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REALIZING TOMORROW'S TECHNOLOGY

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Small Scale Yielding Model for Fracture Mechanics

J. Thomas, J.S. Crompton, and K.C. Koppenhoefer

Overview

- Motivation
- Theory
- Use of COMSOL Multiphysics
- Results
- Summary

Motivation

- **Fracture mechanics – study of cracked structures**
- **Structural failures often produced by propagation of existing defects**
- **Calculate safe life for structures containing defects**

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Motivation

- **Determine the capabilities of COMSOL Multiphysics for conducting computational fracture mechanics**
- **Compare results with well-established computational fracture mechanics software**

Theory

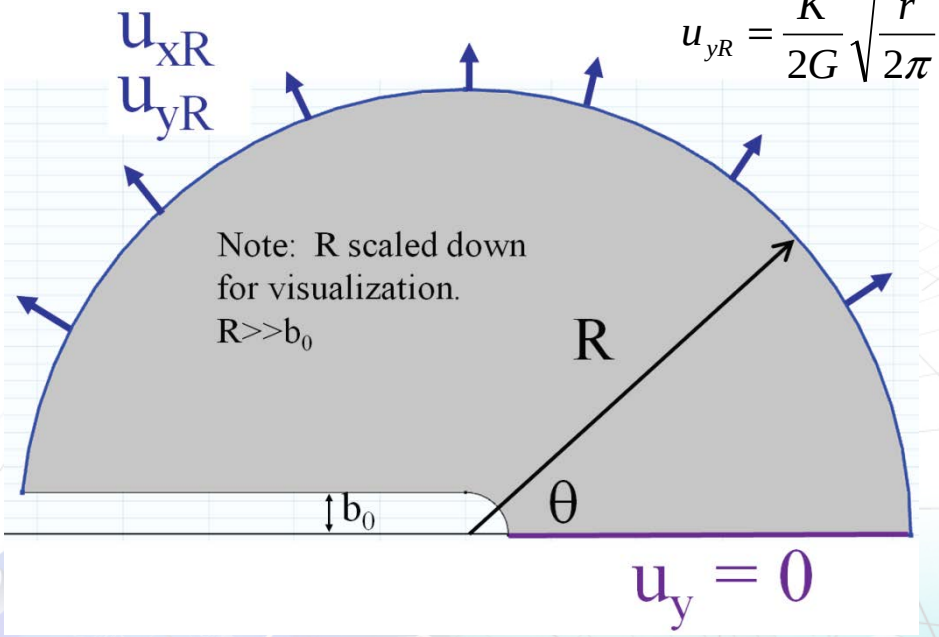
- **Early work – single parameter to describe crack tip stress/strain**
  - ***K* for linear elastic fracture mechanics**
  - ***J* for elastic-plastic fracture mechanics**
- **Computational fracture mechanics – calculate crack tip stress/strain**
  - **Calculate single parameter for complex structures**
  - **Determine effect of finite length scales on crack tip stress/strain**

Small Scale Yielding Model

- **Plastic zone small compared with all length scales**
- **Used as a reference solution in computational fracture mechanics**

$$u_{xR} = \frac{K}{2G}\sqrt{\frac{r}{2\pi}}\cos\left(\frac{\theta}{2}\right)\left[\kappa - 1 + 2\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$u_{yR} = \frac{K}{2G}\sqrt{\frac{r}{2\pi}}\sin\left(\frac{\theta}{2}\right)\left[\kappa + 1 - 2\cos^2\left(\frac{\theta}{2}\right)\right]$$



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Modeling Methodologies

- **Linear elastic fracture mechanics (LEFM)**
  - **Elasticity dominates the solution**
  - **Singularity at crack tip defined by stress intensity factor (*K*)**
- **Elastic plastic fracture mechanics (EPFM)**
  - **Ramberg-Osgood -** $\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n$
  - **Small strain plasticity**
    - $\varepsilon = \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathbf{T}}\right)$
  - **Large strain plasticity**
    - $\varepsilon = 1/2\ [\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathbf{T}} + (\nabla \mathbf{u})^{\mathbf{T}}\ \nabla \mathbf{u}]$

