Evaluation of Instability of a Low-salinity Density-dependent Flow in a Porous Medium

Y. T. Habtemichael[1], R. T. Kiflemariam[2], H. R. Fuentes[1]
1. Department of Civil and Environmental Engineering, Florida International University, Miami, FL, USA
2. Department of Mechanical and Materials Engineering, Florida International University, Miami, FL, USA

Introduction

Seawater intrusion into coastal aquifers is usually modeled by using transport models that account for the effect of variable-density on flow. When a more dense fluid is moving along in a body of less dense fluid, fingering instabilities develop and play important role in the mixing and dispersion processes. The development of instabilities for different Péclet and Rayleigh numbers is investigated by changing the grid size and density contrasts, respectively.

Computational Methods

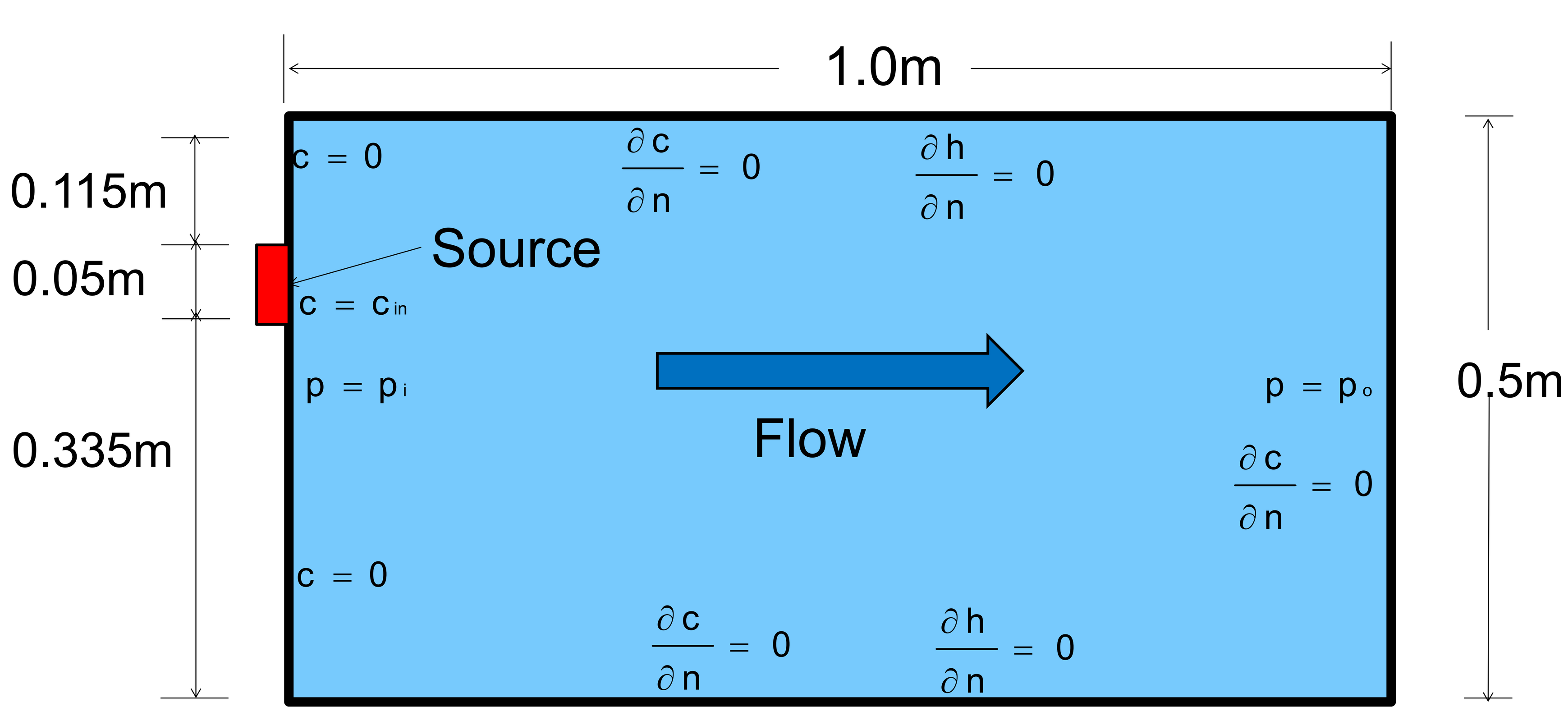


Figure 1. Configuration of the model domain (Ibaraki, 1998)

