Modelling of Ultrasonic Transducers and Ultrasonic Wave Propagation for Commercial Applications using Finite Elements with Experimental Visualization of Waves for Validation.

D. Andrews

Cambridge Ultrasonics Ltd, Church Farm Barn, Horse Ware, Over, Cambridgeshire CB24 5NX, UK.
camben@cambridgeultrasonics.com

Abstract: Finite element (FE) modelling of ultrasonic propagation using Comsol Multiphysics, or indeed any other FE software, inevitably involves large computations because two or three dimensional time-stepping models are needed to predict the behaviour of these kinds of travelling mechanical waves. Comsol can be used to create video of waves in these models, which is both attractive and informative. Unfortunately, for time-stepping solutions, it is possible for numerical instabilities to grow sufficiently large to dominate the solution so that the eventual FE prediction does not match physical behaviour. Any design of transducer that is based upon such flawed FE models alone is unlikely to perform as expected and will almost certainly result in delayed and costly development. It is valuable, if not essential, therefore, to have independent experimental observations of wave propagation that can give a good degree of validation of FE predictions. One approach to providing experimental validation is to use receiver transducers, such as hydrophones (in water) and microphones (in air), to sample the ultrasonic wave at a point (or integrated over some region) but the disadvantage is that receiver signals do not provide views of the propagation path that can be directly compared with Comsol videos. However, another approach to validation, which is less well-known but highly effective, is to render visible ultrasonic waves in transparent materials using a combination of stroboscopic illumination and contrast enhancement, for example: polarized light or schlieren or shadowgraph techniques, and by these means create videos and still images that can be compared directly with Comsol videos and still images. The combination of FE modelling with successful experimental visualization provides a high degree of confidence in the FE models and allows rapid development of complex ultrasonic transducers to proceed efficiently and cost-effectively. Examples of video and still images of FE predictions and examples of experimental visualization of ultrasonic waves propagating from transducers are presented and compared to illustrate the usefulness of this kind of validation.

Keywords: ultrasound, sound, transducer, propagation, schlieren, visualization, validation, FE.

1 Introduction

Ultrasound has a growing number of applications that can be divided into classes:

- Material processing – using high power ultrasound.
- Echo-location – generally using low-power ultrasound, generating some form of image from ultrasonic echoes.
- Communications –may use low power or high power.

Some noteworthy examples of applications are:

- Medical imaging and treatment – imaging tumours and using high power ultrasound to destroy the tumour cells.
- Sonar imaging underwater.
- Sonochemistry –creating high pressures and temperatures to promote chemical reactions.
- Collision avoidance of vehicles.
- Seismic geological exploration.
- Non-destructive testing of materials.

Cambridge Ultrasonics (CU) is a research and development consultancy business with skills in physics, mathematics, electronic engineering, software engineering and ultrasonic technology.

Of importance in any ultrasonic system is the ultrasonic transducer. It is used for either: (a) generating ultrasonic waves or (b) receiving ultrasonic waves and converting them into electrical signals. The design of transducers is important because their performance affects the performance of the entire ultrasonic system.

Transducer can generate ultrasonic waves with special characteristics, for example:

- Collimated plane waves.
- Omni-directional waves.
- Focussed waves.
- Waves in a controlled burst or chirp.
- Waves with controlled frequency and amplitude.

In most cases receivers are required to generate electrical signals that are faithful representations of the incoming waves. Consequently they must generally have these desirable properties:

- Good sensitivity.
- Matched to the spatial distribution of wave-fronts expected to arrive.
- Matched to the frequency distribution of wave-fronts expected to arrive.
- Sufficient bandwidth for the task.

2 Ultrasonic transducer design and development

Since the advent of inexpensive, powerful computers it has become possible to perform FE and finite difference modelling of ultrasonic wave propagation and of transducers. The multiphysics capability of Comsol allows problems involving: piezoelectric physics, linear and non-linear mechanical wave propagation, heat generation and thermal diffusion to be solved simultaneously, which is an advantage. Some difficulties associated with solving the wave equation and the associated equations of state for the propagation of elastic waves through solids and fluids are considered next.

