COMSOL Multyphisics Super Resolution Analysis of a Spherical Geodesic Waveguide Suitable for Manufacturing

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Abstract: Recently it has been proved theoretically (Miñano et al, 2011) that the super-resolution up to $\lambda/500$ can be achieved using an ideal metallic Spherical Geodesic Waveguide (SGW). This SGW is a theoretical design, in which the conductive walls are considered to be lossless conductors with zero thickness. In this paper, we study some key parameters that might influence the super resolution properties reported in (Miñano et al, 2011), such as losses, metal type, the thickness of conductive walls and the deformation from perfect sphere.
We implement a realistic SGW in COMSOL multiphysics and analyze its super-resolution properties. The realistic model is designed in accordance with the manufacturing requirements and technological limitations.

Keywords: Super-resolution, Microwave, Spherical Geodesic Waveguide, positive refractive index, COMSOL, Perfect Imaging.

1. Introduction

The first prove for a perfect imaging system with positive refractive index, inside the framework of Geometrical Optics, has been developed by Maxwell in 1854 [1]. He designed a gradient index lens that can image any point inside the lens to another point outside the lens. This lens is called Maxwell Fish Eye lens (MFE) and the refractive index profile is following the relation below:

$$n = \frac{n_0}{1+\left(\frac{r}{R}\right)^2}$$

where R is the radius of the lens and the r is the radial coordinate of a point inside the lens. Although Maxwell calculated the fish eye lens in the XIX century, due to its complicate structure (gradient index profile), it has not been manufactured until present time. An increase interest in Maxwell fish eye emerged recently due to work of Leonhard.

In 2009 Leonhardt [2] demonstrated that the MFE lens in two dimensions (2D) perfectly focuses radiation of any frequency between the source and its image for 2D Helmholtz fields (which describes TE-polarized modes in a cylindrical MFE, i.e., in which electric field vector points orthogonal to the cross section of the cylinder). This means that the MFE is a perfect imaging system in the Wave Optics, as well. Ma [3] constructed an experiment using a cylindrical MFE and confirmed Leonhardt's theoretical work. In 2010 Miñano and his group suggested another device, equivalent (having the same optical properties) to the MFE, Spherical Geodesic Waveguide (SGW)[4]. The SGW is a very thin spherical metallic waveguide filled with a non-magnetic material and isotropic refractive index distribution proportional to 1/r. When the guide is very thin, the refractive index profile can be assumed constant with a very good approximation.

Up to this point all the results corresponding to the Spherical Geodesic Waveguide were mainly theoretical and based on assumption that the metallic walls are perfect conductors. In this paper we are mainly trying to predict and tackle the problems that are caused by the manufacturing process or by technological limitations, such as cost, size of the waveguide, conductivity or material selection and finally, suggesting a suitable design for prototyping. We also need to study the influence of all those limitation on the super-resolution properties. In [5] the super-resolution up to $\lambda/500$ was reported using an ideal SGW with a diameter of 2 meters and the air gap of 5mm. Concerning the cost and limitations of manufacturing we have decided to reduce the size of the sphere and increase the gap between the metallic spheres. Here we will present the COMSOL simulations for few different designs. All the models will have the same dimensions of the sphere (the diameter is 30 cm), and the same air gap (5mm). At first, we keep the perfect conductor condition to see the influence of the sphere shrinking. The second step is to study the influence of the metallic conductivity on super-resolution properties. In this step, unlike the previous works, we will no longer use a perfect conductor, but a real material.

In the third design, we will introduce a system that allows the drain to move smoothly over the surface of the sphere without losing its electrical contact. Finally, the forth design contains the insulator balls, which keep the inner and outer spheres concentric. This model is built to study the effect of those balls on super-resolution properties of the system.

3. Use of COMSOL Multiphysics

The Spherical Geodesic Waveguide consists of two concentric metallic spheres with different radius. The gap between the spheres creates the waveguide. In the prospective experiment we use 3 small balls to hold the inner sphere at the center of the outer sphere; these balls have to be made of a nonconductive material. The SGW is made in 3D RF model in COMSOL. Table 1 shows the physical and geometric properties of the sphere

R_{Min}	R_{Max}	L	D_e	D_i	θ	C
145	150	20	20	10	0-5	3.5x107
mm	mm	mm	mm	mm	Degree	S/m

Table 1 Physical and geometrical properties of the spherical geodesic waveguide

Where the R_{Max} and R_{Min} are outer and the inner radius of the spherical waveguide, the C parameter is the conductivity of the metallic walls. The conductive walls are made of aluminum with the thickness of 2 mm. The thickness of the waveguide is 5 mm which is the same as the diameter of the insulator balls. The skin depth of the aluminum in the frequency range of our interest (1 to 2 GHz) is less than $3\mu m$, therefore 2 mm wall thickness guarantees that there is no leakage of the energy from the waveguide.

