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Analysis of Super Imaging Properties of Spherical Geodesic Waveguide Using COMSOL Multyphisics

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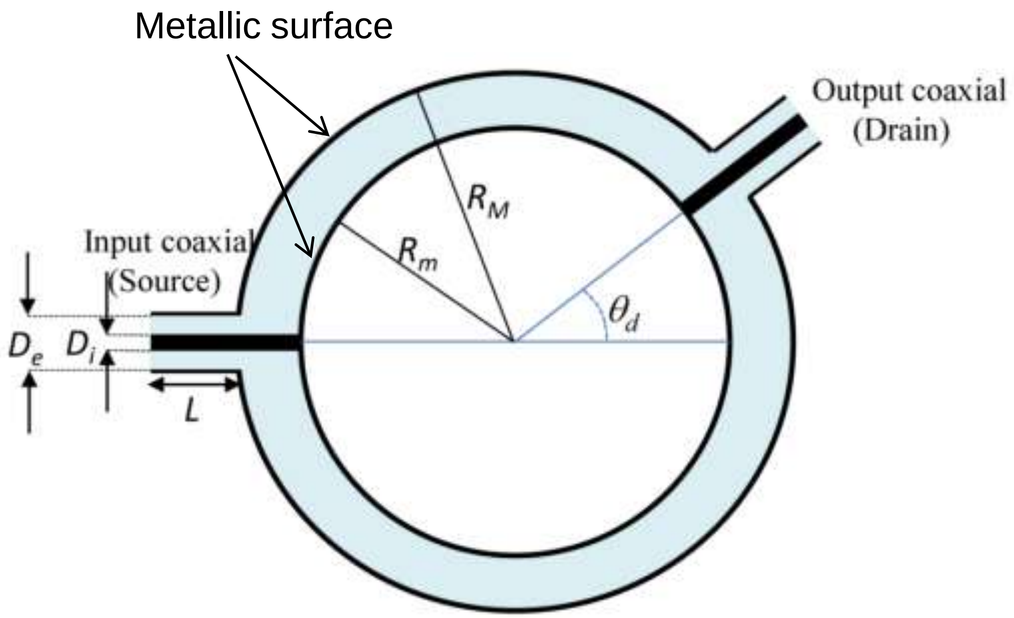
Outline

1. Introduction
2. SGW for extended source and drain
3. Simulations in COMSOL Multiphysics
4. Conclusions

Spherical Geodesic Waveguide

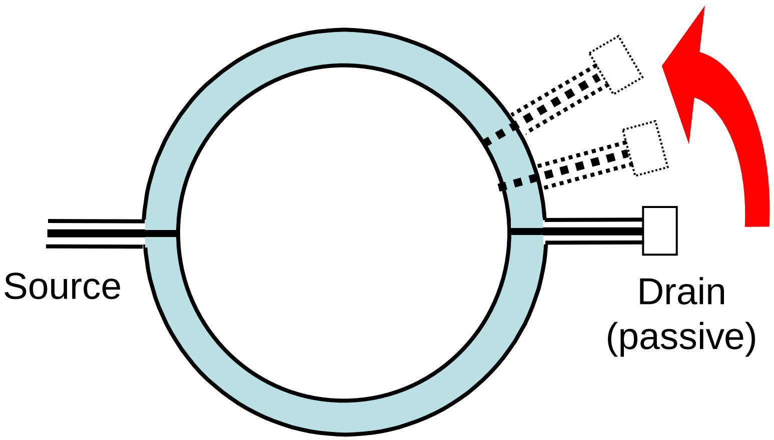


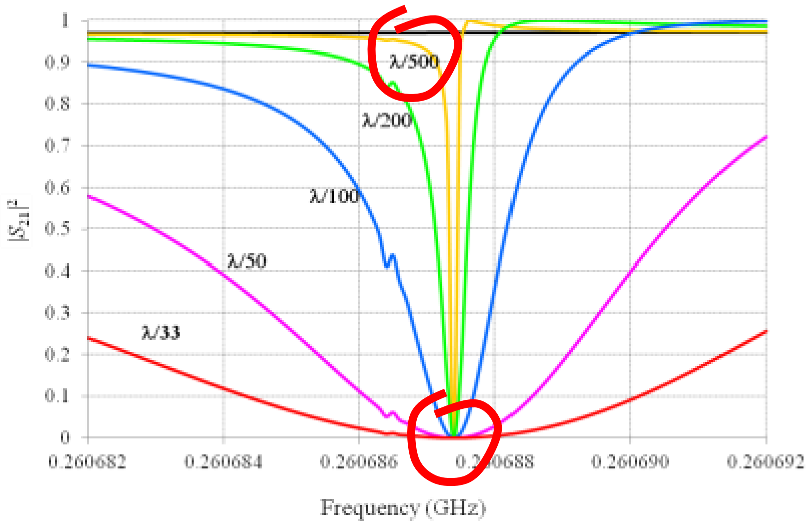
$$D_e = 10 \text{ mm} \quad D_i = 5 \text{ mm} \quad L = 20 \text{ mm}$$

