is solved. The density ρ, the bulk modulus K, and the angular frequency ω enter the equation as

$$\frac{1}{\rho}\nabla^2 p + \frac{\omega^2}{K} p = 0$$

The equation is here written in homogeneous form without any sources, as this is the typical case with the excitation being present via boundary conditions. For the pure acoustics domains, the density is set to $1.2\ \mathrm{kg/m^3}$, and the bulk modulus is $1.82\ \mathrm{kPa}$. In the optimization domains, however, these material values are dictated by the design variable ξ, and for the present cases these values were described via a RAMP interpolation scheme [1] as

$$\rho(\xi) = (\rho_0^{-1} + \xi(\rho_{Hi}^{-1} - \rho_0^{-1}))^{-1}$$

and

$$K(\xi) = (K_0^{-1} + \xi(K_{Hi}^{-1} - K_0^{-1}))^{-1}$$

Here, the zero subscript indicates the values mentioned above, whereas the 'Hi' subscript indicates a much higher value. As In the extremes, the resulting fluid will either be air for $\xi = 0$ or some very dense and very incompressible fluid for $\xi = 1$, and the latter is intended to emulate a rigid structure, leading to a large impedance jump at the interfaces between these binary values. That way, the design variable field can be exported as a mechanical structure by only including design values above a certain value, typically 0.5.

The interpolation scheme is such that it penalizes design variables that are not close to the extremes of 0 or 1, but still the final design variable field is likely not to end up being binary. Additional filtering can further help towards this goal of sharp interfaces between 'air' and 'structure'. First, the initial design variable field is defined such that it can vary continuously in the optimization domains Ω_d with the design variable ξ_c as

$$0 \leq \xi_c(\boldsymbol{x}) < 1 \qquad \forall \boldsymbol{x} \in \Omega_\mathrm{d}$$

Next, an optional filtering can be applied to this field to mitigate certain mesh dependencies via a Helmholtz-like filter strategy [2] via ξ_f as

$$-r\nabla^2\xi_f + \xi_f = \xi_c, \qquad \frac{\partial \xi_f}{\partial n} = 0$$

Finally, an optional projected design variable ξ_p with the purpose of further promoting binary designs is found via the filtered design variable [3] as

$$\xi_p = \frac{\tanh(\beta\eta) + \tanh(\beta((\xi_f - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))}$$

This projection filter pushes the design variable towards the extremes of 0 and 1 around some projection point η, typically 0.5, with a projection slope β dictating how significantly the projection is applied. A continuation strategy is utilized where β is ramped up gradually.

Even with these filtering strategies the resulting field is likely not binary, and this will be touched upon later under the Testing Final Designs section.

An objective function to be minimized combining all desired targets t_n with each their weight w_n is defined as

$$\min_{\xi}: \Phi\left(\sum_n w_n t_n\left(p(\boldsymbol{x}, \xi_p)\right)\right)$$

The objective function Φ contains targets that are typically dependent on sound pressure p, for example in terms of sound pressure levels, intensities, particle velocities, or combinations thereof, across a certain frequency range and with spatial dependencies, such as for example on-axis sound pressure or directivity in general. More details about acoustical topology optimization are found in the references [4], [5], [6], [7].

Implementation

The optimization setup will typically utilize the Pressure Acoustics, Frequency Domain Interface along with the Topology Optimization Interface. The described interpolation scheme and filters for the design variables are directly available in the Topology Optimization node, but may alternative be defined manually. The MMA method is used in the optimization solver, although alternatives have appeared in recent versions. Typically, the projection value β is ramped up during the optimization. Also, the optimization routine may stall for certain value combinations, so one might need to watch the optimization progress and adjust parameters such as thresholds and move limits to get to a final geometry, and this manual approach will differ from case to case.

It should be noted that for topology optimization, connectivity is not ensured, and so there can be parts 'floating' in free air. A new so-called 'Milling' constraint is now available in COMSOL Multiphysics, but that may be overly restrictive for some designs, as one can imagine a single connected part that cannot be milled. For client cases, one needs to discuss how disjointed parts can be connected without disrupting the overall functioning of the optimized part, but it is expected that future versions of the software will have alternative approaches towards connectivity.

