A Design-of-Experiments Approach to FEM Uncertainty Analysis for Optimizing Magnetic Resonance Imaging RF Coil Design

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Abstract: Using the RF module of COMSOL, we compute the magnetic flux density norm (BN) profiles for frequencies in the 76 to 100 MHz range, inside of a prototype birdcage coil, courtesy of Japan's National Institute of Radiological Sciences (NIRS), loaded with a cylindrical water phantom. At the first resonance of the lumped port impedance of the NIRS model, the BN profile was found to be highly non-uniform. A dimensionless metric for the non-uniformity of the profile is proposed as a parameter for assessing and improving the design of such a prototype bird cage coil. A statistical design of experiments (DOE) approach to the uncertainty analysis of the COMSOL solution of the NIRS model is used to minimize the BN non-uniformity metric. Significance and limitations of the DOE approach to uncertainty analysis as a design tool for MRI applications are presented and discussed.

Keywords: Birdcage coil design, design of experiments, design optimization, finite element method, magnetic resonance imaging.

1. Introduction

One of the most challenging mathematical modeling problems in modern imaging technology is the analysis and characterization of the interactions of electromagnetic (EM) fields with a biological subject (see, e.g., Jin [1][6], and McRobbie, et al. [2]). Among the numerous computational tools available for studying such interactions, the finite element method (FEM) has been found to be most attractive, partly because of the availability of several general-purpose software packages such as ABAQUS [3], ANSYS [4], COMSOL [5], and MATLAB [6], and a good number of helpful textbooks on FEM in EM (see, e.g., [7-9]). Since FEM-based solutions are inherently approximations of the physical phenomena, all such solutions contain uncertainties (see, e.g., [10-14]), which need to be quantified as documented in the literature during the last twenty years (see, e.g., [15-20]). A by-product of the process of estimating FEM uncertainties using the statistical design of experiments (DOE), as described in books such as [21-24], is the availability of problem-specific information leading to a strategy of assessing and improving the FEM-based computational model. Successful applications of the DOE approach to optimizing a FEM model have appeared in the literature [25-28] using a public-domain software named DATAPLOT [29-30], but they were exclusively applied to problems in structural mechanics. The purpose of this paper is to show with a numerical example that the same DOE approach is applicable in electromagnetics.

2. FEM Solution of a NIRS Birdcage Coil Model using the COMSOL RF Module

Motivated by the observation documented in the executive summary of a 2006 workshop [31] that "*Physical measurement uncertainties may be addressed prior to designing a clinical trial and thus help in reducing the case size and cost of a clinical trial associated with a drug submission to*

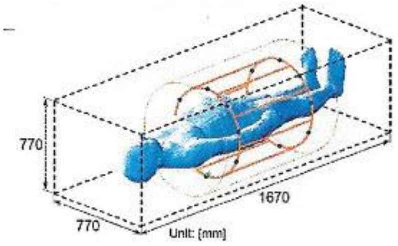


Figure 1. A typical MRI imaging setup with a birdcage coil (courtesy of NIRS and Ref. [32]).

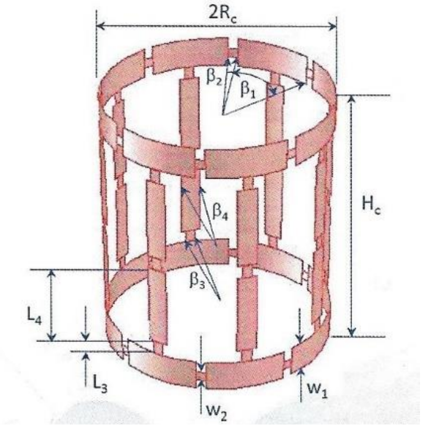


Figure 2. Geometry of a NIRS birdcage coil (courtesy of NIRS, Chiba, Japan, and Ref. [32]).

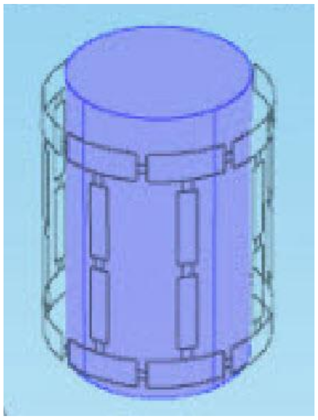


Figure 3. A NIRS birdcage coil loaded with a cylindrical water phantom of diameter equal to 0.8 * (diameter of the birdcage coil).