$$R_M = 1005 \text{ mm} \quad R_m = 1000 \text{ mm}$$

$$n(r) = R_M / r \approx 1$$

Simulation of λ/500 super-resolution for SGW





[J.C. Miñano et al, *New Journal of Physics*, **13**, 125009 (2011)]

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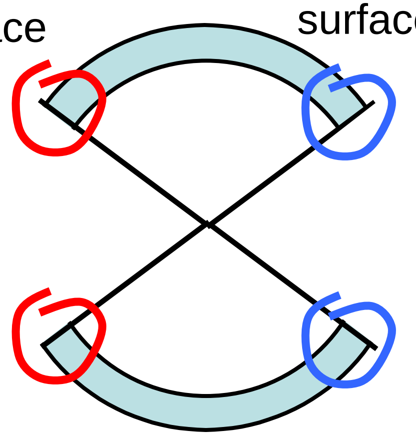
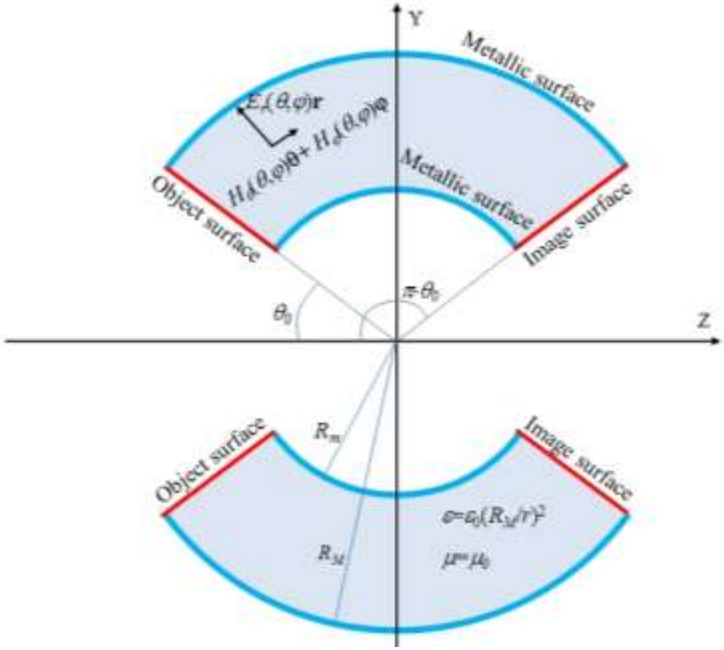
SGW for extended source and drain

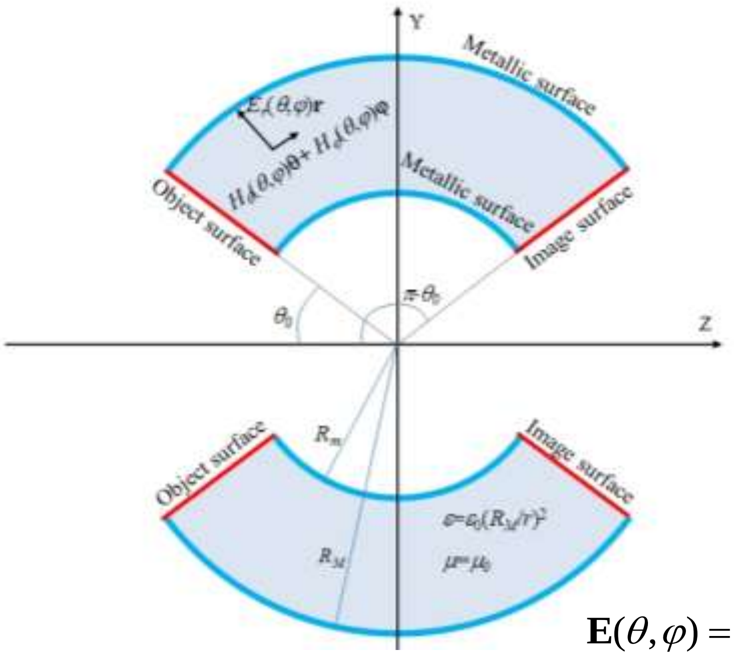




$$\theta_0 = \frac{\pi}{5}, \ \lambda = 0.931138 \text{ m}$$

$$R_M = 1.005 \text{ m}, \quad R_m = 1 \text{ m}, \quad n(r) = R_M / r \approx 1$$

