Modelling the Sound Radiation by Loudspeaker Cabinets

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Abstract:
While musical instruments often rely on a body which resonates on purposefully to amplify the vibration produced by a string or a membrane, such as in a violin or a guitar, loudspeaker cabinets should not contribute at all to the total sound radiation, but aim instead to be a perfectly rigid box which encloses the drive units in charge to transform the electrical signal at their terminal into acoustic waves.
A direct sound radiation measurement of the cabinet contribution alone is almost impossible to obtain because of the far higher masking level of the main drive units radiation, thus only vibration based measurements through accelerometers or no-contact techniques are viable options (this makes even more desirable the capability of accurately modelling inside a fem package the cabinet vibration behaviour and compare different design option for a product optimization). It`s worth mentioning that prototyping curved panels requires an effort which is far greater than prototyping standard “box style” cabinets, thus reducing the total number of prototypes to be tested is of the utmost importance when developing curved panels cabinets like in this case study.
Two phenomena contribute to the total SPL radiated from the cabinet: the acoustic field exciting the wall vibration and leaking by transmission through the wall (which will be the subject of another paper in the near future), and the direct mechanical excitation of the cabinet by the reaction force related to the drive units operation, the subject of this paper.
Modelling steps include measuring and fitting orthotropic material properties, including damping, simplifying the geometry for the purpose of saving computation time, 3D mechanical modelling with curvilinear coordinates system and thin elastic layers, and proper post processing to extract useful data and easy to compare plots.
While proving an effective and powerful tool for optimization of cabinets, where geometries and materials can be easily tested without building the actual cabinets, the understanding of modes coupling and the role of reinforcement panels is an additional benefit. The predictions have been validated with scanning laser Doppler measurements and single point vibration measurements with an accelerometer.

Keywords: acoustics, radiation, resonance, orthotropic properties, thin elastic layers, curvilinear coordinates system.

1. Introduction: problem description

The aim of this project is to estimate the total radiated acoustic output of a loudspeaker cabinet. The ultimate goal, to use finite element simulations to improve loudspeaker enclosure designs.

Lipshitz, Heal and Vanderkooy looked at the impact of cabinet vibration of the perceived sound quality (see [1]), concluding that in some cases cabinet mechanical resonances were audible. Olive’s work on modification of timbre by resonance [2] also demonstrated that low amplitude resonances – typical of cabinet vibration – do affect sound quality. More recently Grande [3] came to the same conclusion, highlighting the impact of the cabinet on the time domain behaviour (spectral decay), while Bastyr and Capone measured the sound pressure level produced by a commercial floor standing loudspeaker with cabinet surfaces large enough to show radiation levels comparable with that of the main drive units in the frequency range 100-300Hz [4].

This acoustic radiation is caused by two phenomena:

- The mechanical reaction force of the electrodynamic transducer (also referred to as the loudspeaker drive unit, or simply drive unit), causing the assembly to vibrate,
- The internal sound pressure – caused by the same drive unit - which excites the cabinet walls and leaks by transmission. This is similar to the way a window or wall transmits noise.

The former is described in detail in section 7. The latter is not considered in this work for the following reasons:

- The internal volume is filled with sound absorbing material, limiting the impact of internal acoustic standing waves,
- Experimental evidence suggests that the mechanical excitation is the main source of cabinet radiation, at least for low frequencies. The definition of a low frequency region depends on the system size and is subject to interpretation, but is typically below 1000 Hz.

As the acoustic output of the enclosure assembly is low compared to the main transducer output, directly measuring it is complex and prone to errors.

A possible hybrid method would rely on accurately measuring the outer surface wall velocity and relying on a FEM or BEM method to estimate the acoustic output in the far field, as investigated in [1] and [3]. Not only is this option difficult to implement in practice (ideally relying on 3D laser Doppler vibrometry) or consuming in terms of computation time but it also requires a working prototype. This makes even more desirable the capability of accurately modelling inside a FEM package the cabinet vibration behaviour and compare different design options for a product optimisation. It`s also worth mentioning that prototyping curved panels requires an effort which is far greater than prototyping standard “box style” cabinets.