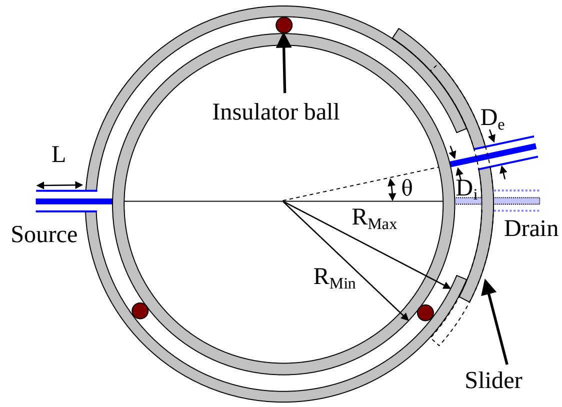


Figure 1. Schematic presentation of the Spherical Geodesic Waveguide (SGW) with coaxial cables (Blue color), the gray color represents the metallic conductive walls and the white color represents the dielectric waveguide. The slider is moving smoothly on the outer sphere to tilt the drain.

In theory, the refractive index of the dielectric gap between the metallic spheres should be:

$$n\ (r) = \frac{an_0}{r}$$

Since the ratio R_{Max}/R_{Min}=1.034 we assume that the refractive index is constant in the waveguide. The material assigned to the dielectric is air with refractive index n=1. This material has been taken from the material library of COMSOL.

When the drain is placed at the antipode point, i.e. $\theta = 0$(see Figure 1), there is an analytical solution for the waveguide modes (see Leonhard). Whereas, when $\theta \neq 0$, the symmetry of the problem is broken, so the wave equation has to be solved numerically. Here, we have used COMSOL to calculate the transmitted power from the source to the drain (both are realized with the coaxial cables). This has been done using the scattering S parameters for different positions of the drain.

The SGW is analyzed using the radio frequency in the range of 1 to 2 GHz. The corresponding wavelength for this range of frequency is varying from 300 to 150 mm respectively. This range is much lower than the next cut off frequency in the coaxial cable:

$$\approx \frac{4c}{\pi(D_e + D_i)} = 70.73GHz$$

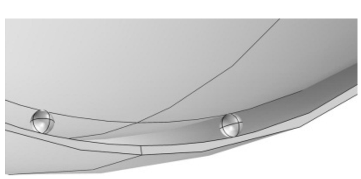
where, D_e and D_i are the outer and the inner diameters of the coaxial cables respectively. The coaxial connector has 20 mm long that guarantee the evanescent modes are negligible in the cable, moreover the ratio of the $D_e/R_{Min}<<1$ which means that the point nature of the source and the drain is modeled successfully.

The number of unit elements in the structure is one of the most important parameters to control the speed and the accuracy of the simulation. Therefore meshing has to be done with a great care. The sphere consists of 53 domains that have been meshed differently depending on their functionality and the electrical field variation. The highest variation of the electrical field occurs close to the coaxial cables. Therefore the mesh density and the control of the mesh in the coaxial cables are very important. Thus, mapping is used to control the number of elements (mesh density) in the cables.

Figure 2 C shows the cross section of the coaxial cable and its meshing structure. In the dielectric part of the waveguide, the change of the electric field is lower, so the mesh elements (tetrahedral) are much bigger. Although the metallic conductive walls are only 2mm thick, they are much thicker than the skin depth. Additionally, we are only interested to measure the power reaching the drain and not in studying the field variation inside the metals. Therefore, we have used Impedance Boundary Condition (IBC) for the guide walls.

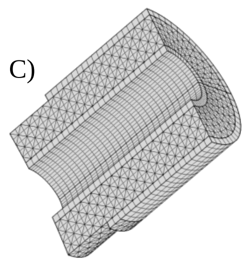
Figure 2 A illustrates the insulator balls that are holding the inner sphere concentric with the outer sphere. They are located close to the drain, in -45 degree respect to the horizontal plane and they are separated by 120 degree about the vertical axis. In order to decrease any unnecessary effect we used only 3 balls, which is sufficient for holding the inner sphere. The electrical field varies rapidly in vicinity of the balls therefore high meshing is required around the balls. Tetrahedral elements were used to mesh the balls and the waveguide. Figure 2 D shows the bottom view of the SGW made in COMSOL after meshing. In the picture we can see the slider, coaxial cable (drain) and the meshing of the insulator balls.