Migration of a low salinity plume (e.g., NaCl) from the source through a porous medium initially saturated with freshwater is modeled using COMSOL Multiphysics (*dl* and *esst* modes).

$$\frac{\partial \varepsilon\rho}{\partial t} + \nabla \cdot \rho u = 0 \qquad \text{(Mass Conservation)}$$

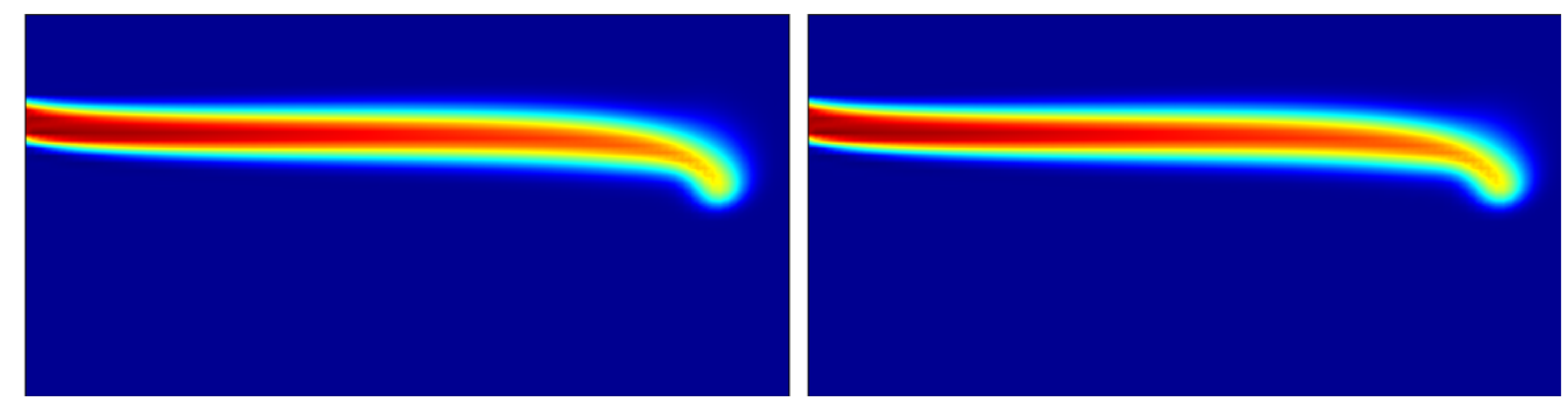
$$u = \frac{\kappa}{\mu}(\nabla p + \rho g \nabla D) \qquad \text{(Darcy's Law)}$$

$$\frac{\partial \theta_s c}{\partial t} + \nabla \cdot (cu) - \nabla \cdot (\theta_s \tau D_{ij} \nabla c) = 0 \qquad \text{(Solute Transport)}$$