Incentive: Tweeter Example

To illustrate the power of acoustical topology optimization, a lens has been made for tweeter, which is a high frequency transducer found in most loudspeakers. In its initial state, there is circular symmetry in geometry, boundary conditions, and fields, so that the acoustical radiation will be independent of any rotation around the tweeter axis. However, it is now attempted to have tweeter 'play upwards', so that the initial on-axis pressure is instead to be found at a vertical angle of 20 degrees. This could be a relevant objective if the tweeter is mounted in a car or a ceiling loudspeaker, to steer the sound towards the user while retaining a certain mounting/installation. A topology optimization routine was set up, and the resulting optimized geometry is seen alongside the tweeter in Figure 1.

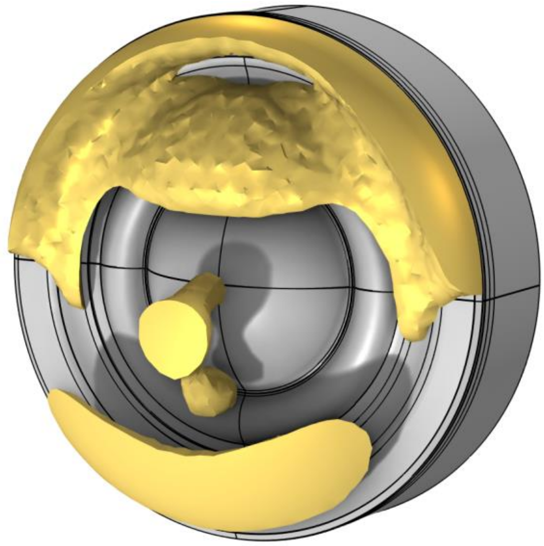

Figure 1: A tweeter has been fitted with a topology optimized lens.

The initial and optimized directivities in vertical direction are shown in Figure 2. It is noted how the sound field has been modified so that at 20 degrees vertical a flat response is found. This would have been a difficult case to solve using traditional engineering methods, as the objective spans a wide range of frequencies and spatial angles, and yet the topology optimized structure is found in half an hour.

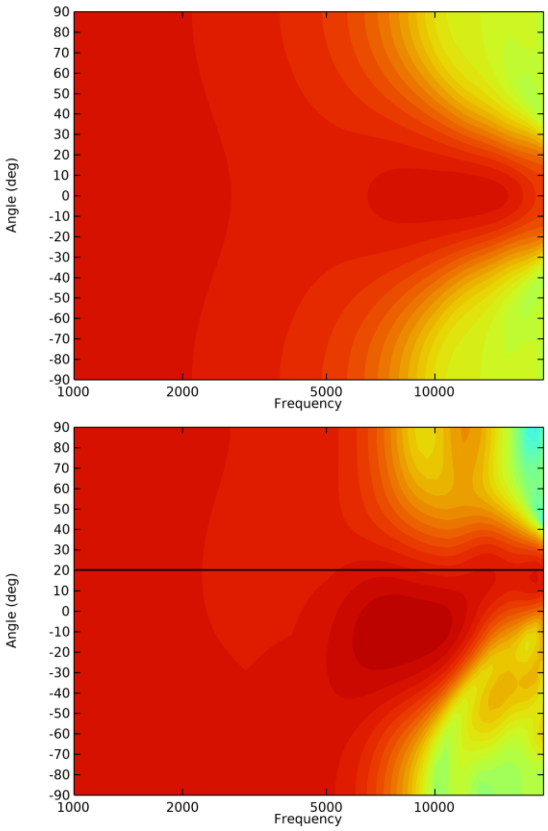

Figure 2: Initial tweeter vertical plane directivity (top) and optimized ditto (bottom) with a line at 20 degrees.

While having refined the underlying topology optimization routine over several years towards product development in the audio industry, additional challenges have presented themselves via client requirements, and as these are not handled in academia, new techniques are described here.