Electric and magnetic fields in the SGW



$$\mathbf{E}(r,\theta,\varphi) = E_r(\theta,\varphi)\mathbf{r}$$
$$\mathbf{H}(r,\theta,\varphi) = H_\theta(\theta,\varphi)\boldsymbol{\theta} + H_\varphi(\theta,\varphi)\boldsymbol{\varphi}$$

$$\mathbf{E}(\theta,\varphi) = \sum_n E_{rn}(\theta)e^{jn\varphi}\mathbf{r}$$

Solution

$$\mathbf{E}(\theta,\varphi) = \sum_n \left[AP_\nu^n(\cos(\theta)) + BQ_\nu^n(\cos(\theta)) \right] e^{jn\varphi}\mathbf{r}$$

$$\upsilon(\upsilon+1) = (k_0 R_M)^2$$

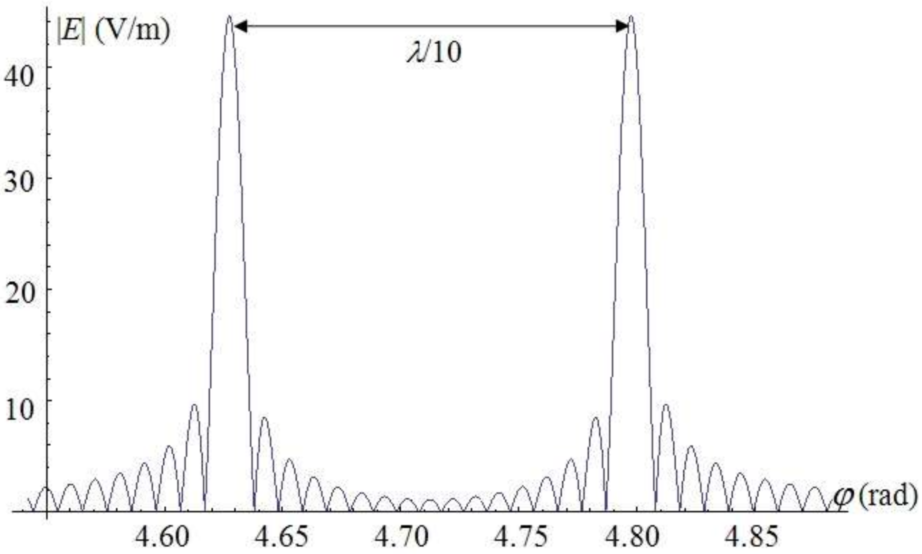
$$\mathbf{E}(\theta,\varphi) = \sum_n \left[E_{fn}F_\nu^n(\cos(\theta)) + E_{rn}R_\nu^n(\cos(\theta)) \right] e^{jn\varphi}\mathbf{r}$$

Perfect drain in the image surface to absorb all the modes

$$F_\nu^n(\cos(\theta)) = P_\nu^n(\cos(\theta)) + j\frac{2}{\pi}Q_\nu^n(\cos(\theta))$$

$$R_\nu^n(\cos(\theta)) = P_\nu^n(\cos(\theta)) - j\frac{2}{\pi}Q_\nu^n(\cos(\theta))$$

Imaging in the SGW



Electric field in the object surface

Electric field is represented by 2000 modes

$$\mathbf{E}(\theta_0,\varphi) = \sum_n A_n e^{jn\varphi}\mathbf{r}$$

Boundary condition in the object surface

$$\sum_n A_n e^{jn\varphi} = \sum_n E_{fn} F_\nu^n(\cos(\theta_0)) \Longrightarrow E_{fn}$$