A)



B)

C)



D)

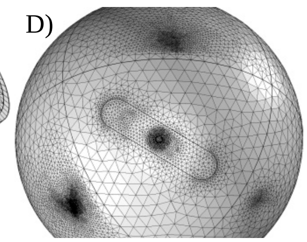


Figure 2 A) The cross section of the SGW made in COMSOL, showing the insulator balls between two spheres. B) The illustration of the SGW with the slider and the drain port on top. C) Cross section of the coaxial cable and the meshing structure. Since we are using impedance boundary condition we did not use the mesh command "convert" for the metallic domains. D) (Bottom view) Due to the rapid field variation in the vicinity of the insulator balls, meshing has to be very dense.

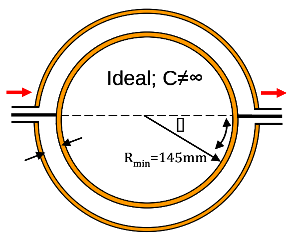
4. Results

We have computed the transmitted power (P_T) using COMSOL for a frequency range between 1 GHz and 2 GHz for different positions of the drain port. The source port is fixed at the point $\theta = \pi$, while the drain port is shifted λ/N (for λ being the wavelength corresponding to the notch frequency) in the neighborhood of the image point, that is, $\theta = 0$, (see Figure 1). Each simulation has been done for a different frequency belonging the range 1-2 GHz.

As we mentioned before we first study the effect of size shrinking (keeping the perfect conductivity condition). A new smaller model respect to the one presented in [5] is proposed to ease the manufacturing process. We have decreased the size of the sphere to 30 cm in diameter, while keeping the same size as in [5] for the waveguide (5mm). In [5] the ratio of the radius of the sphere (1m) to the width of the waveguide (5×10^{-3} m) is $R_{max}/W_w = 200$. In our study this ratio drops to 30.

Figure 3 shows the schematic diagram of the waveguide and the simulation results for 4 different position of the drain. Let us define "resolution" as the arc length (in wavelength units) that a drain port needs to be shifted so $|S_{21}|^2$drops to 10% (not far from the Rayleigh criteria in Optics, which refers to the first null). Using this definition, we can see that this occurs for the four movements. These results show that the size shirking preservs super-resolution properties of the guide. However, we are not able to detect the super-resolution for movement smaller then $\lambda/109$. Here, the notch occurs at a higher frequency comparing to the sphere with 2 m diameter [5].

A)



B)

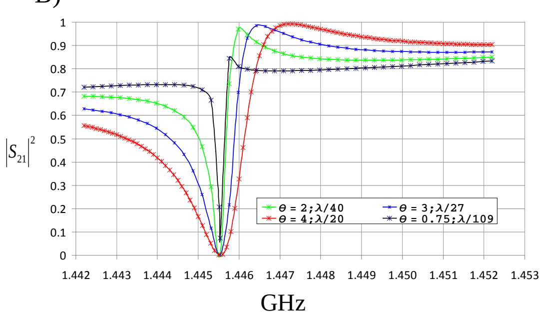


Figure 3. A) Illustrates the schematic diagram of the ideal SGW with the diameter of 30cm B) shows the $|S_{21}|^2$parameter vs frequency for different drain shift

In [5] the notch appeared around 0.26 GHz [5] whereas in this new geometry it happens at 1.4453GHz, which means that the wavelength of the notch was decreased by a factor of almost 5.5 (from 1.15m to 0.207m).

In second step we apply the finite conductivity to the metallic walls. We have used aluminum with the conductivity of 3.5×10^7 S/M. The geometry of the SGW is the same as in Figure 3 (A).

Figure 4 shows the results for this study. The notch frequency is about 1.4451 GHz which corresponds to the wavelength of 20.7cm; this value is very similar to the notch frequency for the waveguide with infinite conductivity. However, we can clearly see that the drop of 10% to 20% in the transmitted power for various frequencies. Additionally, the power does not reach zero even at the notch frequency. According to Figure 4 and our definition of the resolution, we conclude that the resolution has decreased to $\lambda/27$. For a smaller shift we cannot detect the significant power drop.