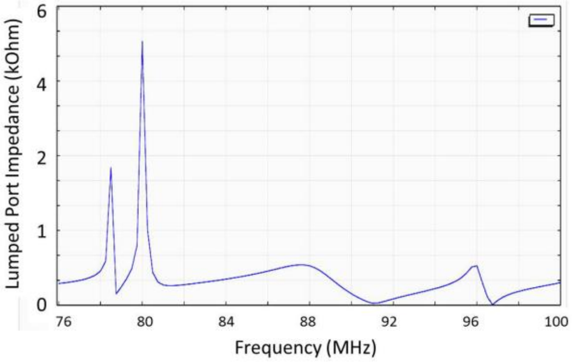
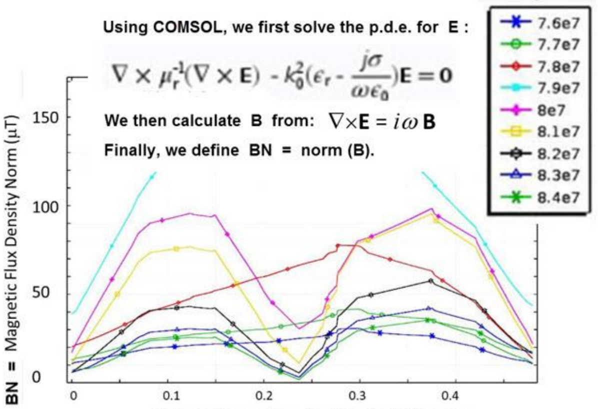


Figure 4. Plot of the lumped port impedance vs. frequency in the 76 – 100 MHz range.

the FDA," we apply a statistical design-of-experiments approach to the uncertainty analysis of the finite element method-based solution of a proposed base design of a magnetic resonance imaging (MRI) RF coil, courtesy of NIRS and ref. [32], as shown in Figs. 1-3, where the input parameters for the base design are as follows:

$R_c = 300$ mm, $H_c = 700$ mm, $w_1 = 80$ mm,
$w_2 = 25$ mm, N = no. of legs = 8, $\beta_1 = 40^o$,
$\beta_2 = 5^o$, $\beta_3 = 5^o$, $\beta_4 = 10^o$, $L_3 = 35$ mm,
R_a = radius of air domain = 1.2 m,
C = capacitance of the port in the middle of each leg = 177 pF, V_0 = excitation voltage at port numbers 1 and 3 = 500 v.



Using COMSOL, we first solve the p.d.e. for **E** :

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = 0$$

We then calculate **B** from: $\nabla \times \mathbf{E} = i\omega \mathbf{B}$

Finally, we define BN = norm (B).

Figure 5. Plot of the magnetic flux density norm (BN) vs. the distance from coil center line.

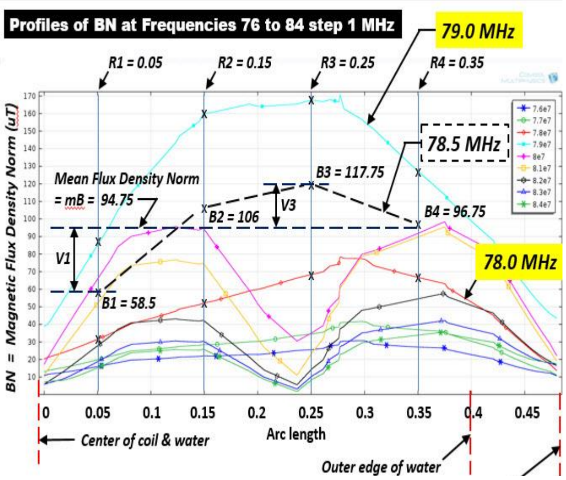
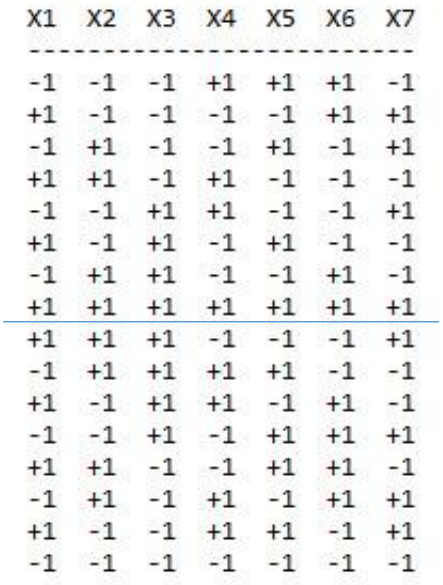