The system considered in this study is the low frequency section of the loudspeaker system shown below:

Figure 1: loudspeaker system considered, B&W 800 Diamond

The transducers of interest are the two 10” units located in the lower enclosure. The low frequency section reproduces the low end of the audio spectrum and is fed through a passive electrical filter similar to a second order low pass at 300Hz.

The following work focuses on assessing the cabinet acoustic output in the 10 to 2000 Hz region. Above this upper frequency the input signal is strongly attenuated and therefore less critical.

4. Materials

4.1 Plywood

The enclosure is a complex assembly:

- curved wood external wrap,
- internal MDF stiffening panels (Matrix™),
- Cast aluminum plinth on steel spikes.

Our material of choice for the cabinet wrap is birch plywood, a stack of wood sheets and binding resins, with the fibres alternatively orientated in each layer. The laminate is bent into shape (the “wrap”) using heat and pressure.

The resin layers are not individually simulated and an equivalent bulk orthotropic formulation is used to describe the plywood behaviour.

Let’s first write the general mechanical stress-strain relation in a matrix form for each layer:

$$\varepsilon_{xy} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\upsilon_{xy}}{E_x} & -\frac{\upsilon_{xz}}{E_x} & 0 & 0 & 0 \\ -\frac{\upsilon_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\upsilon_{yz}}{E_y} & 0 & 0 & 0 \\ -\frac{\upsilon_{xz}}{E_x} & -\frac{\upsilon_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{zx}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

Equation 1: stress strain relation

Let’s assume the layer plane is x-y: because we only consider bending modes, E_z, G_{yz}, G_{zx}, υ_{xz} and υ_{yz} have a negligible impact on the plate behaviour. So, only four elastic constants need to be assessed: E_x, E_y, υ_{xy} and G_{xy}.

Symmetry arguments can also be used. Assuming the wood fibres are in the X direction, the directions Y and Z are then identical:

- $E_y = E_z$
- $G_{xy} = G_{xz}$
- $\upsilon_{xy} = \upsilon_{xz}$

We assume $\upsilon_{yz} = 0$ and $G_{yz} = G_{xy}$.

In order to estimate these parameters, flat plate samples were suspended inside a frame with elastic strings - to replicate a free-free condition – and acoustically excited. The resulting velocity was measured on a regular mesh of points using a POLYTEC laser vibrometer. The first four plate bending modes were identified.
The solid mechanics eigenfrequency COMSOL model for the plate samples was as follows:

- Independent plywood layers,
- Individual coordinate axis for each layer in XY plane, X collinear with fibre direction,

The four bending mode values are then used to estimate a unique set of E_x, E_y, υ_{xy} and G_{xy} parameters.
The figure below shows the first plate bending mode (13 layer laminate), measured and simulated:

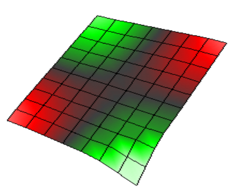

Figure 2: plate measured first bending mode

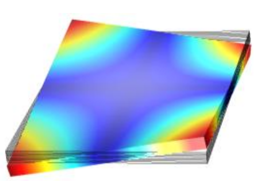

Figure 3: plate simulated first bending mode

The estimated material parameters for each individual layer were:

- E_x = 17.5 GPa
- E_y = 1.5 GPa
- G_{xy} = 1.17GPa
- υ_{xy} =0.03

As simulating each individual layer is unpractical for the complete cabinet assembly, an equivalent orthotropic bulk material was then created. A different equivalent material had to be derived for each laminate configuration.
The first five bending modes were used to match the COMSOL models, with the same assumptions made regarding the non-critical elements of the compliance matrix:

Laminate Configuration	E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	υ_{xy}
21 layers	8.45	10.75	1.16	0.03

Modes, exact solution (Hz)	Bulk approximation (Hz)
592 1225 1679 2154 2416	592 1226 1679 2156 2419

The results above clearly suggest that an orthotropic bulk formulation is suitable to accurately match the plate behavior.