Auxiliary Constraints

Certain geometrical constraints can be needed for client cases that are not readily available in the simulation setup. This can be due to aesthetics, manufacturing, mounting, or for other reasons. It will be demonstrated now what these constraints could be and how they can be implemented using various mapping techniques utilizing the Projection and Extrusion operators in COMSOL Multiphysics alongside mathematical methods.

Affine Transformations Constraints

Affine transformations are a special case of transformations that preserve line parallelism. It includes the cases Translation, Reflection, Scaling, Rotation, and Shear via matrix operations. An example of a Reflection transformation would be to impose a mirror symmetry for an optimization domain, such that horizontally and vertically the geometry will look the same. This is illustrated in Figure 3 where a waveguide lens has been designed for acoustics with two mirror planes, so that a quarter geometry is used, but with the topology optimization domain having an added 45-degrees mirror plane. Without this constraint, one is likely to end up with a design with only quarter symmetry, even when the objective function targets equality between horizontal and vertical variables. With the topology optimization ended, a lens is achieved with three mirror planes with the waveguide having only two, and this lens can subsequently be mirrored into a full 3D geometry.

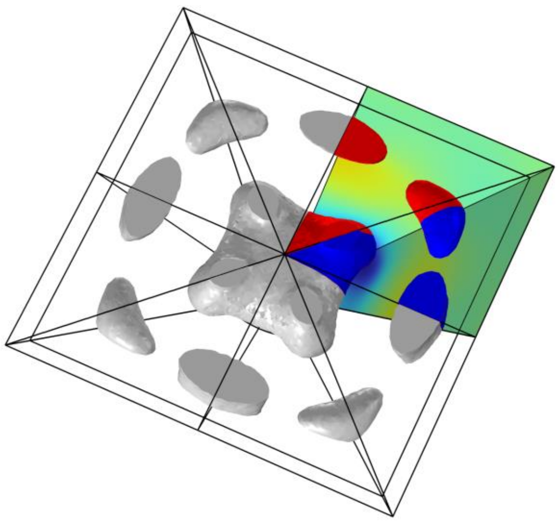

Figure 3: A square waveguide with quarter symmetry and a mirror constrained acoustic lens. The blue parts are the design variables that map to the red parts, with the grey parts completing the lens.

Multiple affine transformation can be combined to achieve some desired constraint. This is demonstrated with a rectangular waveguide where the shape in horizontal direction is required to be retained in vertical direction in a scaled sense. This can be achieved via scaling and mirror operations (remembering that matrix multiplication is not commutative), and a result can be achieved such as the one shown in Figure 4 for a rectangular waveguide, with a resulting acoustic lens.

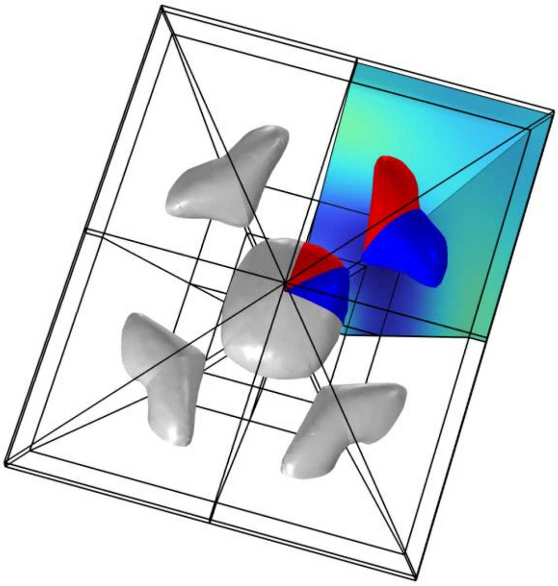

Figure 4: A rectangular waveguide with quarter symmetry and a scale-mirror-scale constrained acoustic lens. The blue parts are the design variables mapping to the red parts, with the grey parts completing the lens.