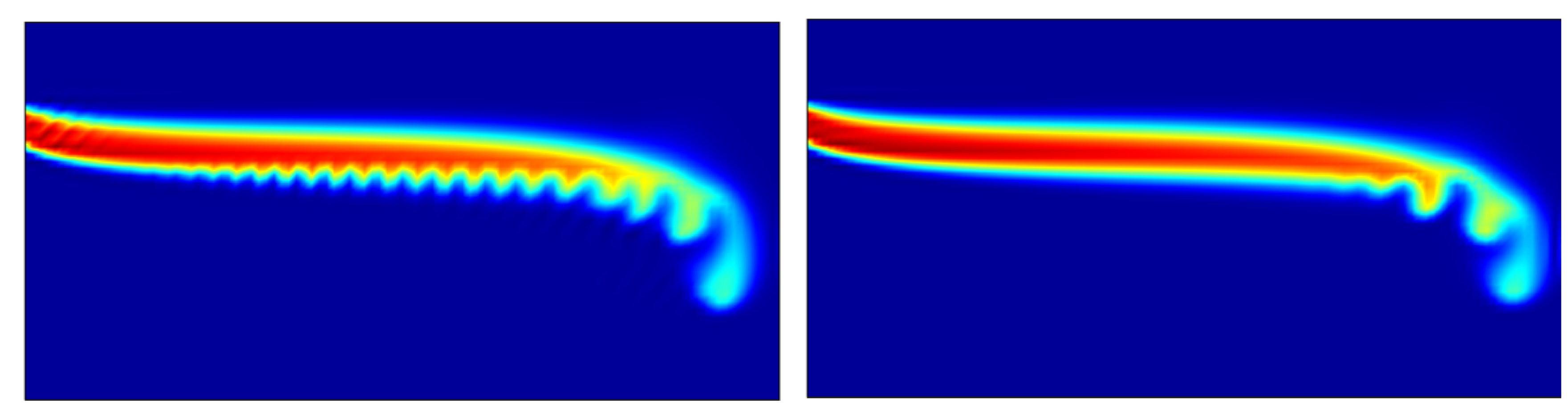
Table 1. Flow and transport equations

Fluid flow is described using Darcy's law and solute transport is given by the advection-dispersion equation. The model parameters are given in Ibaraki (1998). A structured mesh is used with 100 x 50 elements for the coarse grid and 200 x 50 elements for the fine grid.

Results

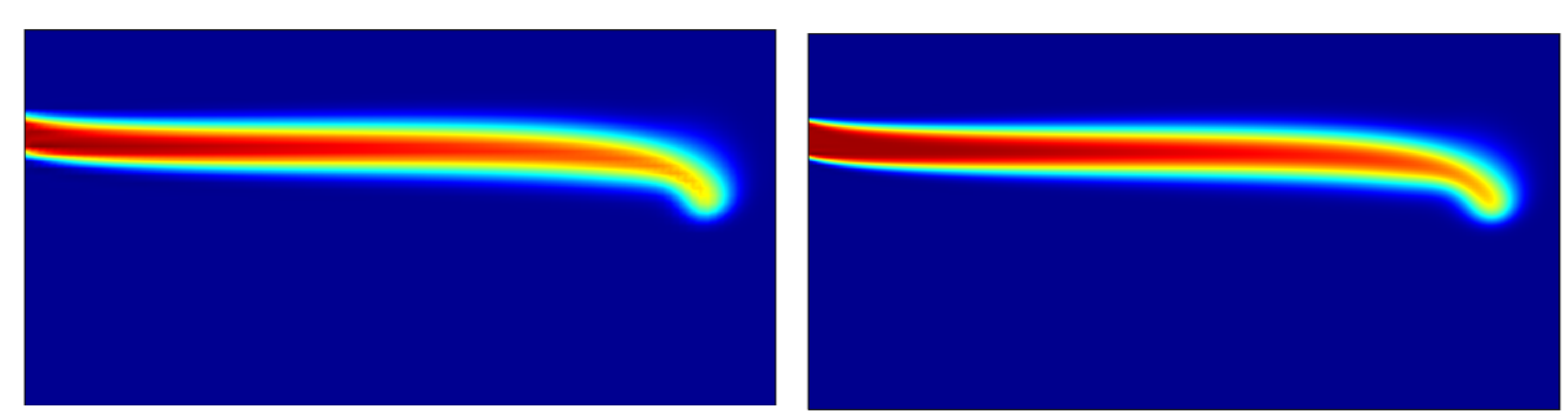


(a) C_{in} = 1000 mg/L, time step: 1800 s (b) C_{in} = 1000 mg/L, time step: 300 s