4.2 Other materials

The other materials used in the assembly are:

- MDF panels,
- Aluminium plinth and drive unit chassis,
- Steel and Neodymium loudspeaker motor,
- Water based glue joint,

All metallic parts use the COMSOL material library default material properties. Neodymium parts, having a negligible impact on the system mechanical modes, use structural steel properties.
The Young's moduli for the MDF panels were estimated by using the technique described in 4.1.

MDF is a non-isotropic material as its density (and therefore modulus) varies across the panel thickness. However, as only bending modes are of interest here, an isotropic formulation is accurate enough to match the first four bending modes.
The plywood and MDF hysteretic damping values were estimated by mechanically exciting beam samples in a clamped-free configuration and using a modal analysis fitting technique (see [5]).
For the water based glue, being a viscoelastic material, a DMA analysis was performed using the temperature-frequency principle (see [6]) to estimate dynamic Young's modulus and loss factor.

5. Glue joint construction

As replicating the exact glue joint geometry is unrealistic, a thin elastic layer formulation is used to approximate the joint's mechanical behaviour. As a truly viscoelastic behaviour cannot be simulated with a thin elastic layer, a single modulus and loss factor figures at 500Hz were used.
A separate COMSOL model was constructed to validate this approach. Two different glue joints are used in this assembly:

- Matrix to matrix panels: a dove tail construction is used with 0.5mm clearance between parts. The glue bead is applied on the outside,
- Matrix panel to curved wrap: a 10mm deep groove is machined in the wrap with a 0.5mm glue layer.

The joints details are shown below:

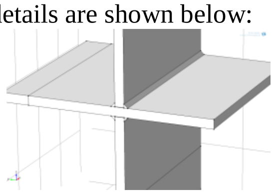

Figure 4: matrix to matrix glue joint

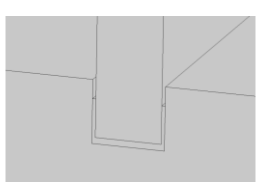

Figure 5: wrap to matrix groove

Both assemblies were modelled with the exact geometry and the thin elastic layer. The stiffness of the thin elastic layer was then adjusted to get a close match for the first six eigenfrequencies.
The approximated stiffness values are subsequently used in the complete assembly.

5. Numerical model

5.1 Description

Since the cabinet is composed of a curved wrap which constitutes the lateral wall, a curvilinear coordinate system was implemented to project the orthotropic properties of plywood along the curved surface:

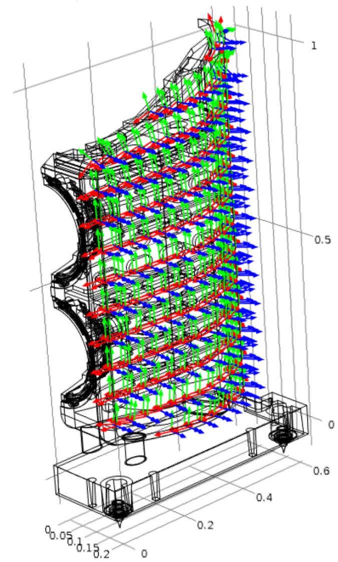

Figure 6: wrap curvilinear coordinate system

The joints between the matrix panels and the bottom, top, front and wrap are simulated through the use of the "Thin Elastic Layer" boundary condition.
The model is constrained with a "Fixed Constrain" boundary condition at the spikes tips.

Following this a 3D model of the cabinet was simulated in three steps:

- A stationary study to solve the curvilinear coordinate system used for the orthotropic wrap (diffusion method),
- A solid mechanics Eigen frequencies study
- A solid mechanics frequency modal domain study
- A solid mechanics frequency domain study.

As the number of modes in the frequency range of interest was low enough, it proved very useful to use the frequency domain modal solver to further reduce computation time.
Due to the left-right symmetry of the system, only half of the structure is used.

