Improving Blood Flow Simulations Using Known Data

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Introduction: Numerical simulations applied to blood flow together with the imaging processing advances are a powerful tool in the prevention and treatment of some diseases. The inclusion of real data in the numerical blood flow simulations allow us to obtain more realistic and accurate results. The techniques that use known data in the simulations are named by Data Assimilation (DA) in the literature. In this work we solve a variational DA problem to numerically reconstruct the blood flow circulation inside a real artery deformed by a saccular aneurysm.

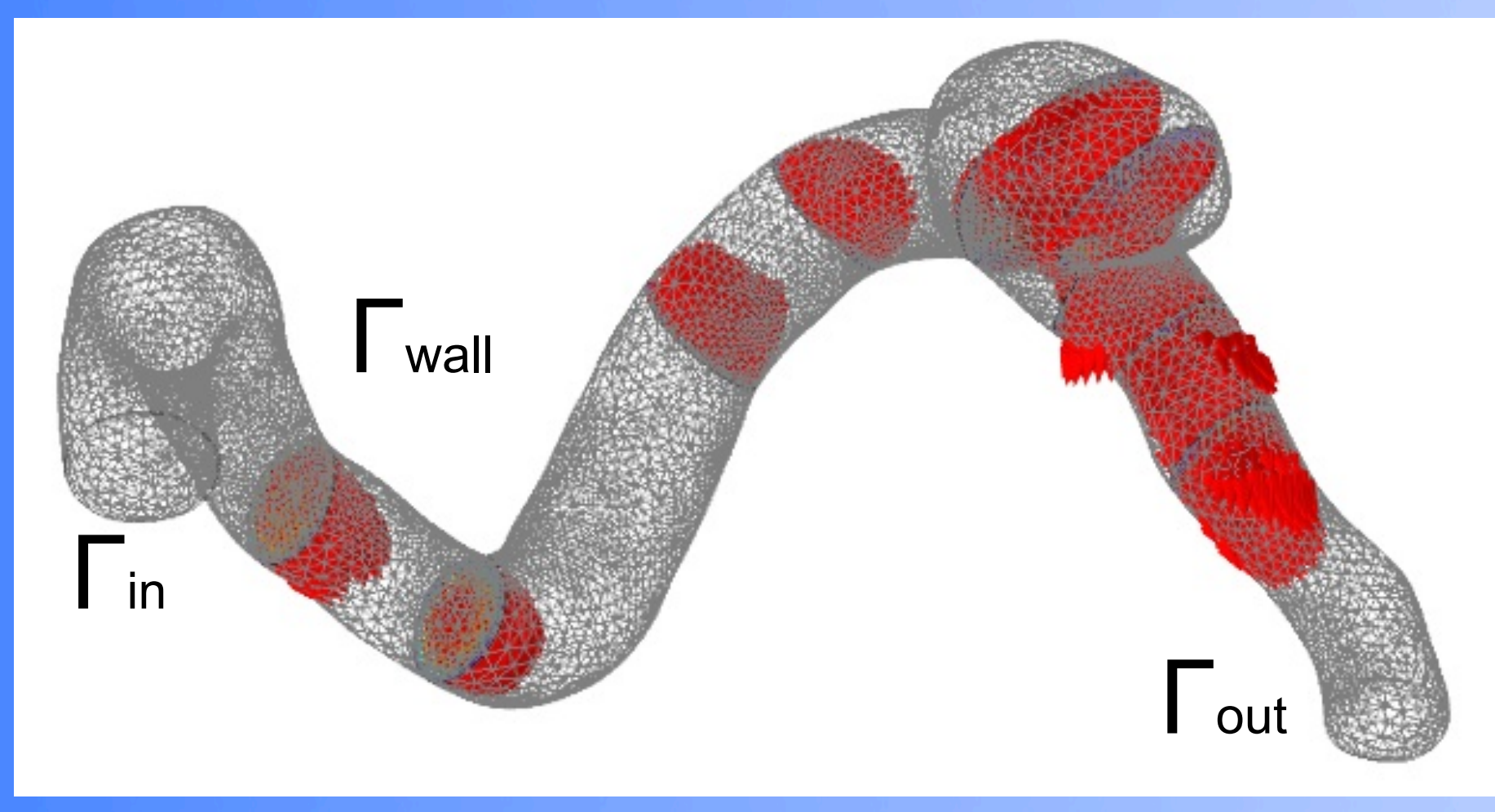

Figure 1. Data regions and boundaries representation.