Consider field in the image surface as

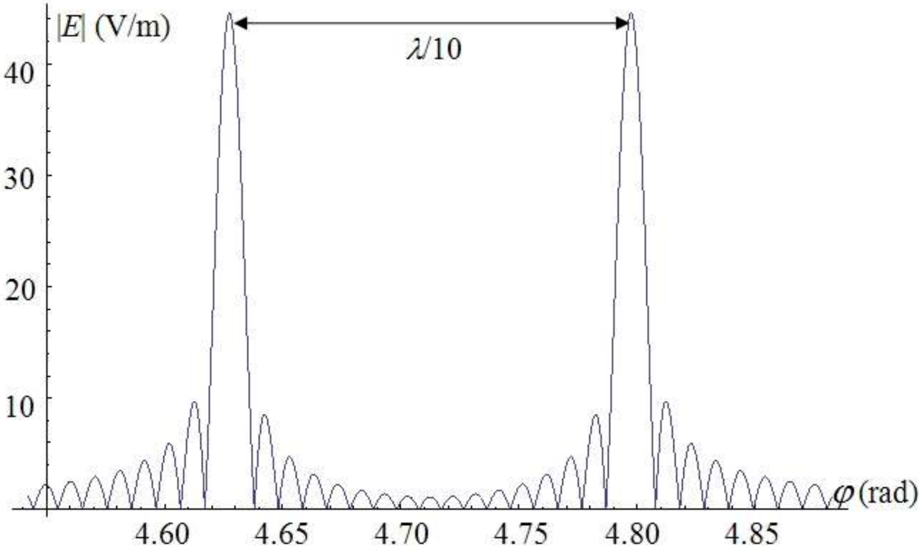
$$\mathbf{E}(\pi-\theta_0,\varphi) = \sum_n B_n e^{jn\varphi}\mathbf{r}$$

Boundary condition in the image surface

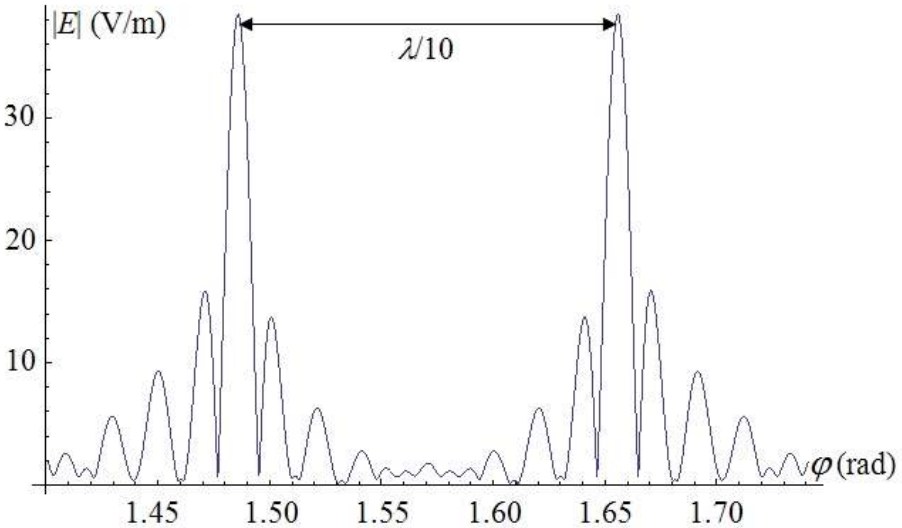
$$\sum_n B_n e^{jn\varphi} = \sum_n E_{fn} F_\nu^n(\cos(\pi-\theta_0)) \Longrightarrow B_n$$

[J.C. González, J.C Miñano, P. Benítez, D. Grabovickic, *ArXiv*, 1204.2672v1 (2012).]