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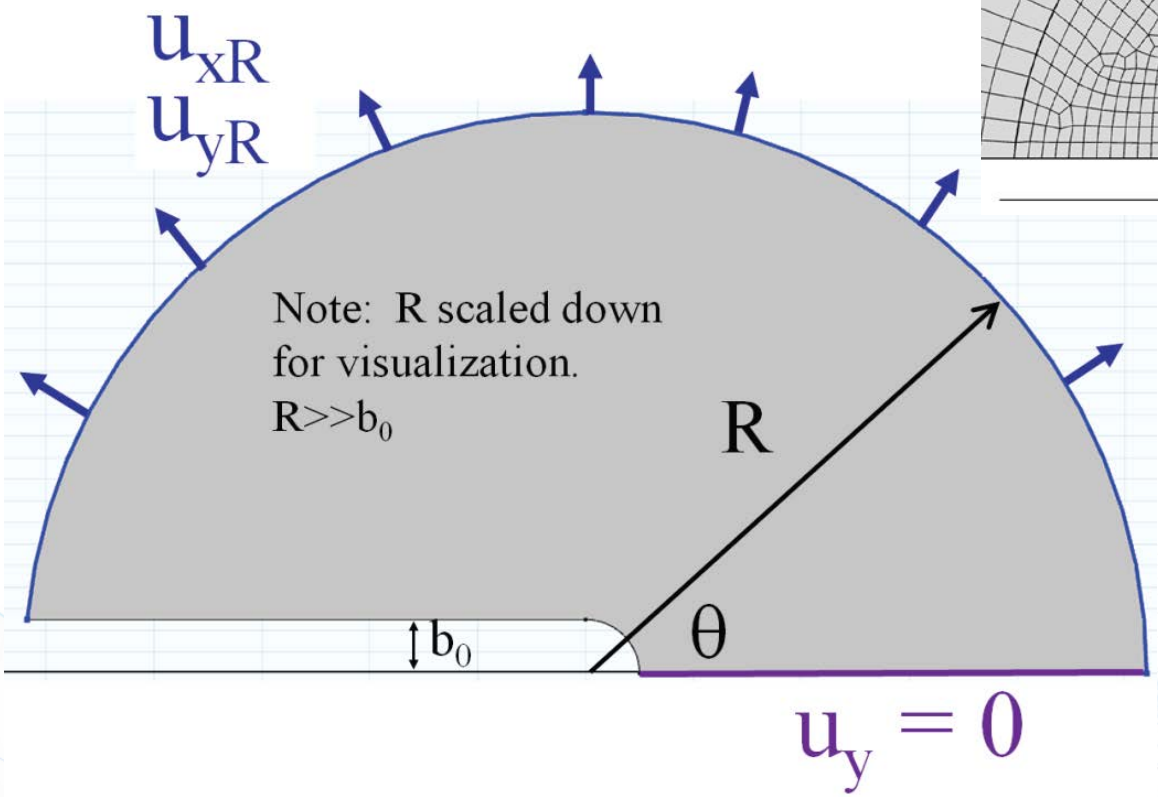
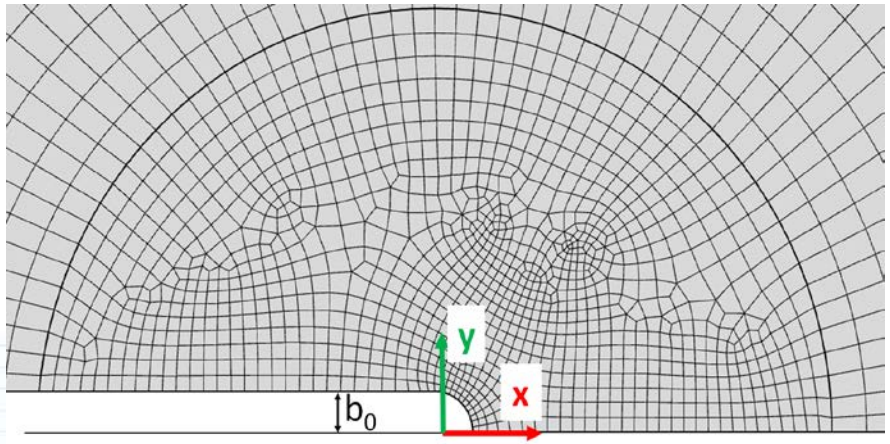
Use of COMSOL Multiphysics