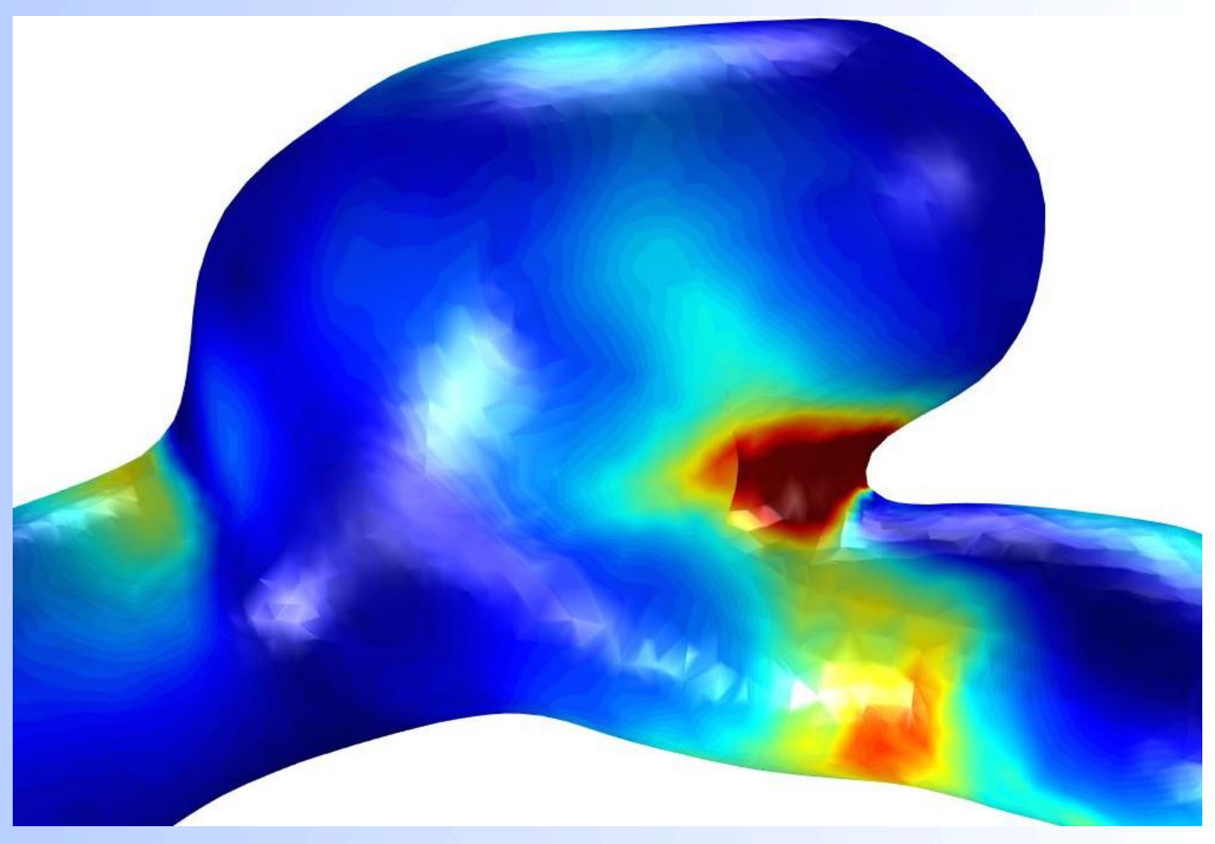

Figure 2. magnitude of wall shear stress representation.

Computational Methods: We use COMSOL Multiphysics® to model the blood flow using a generalized Navier-Stokes system. The geometry representing the real artery is obtained from medical images and imported to COMSOL Multiphysics®. The data to be used in the DA process is generated by solving the forward problem, with realistic parameters and boundary conditions. We then use the Optimization Module to define a cost function to be minimized.

$$J(\mathbf{u},\mathbf{h}) = w_1 \int_{\Omega_{part}} |\mathbf{u}-\mathbf{u}_d|^2\,dx + w_2 \int_{\Gamma_{wall}} |ws - ws_d|^2\,dx + w_3 \int_{\Omega_{in}} |\nabla \mathbf{h}|^2\,dx$$

$$s.a.$$

$$\begin{array}{lll}
-\nabla\cdot(-p\mathbf{I}+\tau)+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}=0 & in & \Omega \\
\nabla\cdot\mathbf{u}=0 & in & \Omega \\
\mathbf{u}=0 & on & \Gamma_{wall} \\
\mathbf{u}=\mathbf{h} & on & \Gamma_{in} \\
(-p\mathbf{I}+\tau)\mathbf{n}=0 & on & \Gamma_{out}
\end{array}$$

The cost functional essentially measures the misfit between the data and the solution at several sections of the artery (see Figure 1). The misfit considers velocity (**u**) and the magnitude of wall shear stress (*ws*) measurements. We include the wall shear stress since it is an important indicator to predict vascular diseases which is highly sensitive with respect to the geometry. Figure 2 illustrates this. We also include a regularizing term to avoid numerical spurious minima. The control variable (**h**) corresponds to the inlet profile at the inlet boundary, *p* stands for the pressure and τ is the extra stress tensor.