It should be noted that while the rectangular waveguide only has 2 mirror planes, one can of course still try to force symmetry in the horizontal and vertical directivity via the targets, while the scale-mirror-scale constraint is aimed at aesthetics without targeting a particular directivity. This horizontal/vertical directivity target could be tried with or without the above scale-mirror-scale constraint, where the former option will be more demanding than the latter.

Finally, affine transformations can also be used to impose repeating patterns in the optimized geometry. This is illustrated in Figure 5, where an acoustic channel without any symmetries has a repeating pattern in a topology optimized structure at its output, while deeper in the geometry the optimization has no geometry constraints and varies even along the height dimension. Note how the acoustic pressure is not repeating, only the optimized structure. This illustrates a potential client requirement where, to a certain depth, the user must not see anything other than a particular pattern.

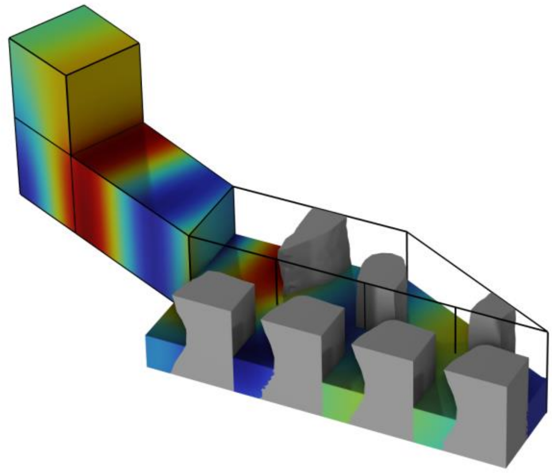

Figure 5. A repeating pattern has been obtained at the output of an acoustic conduit for the topology optimized structure (grey).

General Transformations Constraints

Affine transformations do not suffice for all geometry requirements, and so in COMSOL Multiphysics more general couplings are available that can map inputs to more geometrically complex outputs via user input expressions. One can map between different dimensions, with inherent integrations when mapping from a higher dimension to a lower, or alternatively, mappings can go from a lower dimension to a higher. The latter can for example be utilized to enforce a circular symmetry for the topology optimized structure, without the underlying geometry having such symmetry. The strategy here is to define the topology optimization node in 2D and apply a one-way mapping to the 3D physics, where the acoustical material parameters for the 3D optimization domain take design variable input from the 2D domain.

This is demonstrated by a rectangular waveguide geometry that has two symmetry planes for the geometry, but not necessarily for the boundary conditions and field variables. Nevertheless, a circular symmetry constraint can be enforced for the topology optimized geometry, as is illustrated in Figure 6. The full waveguide is depicted here, but the simulation setup could instead take advantage of half or quarter symmetry, if these are present across geometry, boundary conditions, fields, and targets.

A circular symmetry constraint is quite restrictive when it comes to achieving the desired targets, and in general one should first explore which results are achievable without any auxiliary constraint.

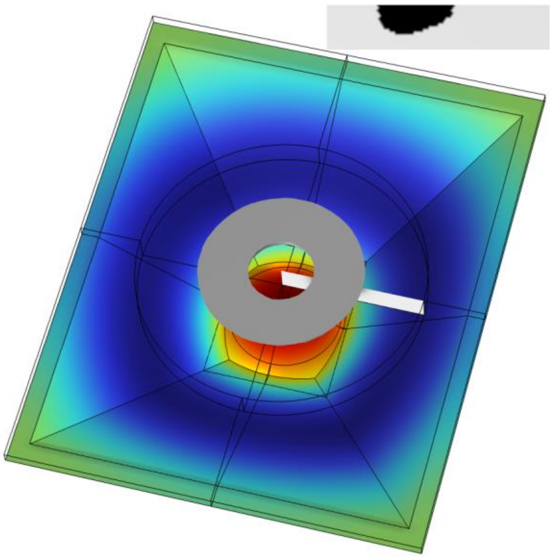

Figure 6: Circular rotation constrained lens for a rectangular waveguide, with the 2D optimized section.