Figure 6. Definition of a dimensionless metric, the non-uniformity coefficient (NUC) of the BN.



```
 X1   X2   X3   X4   X5   X6   X7
 ---------------------------------
 -1   -1   -1   +1   +1   +1   -1
 +1   -1   -1   -1   -1   +1   +1
 -1   +1   -1   -1   +1   -1   +1
 +1   +1   -1   +1   -1   -1   -1
 -1   -1   +1   +1   -1   -1   +1
 +1   -1   +1   -1   +1   -1   -1
 -1   +1   +1   -1   -1   +1   -1
 +1   +1   +1   +1   +1   +1   +1
 +1   +1   +1   -1   -1   -1   +1
 -1   +1   +1   +1   +1   -1   -1
 +1   -1   +1   +1   -1   +1   -1
 -1   -1   +1   -1   +1   +1   +1
 +1   +1   -1   -1   +1   +1   -1
 -1   +1   -1   +1   -1   +1   +1
 +1   -1   -1   +1   +1   -1   +1
 -1   -1   -1   -1   -1   -1   -1
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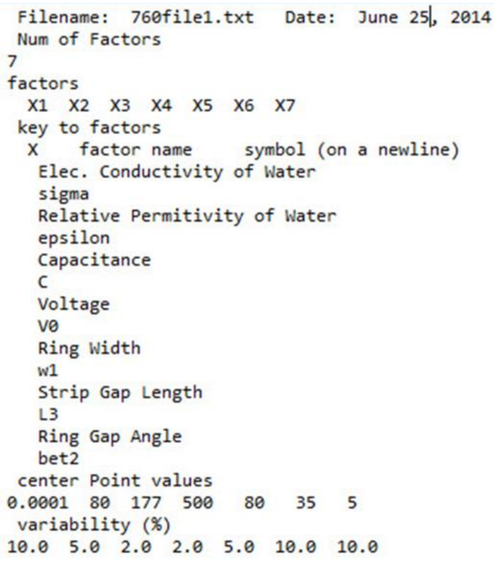
Figure 7. A Resolution IV fractional factorial orthogonal design for a 7-factor, 16-run numerical FEM experiment (Ref.: Box, et al. [21, pp. 426-427]).



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   sigma
   Relative Permitivity of Water
   epsilon
   Capacitance
   C
   Voltage
   V0
   Ring Width
   w1
   Strip Gap Length
   L3
   Ring Gap Angle
   bet2
 center Point values
0.0001  80  177  500   80   35   5
 variability (%)
10.0  5.0  2.0  2.0  5.0  10.0  10.0
```

Figure 8. A data file listing 7 factors, their center point values, and percent variations for a 2-level fractional factorial orthogonal design.

The governing equation of the electromagnetics problem for the base design of the RF coil is:

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = \mathbf{0} \quad , \qquad (1)$$

where $\mathbf{E}$ is the electric field, σ is the electrical conductivity of water (= 0.0001 S/m), μ_r is the relative permeability of water (= 1.0), ε_r is the relative permittivity of water (= 80.0), ω is the circular frequency, and $k_0^2 = \omega^2 \mu_0 \varepsilon_0$ with μ_0 and ε_0 equal to the permeability and permittivity of the free space, respectively.