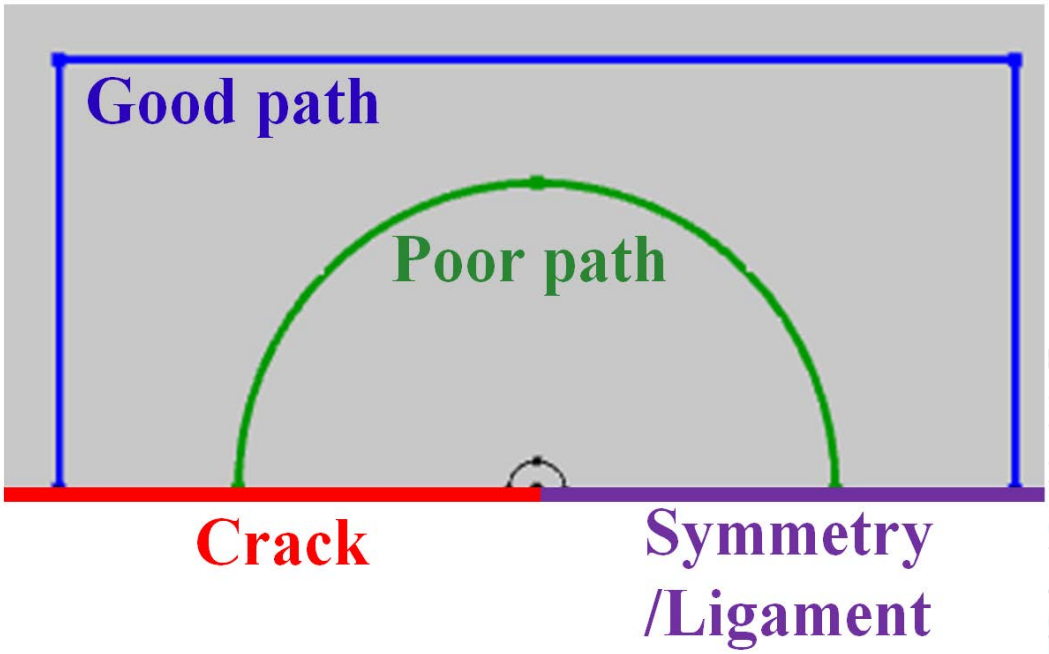




b_0=10μm

Use of COMSOL Multiphysics – J-Integral

$$J = \int_{\Gamma} \left(W_s n_x - T_i \frac{\partial u_i}{\partial x} \right) ds$$