For the rectangular waveguide in question, an ‘elliptical symmetry’ might be more visually pleasing, and this is also possible via a slight modification to the mapping expressions. The result is shown in Figure 7.

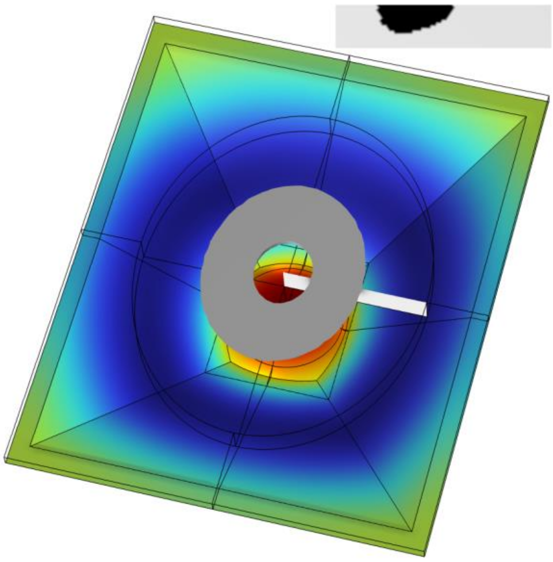

Figure 7: Elliptical rotation constrained lens for a rectangular waveguide, with the 2D optimized section.

Equation-Based Constraints
COMSOL Multiphysics has a wide range of logical expressions that can also be utilized in the topology optimization setup. These can be used for example to control parameters that are normally fixed, such as solver limits, filter parameter, or domain conditions, with certain variables varying as the solver progresses to for example force emphasis on certain parts of the optimization domain. This way the material distribution and/or the filtering taking place can be dynamically controlled within a single study.

Similarly, the governing equations are also accessible in COMSOL Multiphysics, and that allows for alternative setups that are otherwise not possible in the default setup. An example is the current limitation for acoustical topology optimization that the specialized Exterior Field calculation can be included in the objective function with up to 3 mirror planes present, but only if they reside in the XY, XZ, or YZ planes, so that the selected Exterior Field boundary will include at least one eighth of a larger surface that could enclose the full 3D geometry. However, the underlying Kirchhoff-Helmholtz Equation can oftentimes be split into parts, so that even when the selected Exterior Field boundaries are technically not compliant with the above requirements, they can still be used for returning the acoustic field from the missing Exterior Field boundaries by proxy. This way, mirror planes or repetitions can be added via manipulating the equations, without having the geometry explicitly included. Such a setup is seen in Figure 8. Instead of having two mirror planes and a mirror mapping at 45 degrees as for the case in Figure 3, now only 1/8th of the full 3D waveguide is solved for with the Exterior Field equations modified to implicitly include a 45-degrees mirrored contribution to a quarter symmetrical waveguide, which subsequently is mirrored into a full geometry.

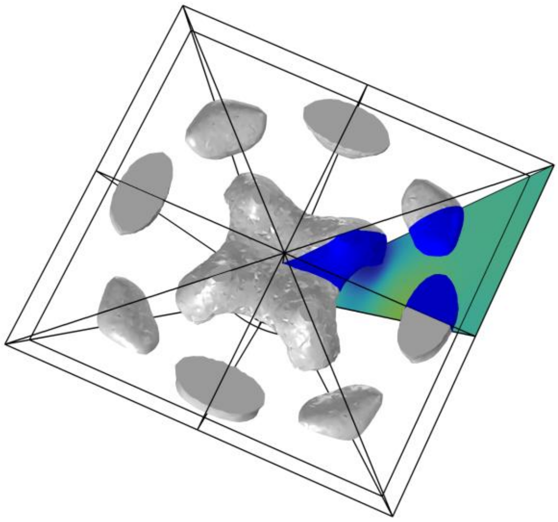

Figure 8: A square waveguide using 1/8th of a full 3D geometry with an equation-based mirror constrained acoustic lens. The blue parts are the design variables, with the grey parts completing the lens.