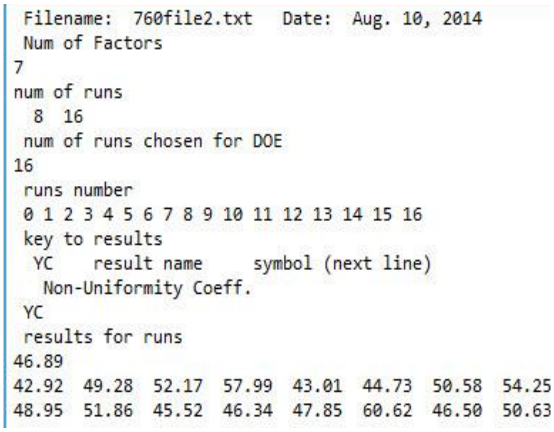
Using the RF module of the finite element analysis software package named COMSOL [5] and the application of the usual lumped port, scattering, and transition boundary conditions, we compute and plot in Fig. 4 the lumped port impedance vs. frequency in the 76 to 100 MHz range, and in Fig. 5 the magnetic flux density norm (BN) in the water phantom as a function of the dimensionless distance from the coil center line. It is interesting to observe that the BN profiles for most frequencies are highly non-uniform, and the first resonance frequency is found to be 78.5 MHz (see Fig. 4).

3. A Design-of-Experiments (DOE) Approach to FEM Uncertainty Analysis

Before we conduct an uncertainty analysis of the FEM solution of the base design so as to develop a design optimization strategy, we need to define a parameter of interest as a metric for optimization. In Fig. 6, we first identify the BN profile at the resonance frequency, 78.5 MHz, by

Table 1. Values of 7 parameters for each FEM run

	X1	X2	X3	X4	X5	X6	X7
	Sigma	Epsilonr	C	V0	w1	L3	bet2
Base Run (00)	0.0001	80	177	500	80	35	5
Unit	*S/m*	*1*	*pF*	*volt*	*mm*	*mm*	*degree*
+ / - variation	10 %	5 %	2 %	2 %	5 %	10 %	10 %
Run No. (01)	0.00009	76	173.46	510	84	38.5	4.5
Run No. (02)	0.00011	76	173.46	490	76	38.5	5.5
Run No. (03)	0.00009	84	173.46	490	84	31.5	5.5
Run No. (04)	0.00011	84	173.46	510	76	31.5	4.5
Run No. (05)	0.00009	76	180.54	510	76	31.5	5.5
Run No. (06)	0.00011	76	180.54	490	84	31.5	4.5
Run No. (07)	0.00009	84	180.54	490	76	38.5	4.5
Run No. (08)	0.00011	84	180.54	510	84	38.5	5.5
Run No. (09)	0.00011	84	180.54	490	76	31.5	5.5
Run No. (10)	0.00009	84	180.54	510	84	31.5	4.5
Run No. (11)	0.00011	76	180.54	510	76	38.5	4.5
Run No. (12)	0.00009	76	180.54	490	84	38.5	5.5
Run No. (13)	0.00011	84	173.46	490	84	38.5	4.5
Run No. (14)	0.00009	84	173.46	510	76	38.5	5.5
Run No. (15)	0.00011	76	173.46	510	84	31.5	5.5
Run No. (16)	0.00009	76	173.46	490	76	31.5	4.5



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 num of runs chosen for DOE
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  YC     result name     symbol (next line)
   Non-Uniformity Coeff.
 YC
 results for runs
46.89
42.92  49.28  52.17  57.99  43.01  44.73  50.58  54.25
48.95  51.86  45.52  46.34  47.85  60.62  46.50  50.63
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Figure 9. A data file listing the results of the center point run and 16 runs of the DOE exercise for uncertainty analysis using DATAPLOT.

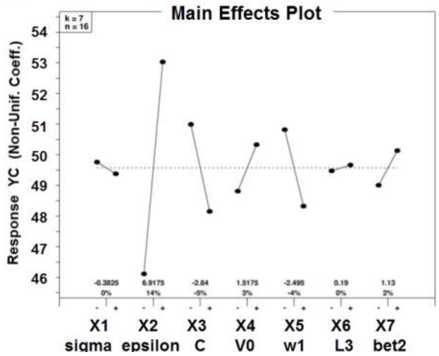


Figure 10. Main Effects Plot of a 7-factor, 16-run 2-level, fractional factorial orthogonal DOE.