3 Mechanical wave propagation

Wave propagation problems are currently solved in FE by dividing the time-domain of the solution into small intervals or time-steps, in much the same way that the spatial-domain is divided into small spatial elements by a mesh. Effectively, the problem is that of solving a 4-dimensional problem if 3-dimensional geometry is used. As the number of dimensions of a FE model increases, the number of nodes generally increases and so too does the time to compute a solution. Consequently, there is always pressure to minimize the number of spatial elements and time-steps to allow calculations to be performed quickly or in a reasonable time.

However, there is an important consideration in both the time-domain and the spatial-domain, which is: does the fineness of time/spatial elements permit the features of the solution to evolve accurately? In FE models of wave propagation this generally depends upon the frequency and wavelength of the ultrasound and is related to Nyquist sampling of signals (Sklar, 1988): if the spatial element size (mesh size) is larger than or equal to half the wavelength then the numerical model will generate errors and likewise if the time-step size is longer than or equal to half the period of the highest frequency of wave then there will be errors. In practice, to obtain good quality results, the spatial-domain and time-domain sampling must be substantially finer than these Nyquist-like limits.

An added problem is non-linear propagation (Beyer, 1974). All materials exhibit non-linear effects during the passage of mechanical waves. Linear waves have infinitesimally small amplitudes. Finite amplitude waves, such as those used in echo-location and also the large amplitude waves used in high power ultrasound, all have non-linear effects associated with them. Gases are compressible and they are weakly non-linear; liquids are virtually incompressible and consequently they are more strongly non-linear than gases and the same is true of solids.

The effects of non-linearity are progressive with distance travelled by the wave: (a) waves that are initially sinusoidal generate harmonics which turn the sinusoid into a saw-tooth wave-form (the compressed half-cycle travels faster than the rarefaction half-cycle), (b) when two waves of different frequencies mix spatially they generate new waves at the sum and difference of their frequencies. The existence of harmonics such as 2nd, 3rd, 4th and 5th and sum frequencies means there is the potential for Nyquist-like sampling problems to emerge; for example, if the mesh size is larger than half the wavelength of the highest harmonic then that harmonic can cause instabilities in the numerical solution. High order harmonics are particularly problematic for FE numerical calculations. Comsol allows the user to select a linear or non-linear solution but the former can introduce substantial errors.

A further problem unique to wave propagation in solids is the diversity of wave-types that solids can support. Compression waves, familiar as sound waves in air, are present but so too are: shear waves, surface waves and variants of surface waves such as Rayleigh, Love and Stoneley waves. It is common for waves of one type to generate waves of another type by a process called mode-conversion, which invariably happens when waves interact with a free surface. The boundary condition of the surface being free of stresses (but not free of displacement) requires that new stresses must be generated on a surface to cancel the (parent) waves when they arrive there. The stress-cancelling surface stresses then become sources of new waves but the sources travel at the speed of the parent waves. Consequently, it is possible for the sources to travel faster (literally super-sonic) than the new mode-converted waves (for example, compression waves travel faster than shear waves and surface waves), with the result that the slower waves must emerge from the surface travelling in a different direction to the waves that caused them. The frequency of the waves always remains unchanged and, consequently, the mode-converted waves must have a smaller wavelength than their parent-wave. It is the reduction in size of the wavelength that can create a difficulty for choosing a mesh size.

The wavelength of waves can also change if waves are transmitted across a boundary between two materials with different wave speeds. The new wavelengths must also be considered when choosing a mesh size.

Wave propagation is generally solved by time-stepping calculations in FE: taking the answer of the previous time-step and using it as the initial condition for the next time-step. Consequently, once a Nyquist-like error emerges it will propagate throughout the solution and it can evolve to dominate the solution. In severe cases, the convergence criterion often fails in the solver and terminates the calculation. In less severe cases a solution is found for all the time-steps but it does not represent physical behaviour accurately. It can be difficult for the user of FE software to detect this kind of error.