This obviously reduces computational time significantly as the geometry has been halved compared to the mirror-mapping constraint setup. The resulting geometry can now be mirrored and rotated to achieve the full 3D geometry to be manufactured. It is also noted that the design is very similar to the previous one in Figure 3, but with small differences, due to general sensitivities in the optimization setup. The above equation-based approach will fully suffice, unless the boundary conditions or fields (or targets) are in fact quarter symmetrical only, without having the added mirror symmetry at 45 degrees.

As a final note it should be mentioned that the constraints shown in this work will most often make the optimization problem more difficult to solve. For example, if only horizontal directivity is included in the objective function, then vertical directivity can be sacrificed to redistribute the energy between the horizontal and vertical planes. However, if at the same time an auxiliary constraint is present that forces similarity between the horizontal and vertical parts of the optimization geometry, this will obviously hinder this redistribution.

Testing Final Designs

Once a somewhat binary optimization field has been achieved, it should be tested how well this approximated via having hard walls at the binary interfaces. The field can be exported as a mesh file and then be imported into a second component. However, this approach will often entail manual manipulation of the field mesh and the existing mesh from the non-optimized geometry, as these will generally be incompatible as shown in Figure 9.

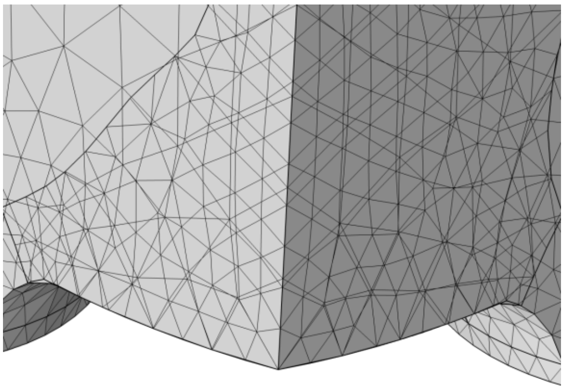

Figure 9: Incompatible and overlapping meshes because of field/mesh export of optimized design and subsequent import into the initial geometry.

Having to join different meshes will often be problematic, as tolerances in the existing domain represented via geometry are different than domains represented via meshes. Also, the manual work involved must be done for each new design, and a joining strategy that worked for one optimized design might not work for the next design to be tested.

To circumvent these issues with mesh imports and repairs, Acculution instead employs a technique where the final optimized field is forced to be binary with a certain threshold value for the split, and then the associated resulting density and bulk modulus are used to emulate the hard walls via this binary field in a secondary study. It is after all the goal of the optimization setup to achieve a functioning design by achieving a binary field to emulate the hard wall interfaces anyway, but instead of strictly achieving this binary field, it is subsequently enforced and tested as is. While the original mesh can be used directly in the binary field technique, a new mesh with refinements and changes to the element order is an option.

This approach has been compared to the mesh export approach, and the results are very similar. The ‘enforced binary’ setup can also include a parameter sweep over the threshold value to quickly evaluate the robustness of the design [5].

Test Case: Waveguide

For a previous case without any symmetry planes, an objective function involving a more homogenous horizontal acoustic radiation in a 90 degrees directivity window is utilized. The geometry is shown in Figure 10, where the black boundary has a normal acceleration boundary condition, and the grey domains constitute the resulting topology optimized geometry with a COMSOL logo included both in the initial state and the optimized ditto.

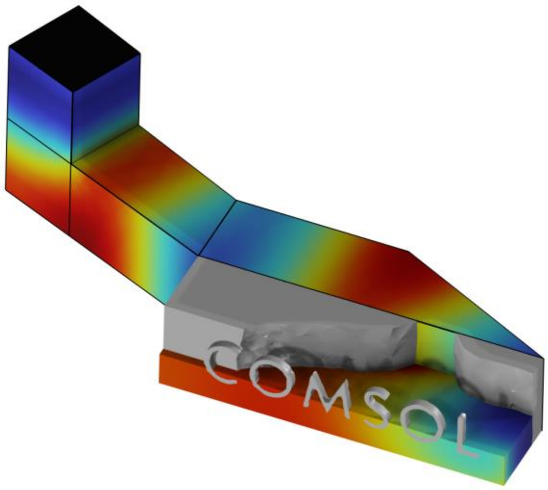

Figure 10: The black boundary has an acceleration input with the gray domains showing the topology optimized design.