Imaging in the SGW

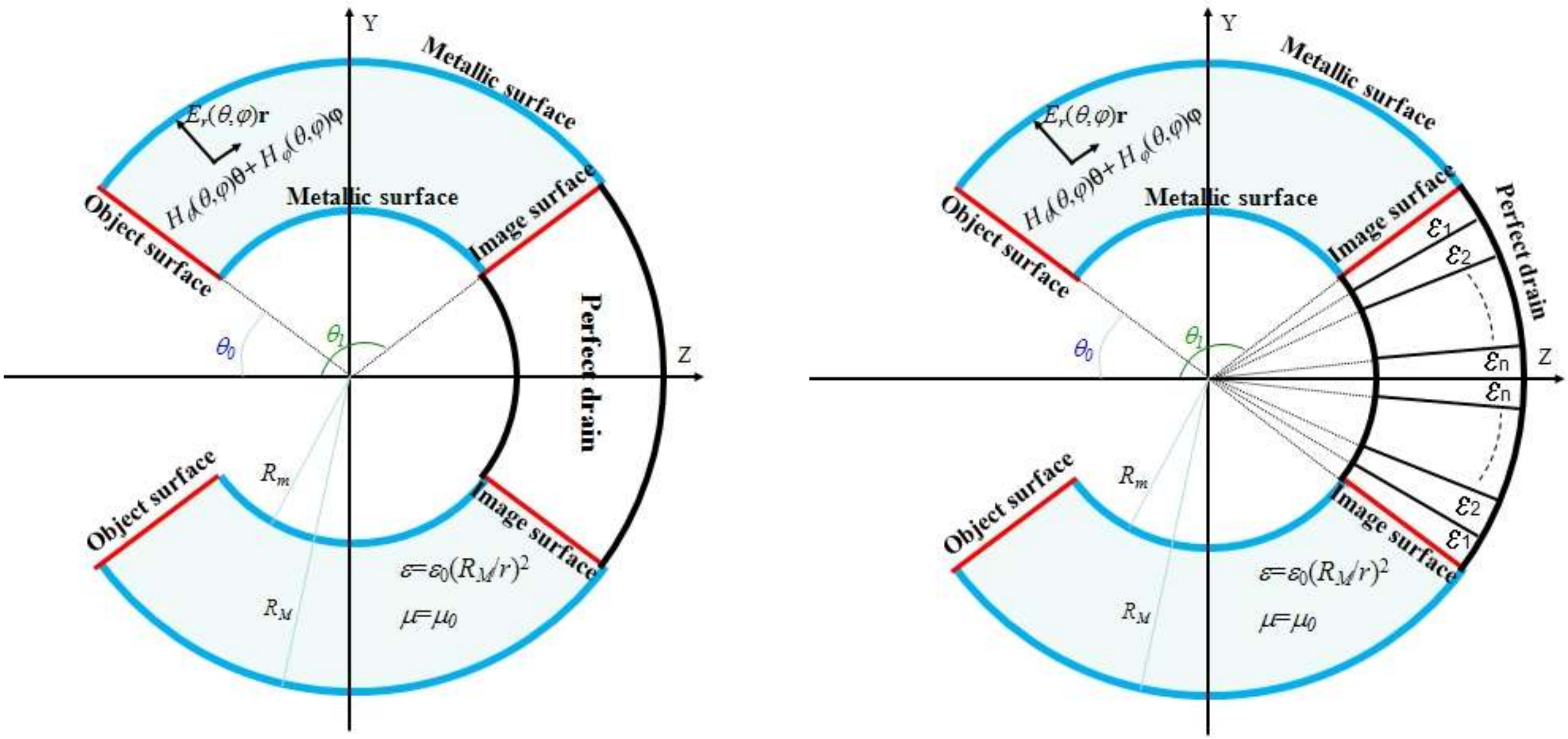


Electric field in the object surface

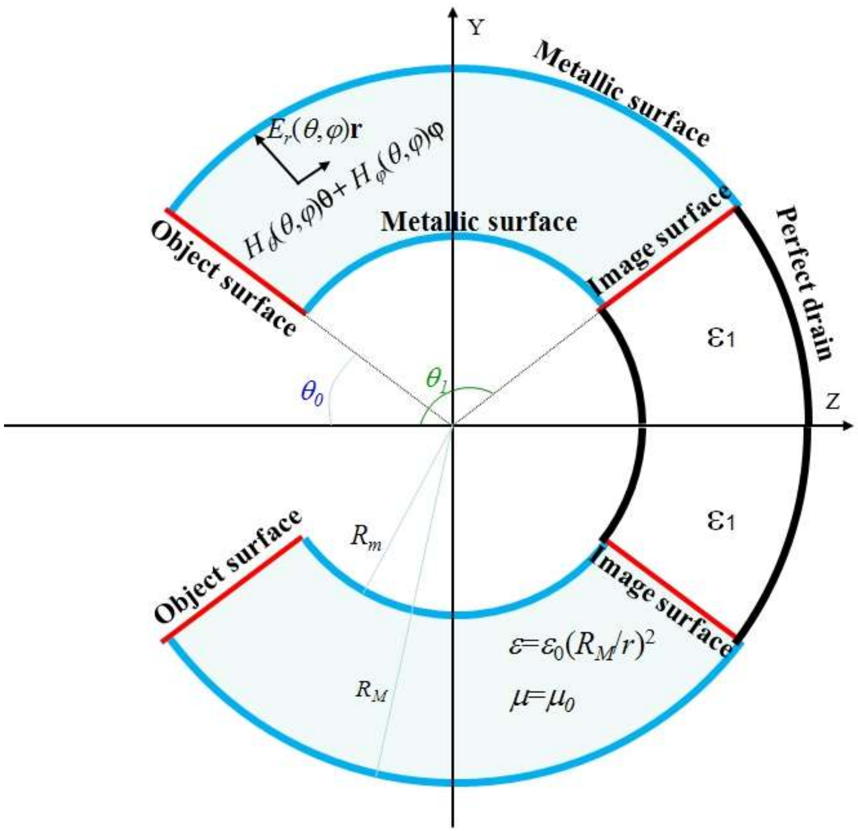


Electric field in the image surface

SGW with perfect drain



SGW and one-layer perfect drain



Electric field in the drain

$$S_\nu^{n_1}(\cos(\theta)) = P_\nu^{n_1}(\cos(\theta)) + AQ_\nu^{n_1}(\cos(\theta))$$