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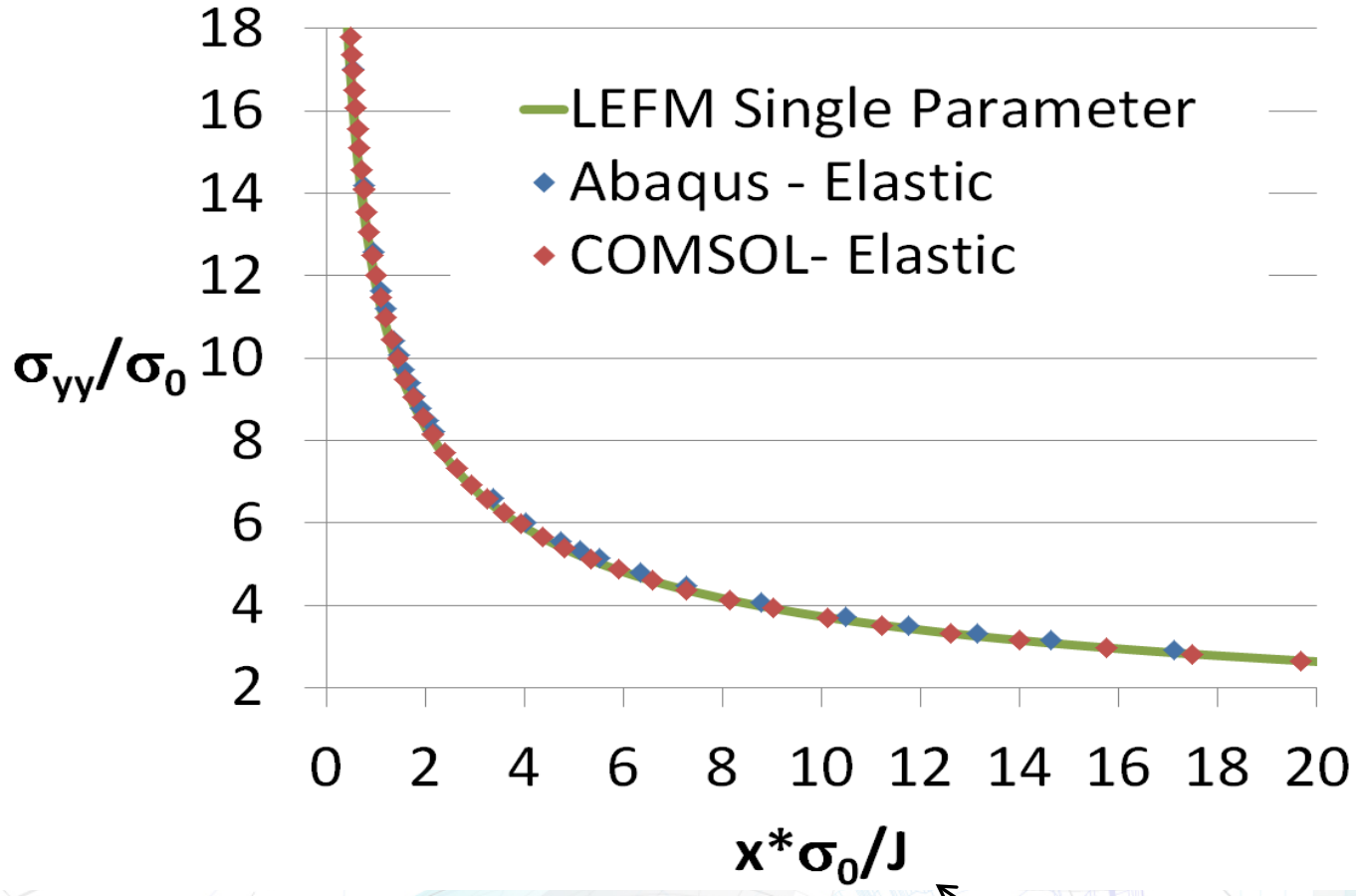
Results - LEFM