5.2 Output quantities

In order to compare measurements performed on prototypes with the models, the acceleration and/or velocity magnitude distribution over the cabinet walls is the most straightforward quantity to examine. Unfortunately it`s not trivial to assess performances of a cabinet in terms of radiated sound solely by looking at the vibration pattern because of complex phase relationships between velocities of different sections of the cabinet.
As usual in engineering optimisation problems, a large number of approximate predictions are more useful than a low number of very accurate predictions, thus the computation of the Rayleigh integral over the main walls proved to be a reasonable trade off to start with, since it could be easily implemented in COMSOL and required a very limited amount of post processing time. This simplification is valid if the radiating boundary is shallow: the Rayleigh estimation is therefore limited to the front flat baffle and the wrap side.

7. Electromechanical coupling, post processing

7.1 Principle

Our forced response simulation assumes a unitary force input applied to the magnetic motor. The reality is more complex as the force generated by the transducer motor system and applied to the structure is highly frequency dependent. This relates to the electromechanical transfer function between the input electrical voltage applied across the transducer terminal and the motor output force. This transfer function was analyzed using a linear lumped parameter electromechanical model which describes the transducer cone (assume infinitely rigid) and the electrical impedance of the coil. This approach allows us to simply post-process the model output quantities with the estimated transfer function.

This lumped parameter approach is well documented (see [7] to [8]) but normally used to describe the low frequency behaviour of loudspeakers.

7.3 Transducers used in cabinet assembly

The transducers used in the final assembly were based on two 10 inch bass units with the cone removed (see below) in order to limit the amount of radiated sound and the related acoustic excitation of the cabinet walls.

Using this method, the assembly is only mechanically excited by the motor reaction force, as for the COMSOL simulation.
The derived voltage to force transfer function derived using the method used in 7.1 is shown below:

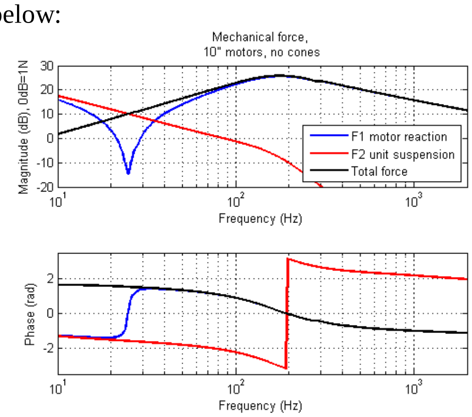

Figure 7: voltage to force transfer function for the cabinet transducers

8. Final assembly, experimental results

8.1 Mechanical behaviour

The purpose built drive units described in section 7.3 were driven while the velocity of each point on a user defined grid over the wrap and front baffle (the two largest radiating surfaces thus the most important) were measured with a Polytec laser Doppler scanning system. The measured modal shape and frequency, on the left, are compared below to the predicted data, on the right, for the front baffle, for the first relevant modes.

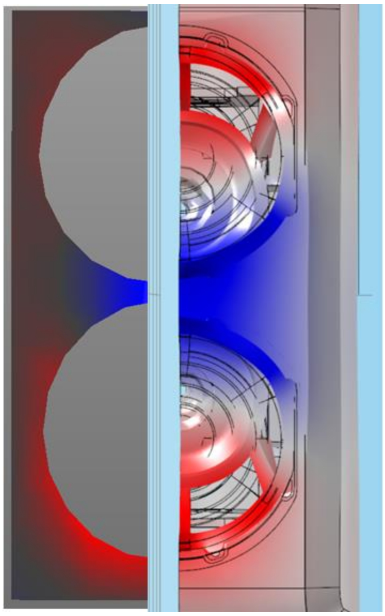

Figure 8: front baffle normal velocity at 222 Hz measured / 281 Hz predicted frequency

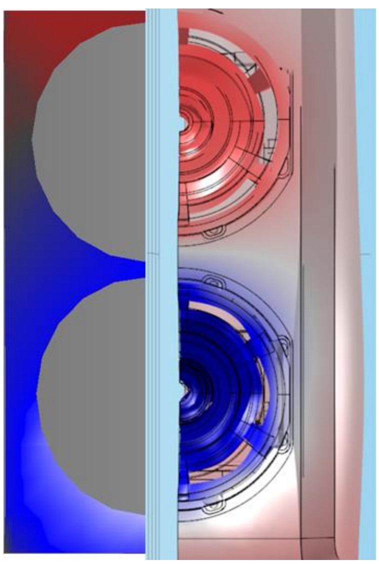

Figure 9: front baffle normal velocity at 313 Hz measured / 466 Hz predicted frequency