It is essential to have a critical attitude towards a numerical solution for ultrasonic and sonic wave propagation problems.

4 Experimental validation methods

It is desirable if not essential that critical review of the results of FE models should be based upon experience gained for experimental observations. Experiments act as validation of the numerical model solutions. Validation could be of a qualitative nature, for example: comparing videos of experimental results of wave propagation with Comsol videos of the same arrangement, or validation could be quantitative, for example: comparing signals from an ultrasonic receiver with an equivalent signal generated using a Comsol model.

Schlieren visualization of ultrasonic waves using high performance light emitting diodes (Andrews & Wallis, 1977) is one advantageous method of validating FE models. It uses pulsed ultrasound, stroboscopic illumination and a method of optical contrast enhancement. Images can be viewed directly by eye but more conveniently by a video camera. Schlieren (Andrews, Study of wavefronts in acoustic diffraction patterns using a stroboscopic schlieren technique, 1982) is a method of optical contrast enhancement; other methods use polarized light or are known as *shadowgraph*. Some features of schlieren visualization are:

- Schlieren imaging can be performed using any transparent material, including: gases, liquids and solids.
- Schlieren visualization generates a 2-dimensional image of a three dimensional field of pressure waves. It integrates optical contrast contributions (caused by ultrasonic waves) along the optical path.
- It is possible to obtain quantitative values from schlieren images.
- Schlieren images can easily be compared with Comsol videos.
- Schlieren can be used not only to evaluate transducers (and validate Comsol models of transducers) but it can also be used to examine ultrasonic wave propagation effects, including: reflection, refraction, mode-conversion, reverberation, transmission, diffraction.

From long experience at CU, schlieren visualization experiments are excellent forms of validation for equivalent FE models.

5 Schlieren visualization

Schlieren visualization experiments were performed using an ultrasonic transducer emitting bursts of waves at 500 kHz into a glass-walled tank of water. Within the tank was placed an aluminium cylinder, located directly in the path of the ultrasonic waves and with its axis parallel to the optical axis. Experiments show that plane waves and circular edge-waves emerge from the transducer and travel through the water. The plane waves then reach the solid cylinder where they are scattered.

In principal, schlieren visualization can be used to generate video images that can be compared with Comsol videos. However, video image quality collected directly from experiments can be of disappointing quality because even high-definition (1920 x 1080 = 2.1 Mpixels) video images have far fewer pixels than, say, the 12 Mpixels that can be collected in a still image. Also, still images are generally of about 1 s exposure time and, because the stroboscope repetition frequency is typically 100 Hz, the image is averaged over many frames so that the signal-to-noise ratio of the still image is usually much superior. Finally, video is generally collected at about 25 frames/s and this frequency can beat with the stroboscopic repetition frequency resulting in slowly moving black lines moving through the video – still images do not suffer from this effect.

CU has been improving schlieren imaging for many years and now uses a computerized method of experiment control so that the camera is used to collect a sequence of still time-lapse photographs that can be used to create video. These videos have the signal-to-noise ratio of still images and have no beat-frequency black bands. Video quality is very much higher.

6 Results

The following time-lapse sequence shows pairs of images of ultrasonic waves travelling in water towards the cylinder: photographs on the left are from schlieren visualization experiments; images on the right are taken from a Comsol FE model (see later). Note the two horizontal lines adjacent to the circle in the FE images are domain boundaries to aid meshing in water and have no physical significance and can be ignored for the purpose of comparison.

The predominant features are ultrasonic wave-fronts with a frequency of around 500 kHz and wavelength of 3.0 mm in water: straight-line features are plane waves (viewed parallel to the plane) and curved features are toroidal waves (in schlieren photographs) or circular waves (in FE model). Wave-fronts travel from the top of the image down towards the bottom (apart from reflected waves) as time advances.