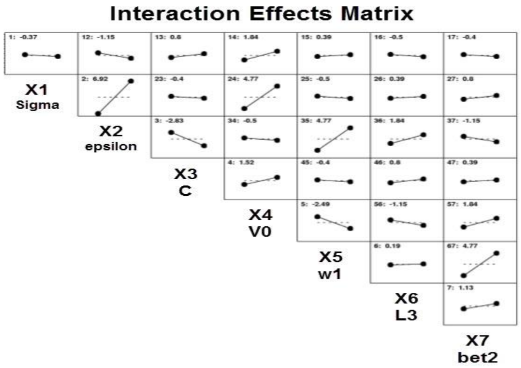


Figure 11. Interaction Effects Matrix of a 7-factor, 16-run, fractional factorial orthogonal DOE.

interpolation, and then measure the coordinates of four points on the profile, *(R1, B1), (R2, B2), (R3, B3),* and *(R4, B4),* such that $R1 = 0.05$, $R2 = 0.15$, $R3 = 0.25$, and $R4 = 0.35$. Denoting the mean magnetic flux density norm by *mB*, we define three quantities as follows:

$$mB = (B1 + B2 + B3 + B4) / 4 \quad ; \qquad (2)$$

$$d_i = Bi - mB\ , \quad \text{for } \ i = 1, 2, 3, 4 \quad ; \qquad (3)$$

$$YC = (\Sigma\, d_i^2\,)^{1/2} / mB, \quad \text{for } \ i = 1, 2, 3, 4\ . \quad (4)$$

Here, *YC* stands for a dimensionless quantity to be named the "non-uniformity coefficient " of the magnetic flux density norm profile.

Out of more than 70 parameters of the base coil design, we select seven as factors for a 2^{7-3} fractional factorial, 2-level, orthogonal design (Fig. 7). The names, values, and % variations of the 7 factors are given in Fig. 8 and Table 1.

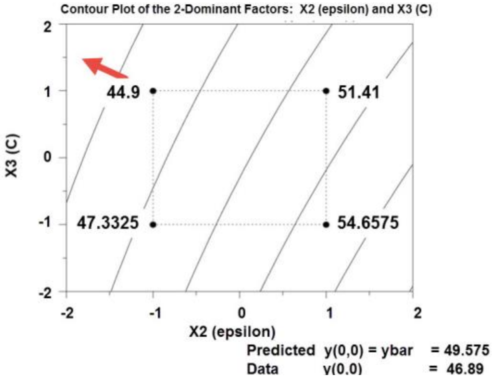


Figure 12. A contour plot of the two-dominant Factors, X2 and X3, of the 7-factor, 16-run DOE.

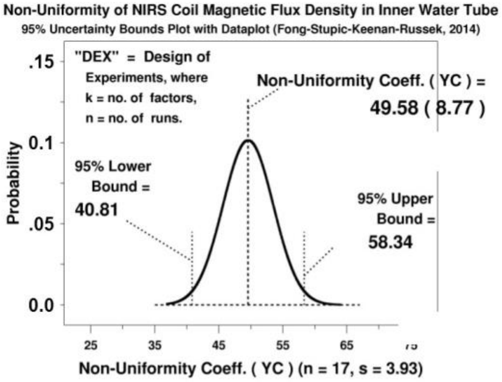
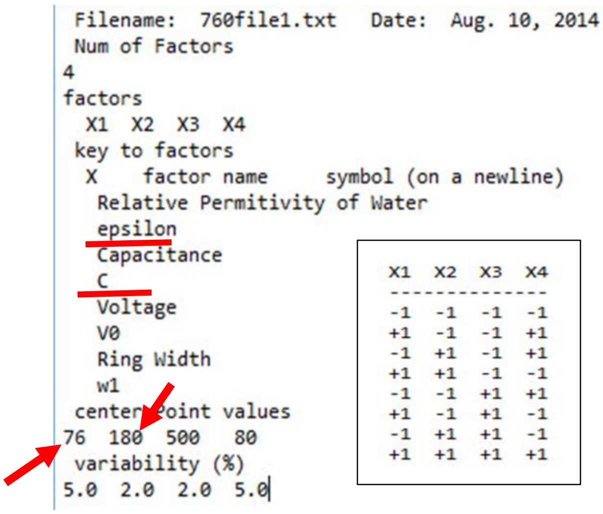


Figure 13. An uncertainty estimate of the non-uniformity coefficient at 95 % confidence level

Using the parameters specified in Table 1 for each of the 16 runs, we compute the BN profiles at their respective resonance frequencies, and their non-uniformity coefficients, *YC*, as listed in Fig. 9. We then conduct an uncertainty analysis of the 16-run plus a center point (the base design solution) experiment, using a computer code written in DATAPLOT [29-30]. The key results of the analysis are given in Figs. 10-13.