Normalized Stress

Normalized Distance ahead of crack tip

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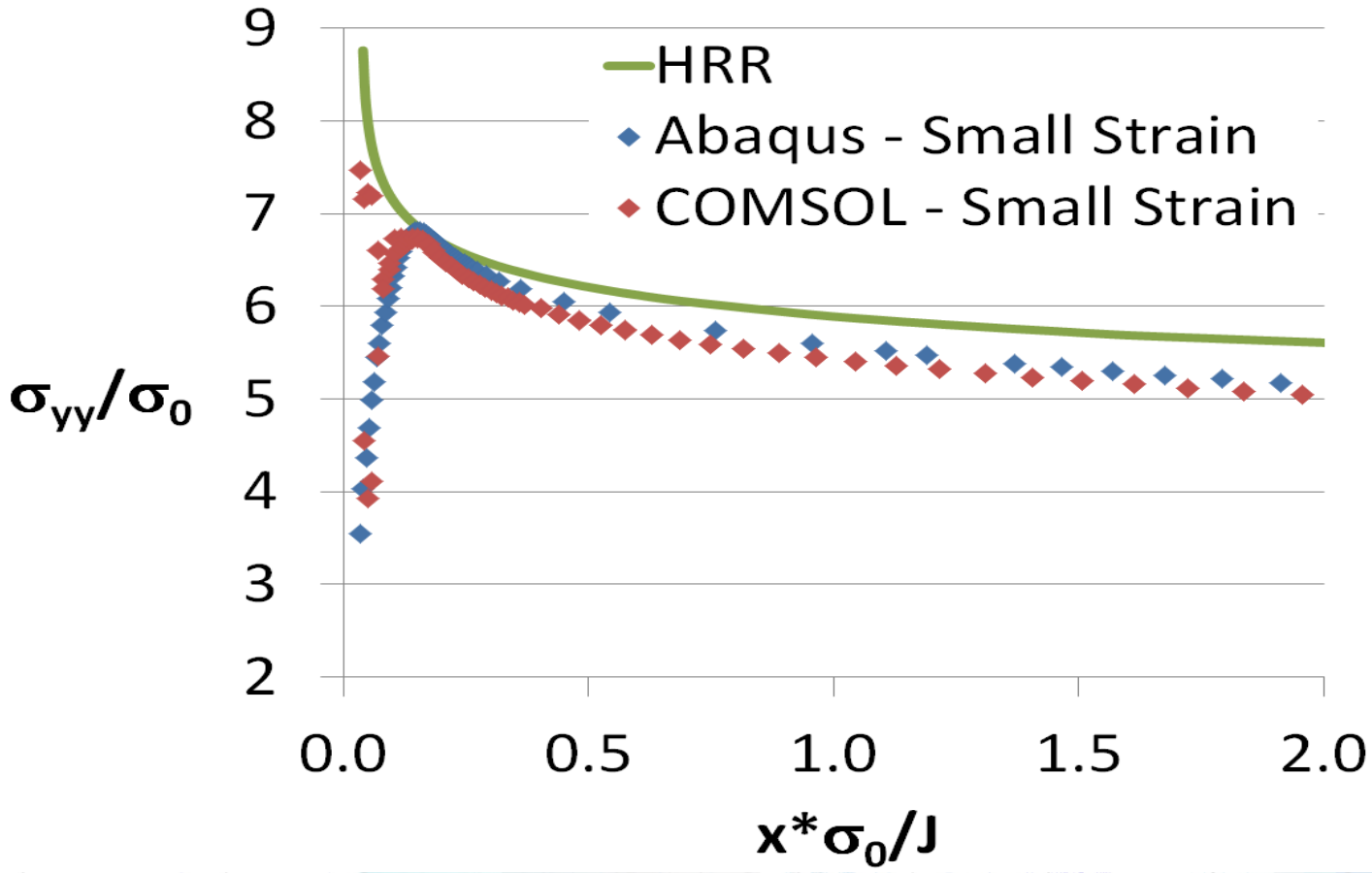
Results – EPFM (Small Strain)