Finite field at θ=π $\qquad A = -2\tan(\pi\nu)/\pi$

The equality of the electric field

and tangential magnetic field at θ_1

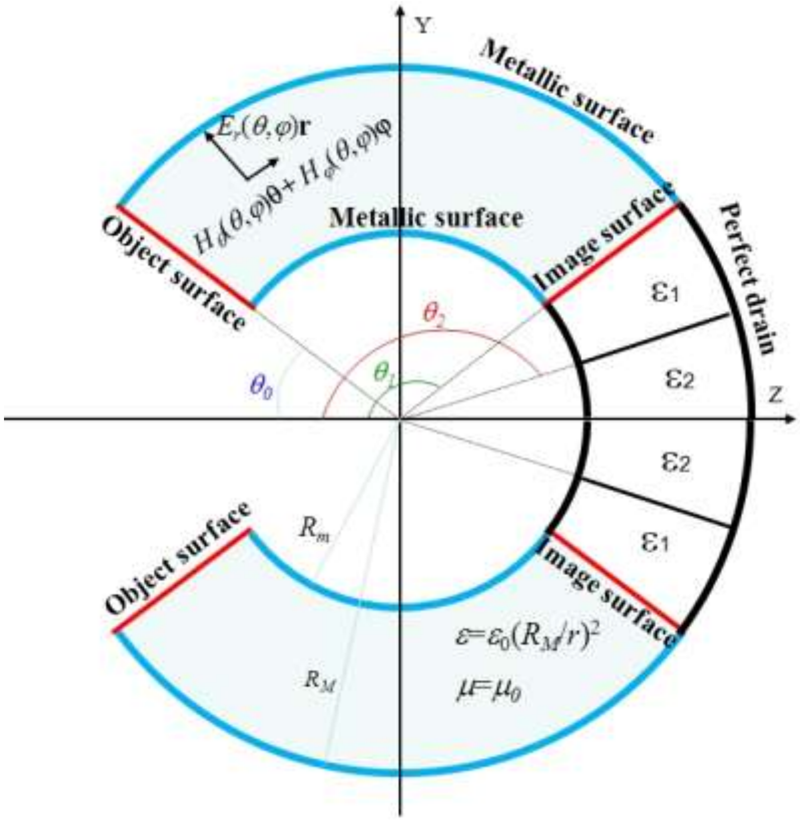
$$E_0 F_{\nu_0}^{n_1}(\cos(\theta_1)) = E_1 S_{\nu_1}^{n_1}(\cos(\theta_1))$$

$$E_0 \frac{dF_{\nu_0}^{0}(\cos(\theta_1))}{d\theta} = E_1 \frac{dS_{\nu_1}^{0}(\cos(\theta_1))}{d\theta} \longrightarrow \nu_1$$

$$\nu_1(\nu_1+1) = \left(\frac{2\pi f}{c} R_M\right)^2 \varepsilon_1 \longrightarrow \varepsilon_1$$

[J.C. González, J.C Miñano, P. Benítez, *New Journal of Physics*, 13 (2011)]

SGW and two-layer perfect drain



Electric field in the object surface

$$\mathbf{E}(\theta_0,\varphi)=e^{j\varphi}+e^{j2\varphi}$$

Electric field in the guide when there is no any reflection

$$\mathbf{E}(\theta,\varphi)=\sum_{n=1}^{2}E_{fn}F_{\nu}^{n}(\cos(\theta))\,e^{jn\varphi}\mathbf{r}$$