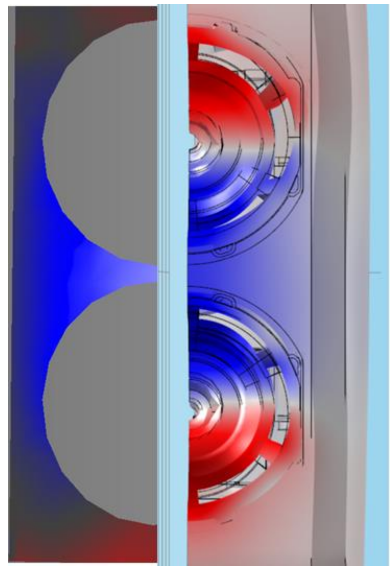

Figure 10: front baffle normal velocity at 603 Hz measured / 690 Hz predicted frequency

As comparing animated simulated forced responses to measured responses can be visually misleading, the measured and predicted acceleration magnitude spectra were also overlaid for two critical locations (cabinet front baffle and middle of side baffle):

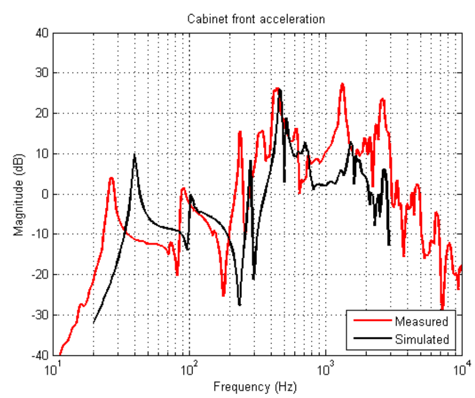

Figure 11: cabinet front acceleration

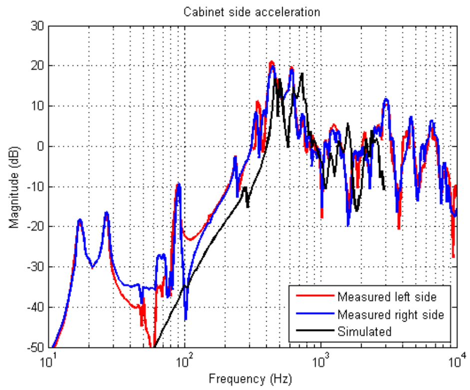

Figure 12: cabinet side acceleration

The following features are visible in these spectra:

- Peaks below 90Hz are related to the rigid body motion of the enclosure on its spikes (acting like springs),
- The 90 Hz peak is a plate bending mode of the plinth,
- Peaks in the 250Hz to 900Hz region are complex modes related to the front baffle and side wrap.

The simulation was able to reproduce these three regions fairly well but systematically over-estimates the modes frequency values by 10 to 20%.
The largest discrepancy was for the wrap acceleration below 100Hz. The left to right symmetry assumption used was clearly not valid for the rigid body modes on the spikes – the left and right side measurements differ by up to 15dB. As each spike was terminated by a small 1mm spherical cap, obtaining a consistent mechanical interface with the floor below was difficult.

Finally, predicted peak amplitudes and Qs are realistic.

8.2 Acoustic output estimation
For a flat vibrating surface of area S mounted on a rigid baffle, the pressure at a point P and at a distance r from the vibrating point can be expressed by the Rayleigh integral

$$p(P) = \int_S 2i\rho\omega v_n G(r) dS$$

Where G(r) is the full space Green`s function defined as

$$G(r) = \frac{e^{-ikr}}{4\pi r}$$

ρ is the density of the medium, ω the angular frequency, v_n the normal velocity.

In practice for a three dimensional radiating surface, the Rayleigh integral “projects” all the radiating elements over a plane and is only a valid approximation for shallow objects.