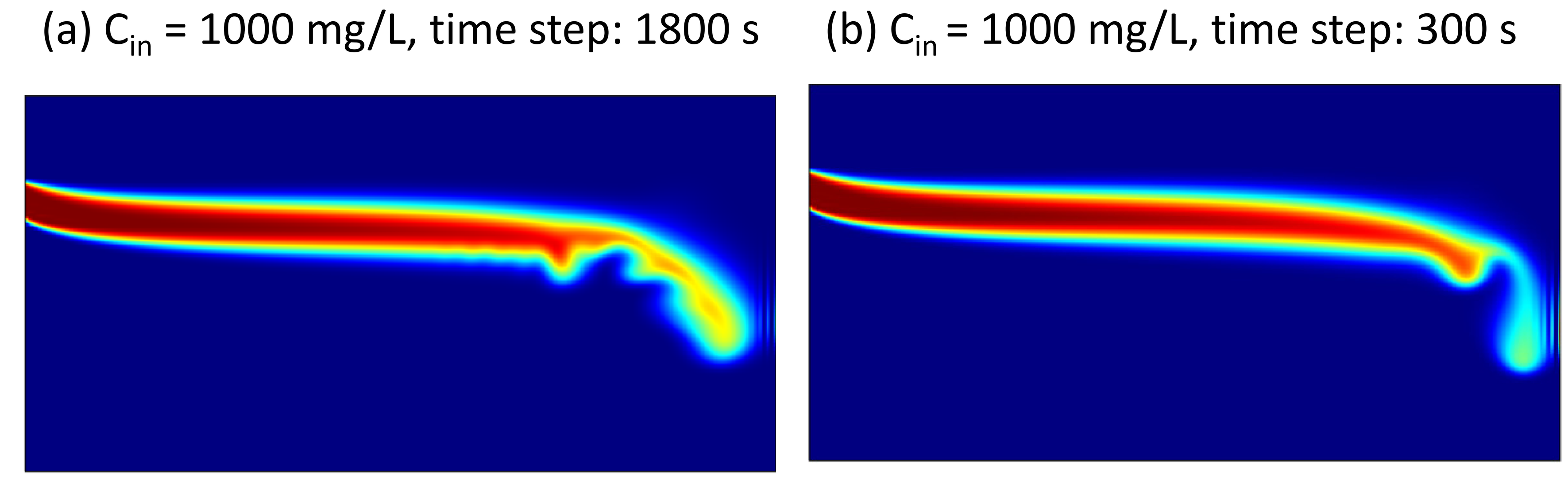


(c) C_{in} = 2000 mg/L, time step: 1800 s (d) C_{in} = 2000 mg/L, time step: 300 s

Figure 2. Concentration distributions after 90 h (grid size: 10mm X 10mm)



(a) C_{in} = 1000 mg/L, time step: 1800 s (b) C_{in} = 1000 mg/L, time step: 300 s



(c) C_{in} = 2000 mg/L, time step: 1800 s (d) C_{in} = 2000 mg/L, time step: 300 s

Figure 3. Concentration distributions after 90 h (grid size : 5mm X 5mm)

Conclusions

For C_{in} = 1000 mg/L, the single unstable 'lobe' is similar for different time steps and mesh sizes. Instability with more than one lobe developed for C_{in} = 2000 mg/L, and this is reduced with finer mesh and smaller time step.

References

1. X. Mao et al., "Three-dimensional model for multi-component reactive transport with variable density groundwater flow," *Environmental Modeling & Software*, vol. 21, pp. 615-628, (2006).
2. M. Ibaraki, "A robust and efficient numerical model for analyses of density-dependent flow in porous media," *Journal of Contaminant Hydrology.*, vol. 34, pp. 235-246, (1998).
3. R. A. Schincariol et al., "On the generation of instabilities in variable density flow," *Water Resources Research*, vol. 30, pp. 913-927, (1994).

Excerpt from the Proceedings of the 2014 COMSOL Conference in Boston