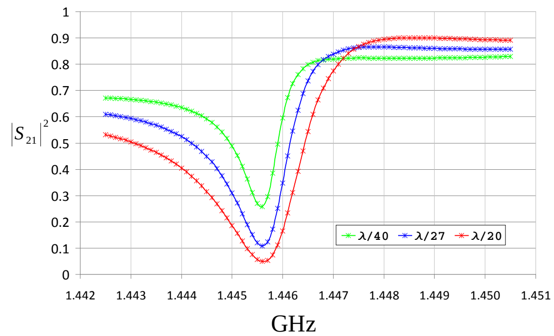


Figure 4. The $|S_{21}|^2$parameter versus frequency and displacement based on $\boldsymbol{\lambda}$. considering finite conductivity

Let us now add a slider to the design. The slider allows the drain to move smoothly on the outer sphere, also providing electrical contact with inner and outer spheres (Figure 1). In order to implement this, we have to create a cut on the wall and cover it as it can be seen in Figure 5 (A). This cut and its cover creates non-uniformity in the waveguide which might cause changes in super-resolution.

Figure 5 (B) shows the results obtained by COMSOL for this case. According to the graphs, the notch occurred at 1.4431GHz, at a slightly lower frequency compared to the previous case presented in Figure 4. However, now there is another small drop at 1.4463GHz. We assume that this occurs due to the non-uniformity caused by the slider. There is not a significant change in the super- resolution properties of the SGW. However, the bandwidth of the notch has been increased.

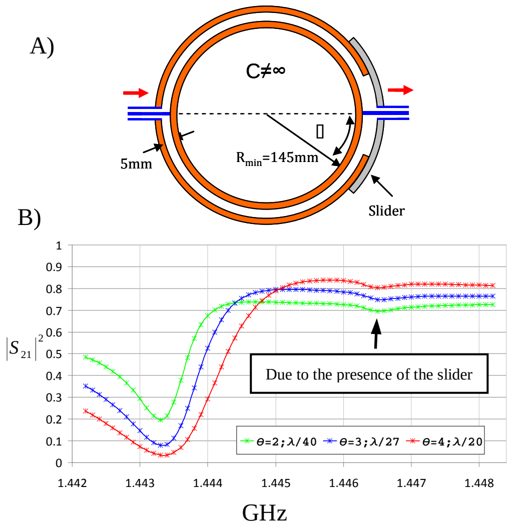


Figure 5 A) Illustrates the schematic diagram of the SGW with the drain slider B) shows the $|S_{21}|^2$parameter vs frequency for different drain shifts, the rippling caused by the slider is visible on the graph.

The last and the final part of this study is the analysis of the complete system suitable for manufacturing. This realistic design contains the slider and the insulator balls. These balls might scatter the waves and change the expected results. Figure 6 shows the super-resolution properties of this design.

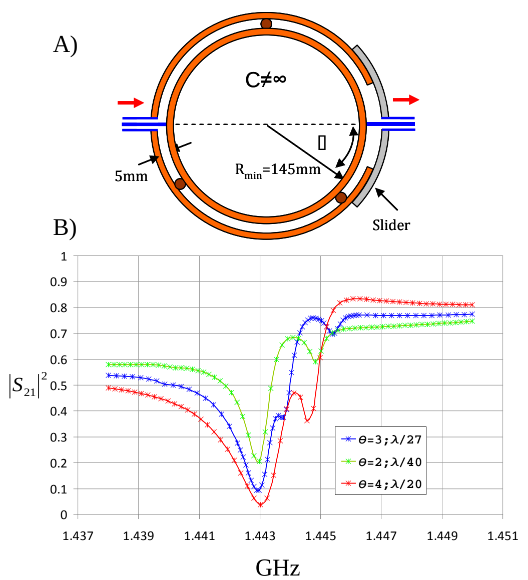


Figure 6 A) Illustrates the schematic diagram of the SGW with slider and the insulator balls (B) The $|S_{21}|^2$ parameter versus frequency and displacement based on $\boldsymbol{\lambda}$.considering all the parameters.

In Figure 6 there are three additional notches (different from the main notch frequency at 1.443 GHz) caused by the presence of the insulator balls. Each ball, regardless of the position of the drain, reflects a portion of energy back to the source and thus creates a notch. The central frequencies of these notches are very close to the notch of the waveguide. The main notch gets dipper and wider by increasing the θ. For a certain value of the angle θ, the main notch overlaps the 3 small notches caused by the insulator balls making them disappear.

The super-resolution property of the system did not change significantly with presence of the insulator balls. The notches that are created by the balls ate located in different frequencies and they are distinguishable from the main notch.

5. Conclusion

According to the presented analysis, we have concluded that the conductivity of the metallic shells dramatically changes the super-resolution properties of the SGW. When the conductivity decreases the resolution decreases as well. The metallic (aluminum) losses cause the resolution drop from about $\lambda/109$ to about $\lambda/20$.

The super-resolution properties of the SGW do not vary much with other geometrical parameters such as non-uniformity in the waveguide, or presence of periodic structure such as insulator balls.

8. References

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