The probe location for the computation of the Rayleigh integral is shown below (small sphere in the bottom right corner). The probe height is in the middle of the cabinet height.

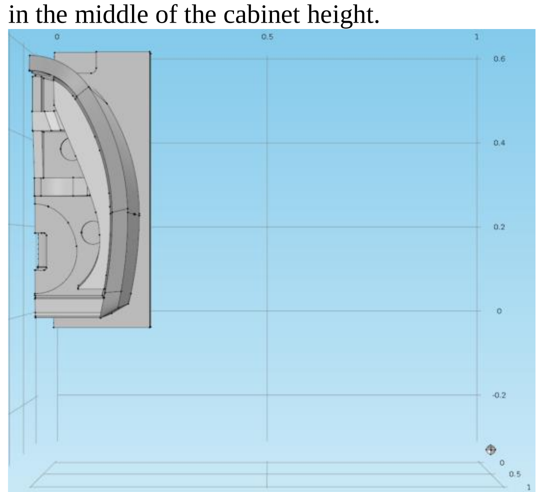

Figure 13: probe location for the Rayleigh integral computation relative to the cabinet

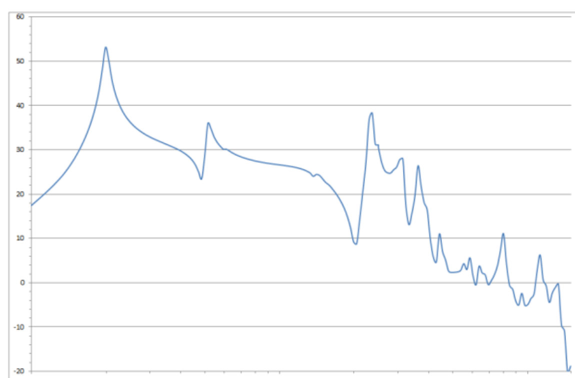

Figure 14: Sound Pressure Level, in dB, radiated by the front baffle

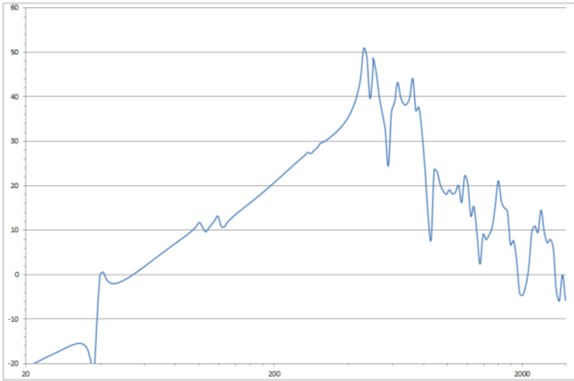

Figure 15: Sound Pressure Level, in dB, radiated by the cabinet side (wrap)

The two predicted SPL plots versus frequency for the same probe location look similar to the acceleration plots (Figures 14-15), thus most modes are radiating.
The crossover filter transfer function is applied to these responses thus the slope above 300Hz is affected by its low pass nature.

9. Conclusions

A complete loudspeaker enclosure model was constructed, including the use of orthotropic materials combined with a curvilinear coordinate system and hysteretic damping. Simulated accelerations show good agreement with measured results; the model is able to accurately predict trends even if overestimating the resonances frequency values.
This model provides a valuable tool for designing improved loudspeaker enclosures.
The minimization of the cabinet radiation in terms of SPL can be pursued by various means: moving the modes to a frequency range where damping material is most effective, moving them out of the audible frequency range, modifying the modal shapes in order to minimize their radiation efficiency and thus the radiated sound pressure.

A different perspective on the problem is analysing the time domain behaviour. Many resonances even if small in amplitude in the steady state response can be clearly perceived as sound coloration because if showing high Q the energy decay in time is quite long. Thus after the excitation has stopped, the drive units SPL decay is quicker and thus is overpowered by the cabinet SPL decay which is slower.

Further work should improve the accuracy of modes frequency prediction and evaluate through simplified BEM formulations the total acoustic power radiated by the cabinet.

10. References

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11. Acknowledgements

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