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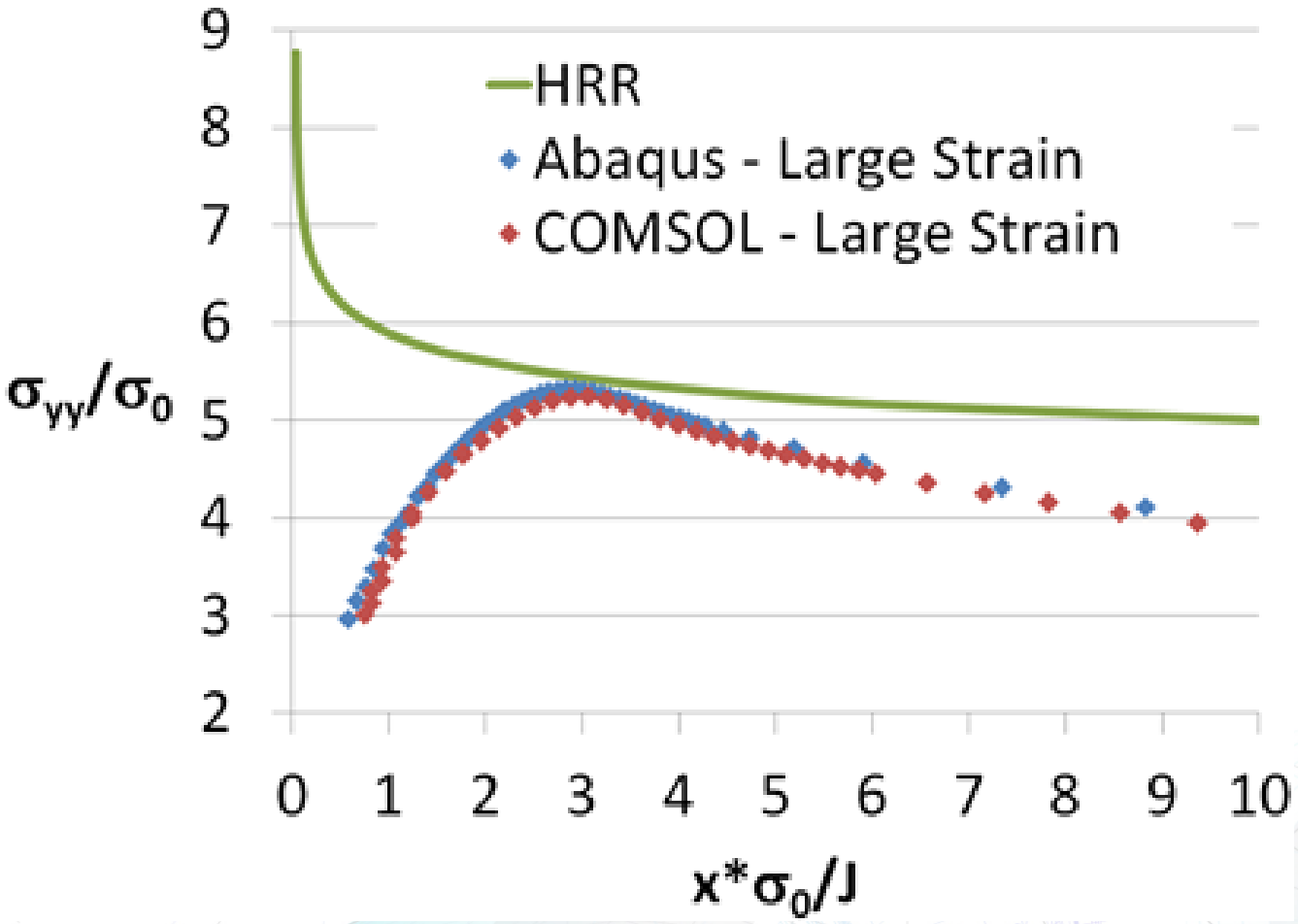
Results – EPFM (Large Strain)



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Summary

- COMSOL Multiphysics provides the capabilities needed for computational fracture mechanics
- Software accurately calculates the near tip strain/stress fields for
  - LEFM
  - EPFM (small and large strain)

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