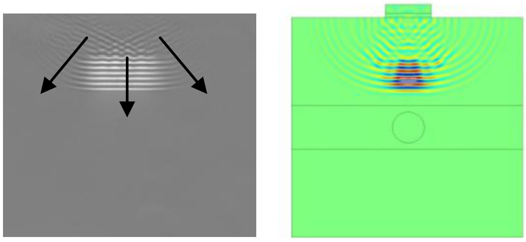

Figure 1 25.0 μs after launch of waves. Ultrasonic waves at 500 kHz emerging from an ultrasonic transducer in water. The photograph on the left was rendered visible using schlieren visualization. The image on the right is from a FE model. The arrows show the direction of motion of the waves.

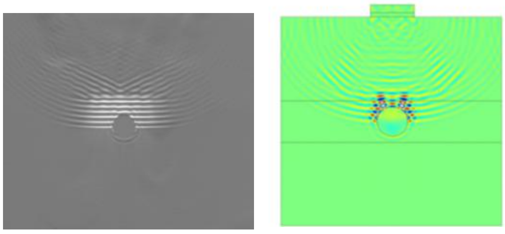

Figure 2 36.5 μs after launch of waves. Ultrasonic waves have reached a solid aluminium cylinder viewed along its axis. Some waves have been transmitted through the aluminium and can be seen below the cylinder as they are re-transmitted back into the water.

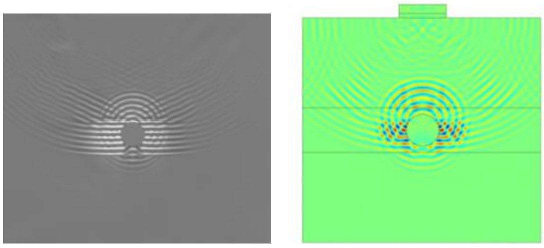

Figure 3 43.0 μs after launch of waves. A burst of ultrasonic waves is starting to pass an aluminium cylinder. Reflected circularly expanding waves can be

seen above the cylinder and circular waves can be seen below the cylinder, the latter waves caused by transmission though the cylinder and diffraction around it.

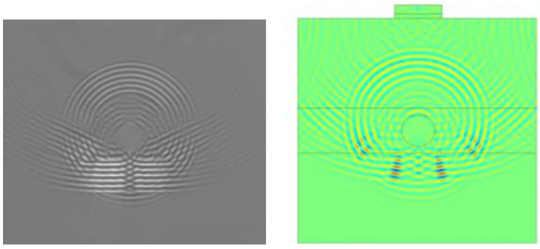

Figure 4 52.0 µs after launch of waves. A burst of ultrasonic waves has passed a solid aluminium cylinder and have been scattered by it. The pulse of waves reflected lasts longer than the incident burst showing that the cylinder has trapped some waves within it that are reverberating and periodically leaking energy back into the water.

In the FE models the colours represent pressure or stress in the range -200 kPa (blue) to +200 kPa (red). Waves are visible in all domains of the model, including physically opaque materials. In the schlieren visualization photographs it is difficult to be precise about how to interpret the contrast features of the image other than to say they are associated with the density changes caused by the presence of pressure waves in the water.

The FE model agrees closely with the experimental schlieren visualization images. This process validates the FE model.

7 Comsol model

Figure 5 shows the geometry used in the 2-dimensional FE model. Excessively long computation time makes it unattractive to work with 3-dimensional models in this case. Also, the model was restricted to linear material properties and a linear wave equation and constitutive relations.

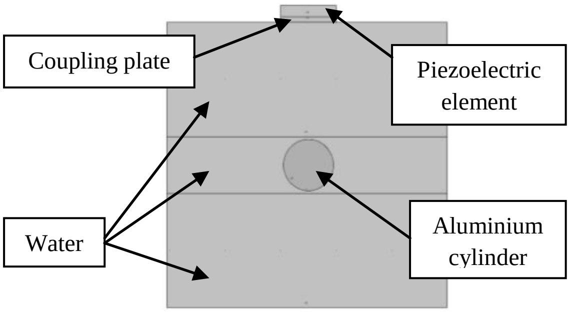

Figure 5 Geometry of the FE model.