The horizontal radiation pattern is shown for a few frequencies in Figure 11 for the initial and the optimized (via subsequent ‘enforced binary’ testing) geometry, and it is seen how the pattern is improved after the optimization.

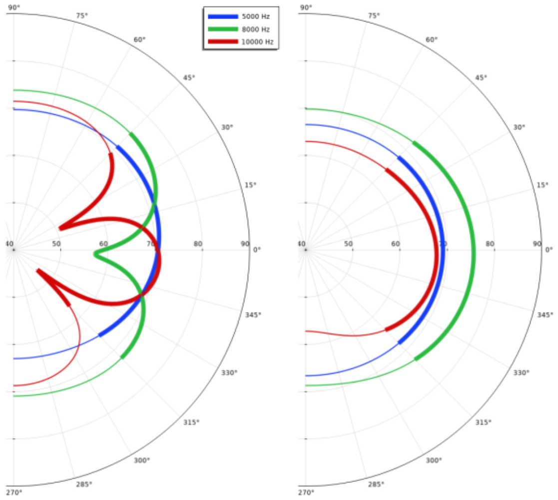

Figure 11: The horizontal radiation pattern before (left) and after optimization (right) for three frequencies with the focus being on ± 45 degrees (bold lines).

A different view is seen in Figure 12, where the sound pressure levels before and after optimization are depicted at several horizontal angles.

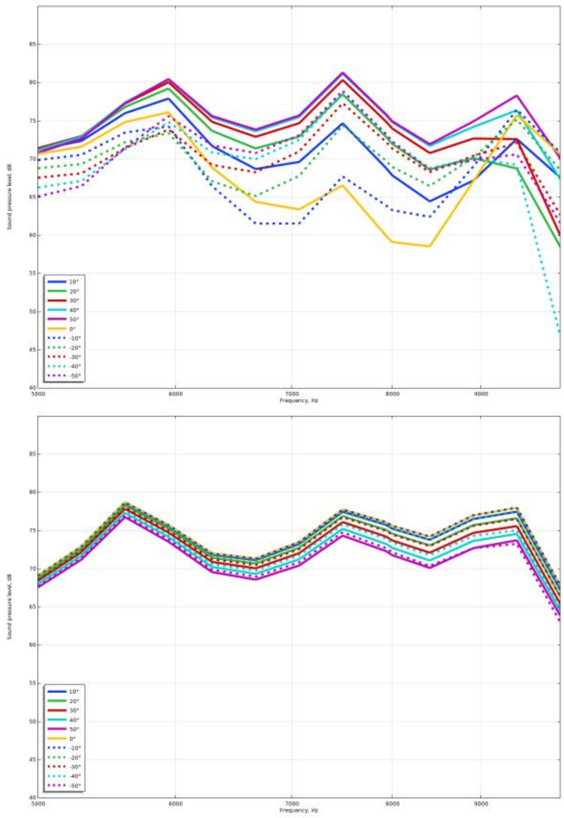

Figure 12: Sound pressure levels at certain horizonal angles before (top) and after (bottom) optimization.

The sought-after targets have been met satisfactorily, and so waveguide design in cars, mobile phones, or similar, is just one branch of acoustic product development, where this topology optimization technique can be extremely useful.

Conclusions

It has been shown how acoustical topology optimization has reached a maturity level allowing for products to be designed directly using this technology. The results will most-often never be achievable with any other technique due to the combinatorics involved with vast solution spaces across the frequency domain and the spatial ditto, and yet, once a robust optimization setup has been established, extremely complex solutions can be found in a matter of hours.

New methods have been developed that further support the requirements from the industry, such as enforced aesthetics or mounting conditions, and it has been demonstrated how to achieve this via mapping methods and mathematics manipulations in COMSOL Multiphysics.

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