The equality of the electric field and tangential magnetic field at θ1 and θ2

$$E_{0n}F_{\nu_0}^{n}(l_1)=E_{1in}F_{\nu_1}^{n}(l_1)+E_{1rn}R_{\nu_1}^{n}(l_1)$$

$$E_{1in}F_{\nu_1}^{n}(l_2)+E_{1rn}R_{\nu_1}^{n}(l_2)=E_{2n}S_{\nu_2}^{n}(l_2)$$

$$E_{0n}\frac{dF_{\nu_0}^{n}(l_1)}{d\theta}=E_{1in}\frac{dF_{\nu_1}^{n}(l_1)}{d\theta}+E_{1rn}\frac{dR_{\nu_1}^{n}(l_1)}{d\theta}$$

$$E_{1i0}\frac{dF_{\nu_1}^{0}(l_2)}{d\theta}+E_{1r0}\frac{dR_{\nu_1}^{0}(l_2)}{d\theta}=E_{20}\frac{dS_{\nu_2}^{0}(l_2)}{d\theta}$$

$$l_1=\cos(\theta_1),\quad l_2=\cos(\theta_2)$$

$\Longrightarrow \varepsilon_1,\varepsilon_2$

$\varepsilon_1=1.07003-0.398973\cdot i$

$\varepsilon_2=1.84269-1.66792\cdot i$

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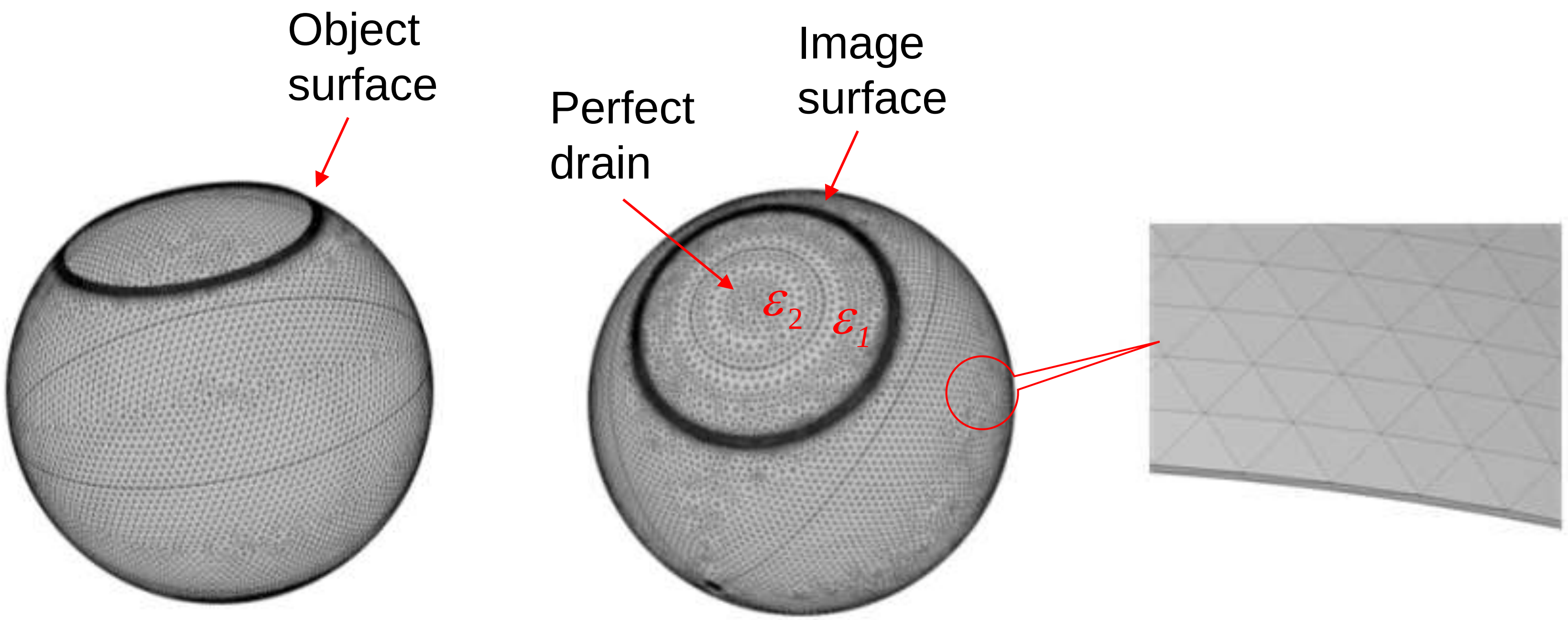
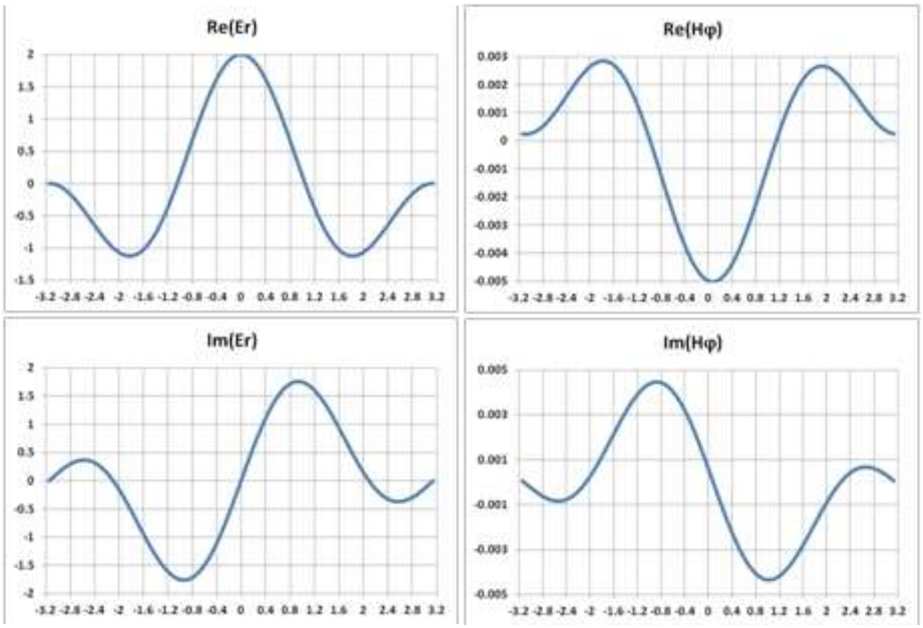
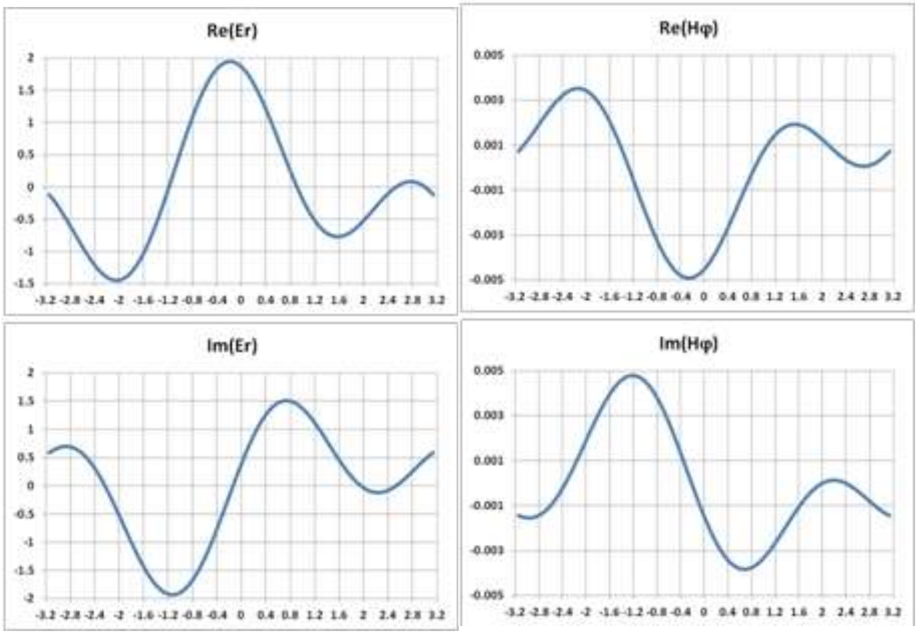
Meshing in Comsol



Simulation results in Comsol



Electric and magnetic fields in the object surface



Electric and magnetic fields in the image surface

The same results are obtain analytically considering that there is no any reflection at the image surface

$$\mathbf{E}(\theta,\varphi) = \sum_{n=1}^{2}\left[E_{fn}F_{\nu}^{n}(\cos(\theta)) + \cancel{E_{rn}R_{\nu}^{n}(\cos(\theta))}\right]e^{jn\varphi}\mathbf{r}$$

→ We have constructed a drain perfectly absorbing two modes

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Conclusions

- Simulations of the SGW show super-resolution up to λ /500 at microwave frequencies for a point source.
- SGW for extended objects images two dirac delta functions separated by λ /10, when a theoretical perfect drain (absorbing all the modes) is used
- The perfect drain can be realized using a drain containing a multi-layer structure having different permittivities in each layer
- We have presented the procedure for calculating the multi-layer drain capable to absorb *k* modes.Up to now, we have solved it only for *k*=2