The dimensions are a follows:

- Water tank – 101 mm x 100 mm (overall).
- Aluminium cylinder – 14 mm diameter.
- Piezoelectric disc – 20 mm x 4 mm.
- Coupling plate – 20 mm x 2 mm.

The model made use of Comsol's Transient Acoustic-Structure interface and the Piezoelectric Devices interface.

The combination of the piezoelectric element and coupling plate form an ultrasonic transducer.

The mesh has approximately 88,000 elements and is too fine to show here.

The piezoelectric disc was of the soft ferroelectric variety rotated so that its poling z-axis was parallel to the y-axis of the model geometry. It was configured with two electrodes: one on the surface in contact with the coupling plate, kept electrically grounded, and the other on the uppermost surface that was driven by a tone-burst excitation voltage of 100 V amplitude at 485 kHz for 10 µs. Using manufacturers' data for the soft ferroelectric material predicts a thickness mode of resonance for the piezoelectric disc of 485 kHz so the excitation is chosen to maximize the output of the transducer and to produce a burst of approximately five ultrasonic wave-fronts.

Domain	Typical mesh size (mm)	Type of mesh
Piezoelectric element	0.2	Mapped (square)
Coupling plate	0.5 x 0.2	Mapped (rectangular)
Aluminium cylinder	0.5	Triangular
Water (uppermost in contact with transducer)	0.2 to 0.5	Mapped (rectangular)
Water (in contact with cylinder and lowermost section)	0.02 to 0.5	Triangular

The solver was set to solve for times between 0 µs and 50 µs with time steps of 50 ns.

The speed of sound in water is 1500 ms^{-1} and calculation shows that the wavelength of ultrasound in water should be 3.1 mm at 485 kHz. The speed of compression waves in aluminium is 6400 ms^{-1}

and calculation shows the wavelength should be 13.2 mm. The speeds of compression waves in the piezoelectric disc and the coupling plate are both higher than the speed of sound in water

As discussed earlier, the mesh spacing should be less than half of 3.1 mm in water and half of 13.2 mm in aluminium. The table above shows that typical mesh sizes were between 3x and 7x smaller than the minimum required in water and between 10x and 20x smaller in aluminium. The mesh size used in the FE model is therefore substantially smaller than the wavelength, which is an essential and satisfactory situation for modelling wave propagation effects in these materials. There is a similar situation in the piezoelectric element and the coupling plate.

However, one must also consider the possibility of shear and surface waves being generated in the solids with wave speeds approximately 60% of the compression wave speed and, hence, wavelengths of 60% of the compression wavelength. For these slower waves calculation shows the mesh size used in the FE model should still be between 3x and 12x smaller than the Nyquist-like limit – again an acceptable conditions.

The period of the ultrasound is 2.06 μs. Consequently, the time-step should be less than half of this value and the time-step used in the FE model of 50 ns satisfies this requirement by 20x, which should be acceptable.

8 Video

Examples of high quality video from schlieren visualization experiments and Comsol videos are available on CU’s web-site.

www.cambridgeultrsonics.com

9 Results for poorly-configured models

Hitherto well-configured FE models have been constructed with sufficiently fine meshes and short time-steps and results agree well with validating experiments. For the sake of illustration this section will show poorly-configured models.

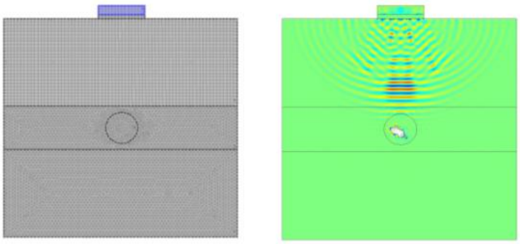

Figure 6 Coarse mesh solution at 25.0 us. Compare with figure 1.