In Fig. 10, we observe that the relative permittivity of water (X2) and the port capacitance (X3) are dominant. In Fig. 12, we show a contour plot of the two dominant factors such that a strategy of design optimization is indicated by a red arrow in the direction of smaller *YC*. In Fig. 13, we observe that the 95 % confidence interval estimate of *YC* is given by 49.58 (8.77). We also observe in Fig. 11 that there are several interaction effects, but they could be ignored in developing a first-order design optimization strategy.

We now apply the strategy to introducing a new coil design with a changed epsilon and C as shown in Fig. 14. Instead of 7 factors, the new experimental design for the second coil uses only 4 factors and 8 runs as shown in Fig. 14. The uncertainty analysis results, as shown in Figs. 15 and 16, yield a better interval estimate, *YC-2* , i.e., 48.32 (1.9), as compared with the first *YC*.



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   epsilon
   Capacitance
   C
   Voltage
   V0
   Ring Width
   w1
 center point values
76  180  500   80
 variability (%)
5.0  2.0  2.0  5.0
```

```
X1  X2  X3  X4
--------------
-1  -1  -1  -1
+1  -1  -1  +1
-1  +1  -1  +1
+1  +1  -1  -1
-1  -1  +1  +1
+1  -1  +1  -1
-1  +1  +1  -1
+1  +1  +1  +1
```

Figure 14. The data file of the second DOE with a reduced number of factors in order to minimize the non-uniformity coefficient of the NIRS coil.

4. Significance and Limitations of the DOE Approach to Uncertainty Analysis as a Tool for Design Optimization

The uncertainty analysis of the finite element method-based solution of the RF coil using a design-of-experiments approach is significant in the sense that it offers a coil designer a first-order strategy for design optimization. However, the approach is limited in the sense that it requires the user to exercise judgment in selecting a relatively small number of parameters for implementation. In case of doubt, one can, nevertheless, try several schemes to achieve an ultimate objective of design optimization.

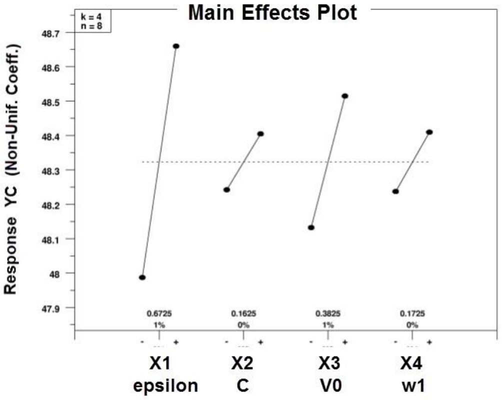

Figure 15. The Main Effects Plot of a 4-factor, 8-run, 2-level fractional factorial orthogonal DOE.

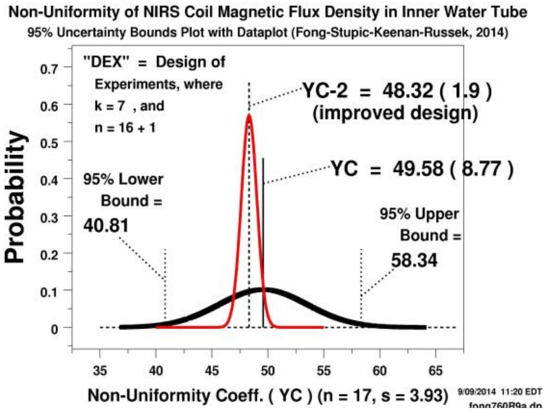

Figure 16. Uncertainty estimates of the two non-uniformity coefficients, *YC* of the first design and *YC-2* of the improved design (in red).

5. Concluding Remarks

Using a numerical example of a simple MRI RF coil design, we have demonstrated that it is feasible to generate a design optimization strategy by applying a design-of-experiments approach to an uncertainty analysis of COMSOL-RF solution results. This should open the door for assessing and improving the design of many state-of-the-art MRI RF coils, as, for example, presented by Suga, Saito, Takahashi, and Ito [33], and Gurler and Ider [34].

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