Figure 6 shows results from a FE model with the same geometry, materials, boundary conditions, time-step and excitation as used in the model shown in figures 1 to 4 but the mesh is of size 0.5 mm to 1.5 mm – this is still within the Nyquist-like limit but only just. Computation time is substantially shorter because there are many fewer nodes in the mesh but the results are poorer and it is the comparison with validation experiments that allow one to make a judgement on quality. An example of one error is apparent inside the solid aluminium cylinder where there is a white patch (Comsol uses white to indicate a value, stress in this instance, that has gone outside the colour range between blue and red); although validation experiments do not directly reveal the state of stress in the aluminium, it is physically unreasonable to expect such a large stress to be generated before the ultrasonic pulse has reached it. Another more subtle example of a problem is the presence of wave-fronts ahead of the starting pulse in the main burst – the validation experiments show they should not be present (see figure 1).

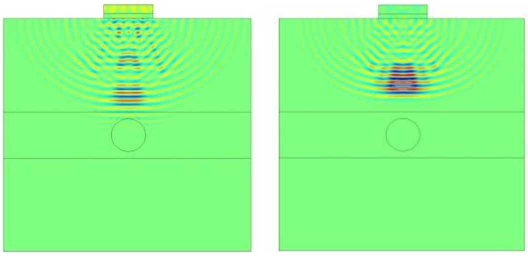

Figure 7 Comparison of results from a poorly-configured FE model (left) and a well-configured model (right - previously shown in figure 1).

Figures 7 and 8 show examples of results for a poorly-configured model, the same as that used in the previous poorly-configured model but with one difference: the time-step interval has been increased from 50 ns to 1 μs, which is at the Nyquist limit. Comsol’s time-dependent solvers (generalized alpha and BDF) force smaller time-steps than 1 µs, particularly at the start time of the model when the excitation force has a non-continuous first-derivative in time. Consequently, it is difficult to assess the true impact on the model of 1 µs time-steps. Probably the solver would fail to converge with such large time-steps and the Comsol time-dependent solver compensates by forcing smaller time-steps.

Failures observed in ths poorly configured model are:

- Unexpectedly high and irregular stresses in the cylinder.
- Wave-fronts appearing ahead of the first wave-front, producing an erroneous dispersion effect.
- Generating an incorrect wave-speed in water due to the above effect.
- Waves appear to reverberate inside the transducer for much longer than either the validation experiments show or the well-configured model predicts.

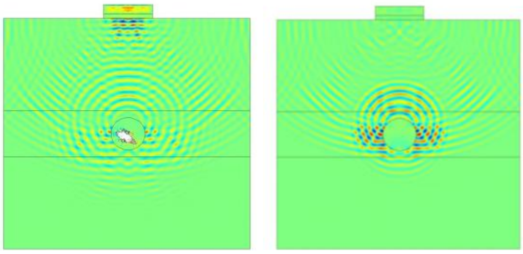

Figure 8 Comparison of results from a poorly-configured FE model (left) and a well-configured model (right - previously shown in figure 3).

Despite the failures, the poorly-configured model still produces results that have some physical correctness. On the one hand this is a credit to the abilities of the Comsol software but it can also represent a problem to the designer of ultrasonic transducers. Just because the FE model has completed without a failure of convergence does not guarantee that the results are physically correct even if they do have features that appear to be physically plausible.

10 Conclusions and discussion

Comsol can be used to solve computationally intensive FE models of ultrasonic wave propagation. Consequently, FE software can be used advantageously to help in the development of novel ultrasonic transducers for use in new applications of ultrasound.

It is highly desirable to compare results from FE modelling with validation experiments to prevent subtle and gross failures of models from being used as the basis of new ultrasonic systems. Photographs and video from schlieren visualization experiments are very well-suited to validating FE models of ultrasonic propagation.

Once a simple FE model is validated by schlieren visualization it is generally possible to introduce additional, novel features to the FE model (to the transducer for example) and the user can have confidence that new FE results will probably be physically correct without further validation, however, the user must still proceed with caution and healthy scepticism.

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