Results: To check the robustness of the DA approach, we test it for data that was obtained with different inlet profiles: parabolic, semi-flat and flat. The Figure 4 represents the adjustment of the obtained control to the pretended inlet in the tree profiles considered.

Cost function parameters: $(w_1, w_2, w_3) = (10^2, 10^2, 10^{-3})$;
Finite elements: stabilized P1-P1;
Degrees of freedom for **u**: 180789.

$$\mathrm{Re} = \frac{\left(\int_\Omega |\mathbf{u}-\mathbf{u}_d|^2 dx\right)^{1/2}}{\left(\int_\Omega |\mathbf{u}_d|^2\, dx\right)^{1/2}}$$

Relative Error

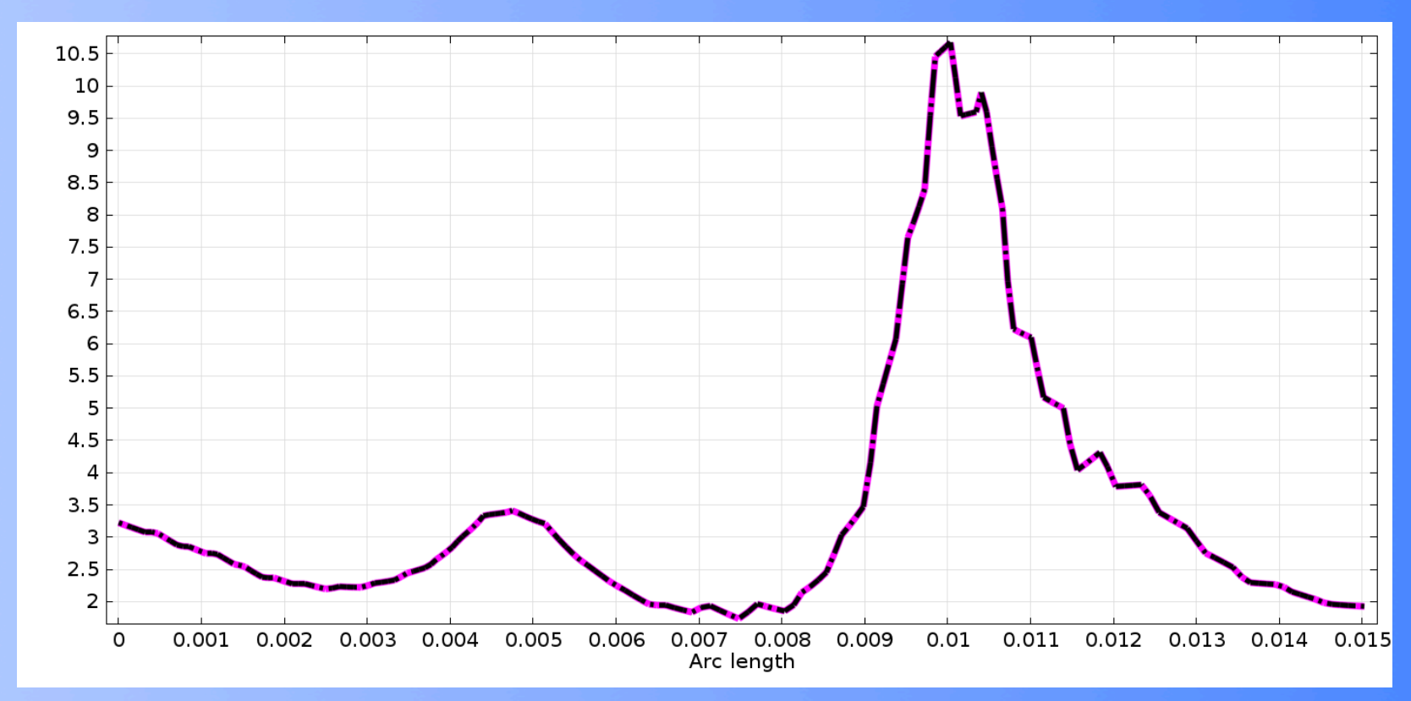

Figure 3. Dash dot – pretended *ws*; solid – controlled *ws* in the saccular region.

Profiles	Parabolic	Semi-flat	Flat
Re	0.002066	0.003636	0.004843

Table. Relative Errors for the different inlet profiles.

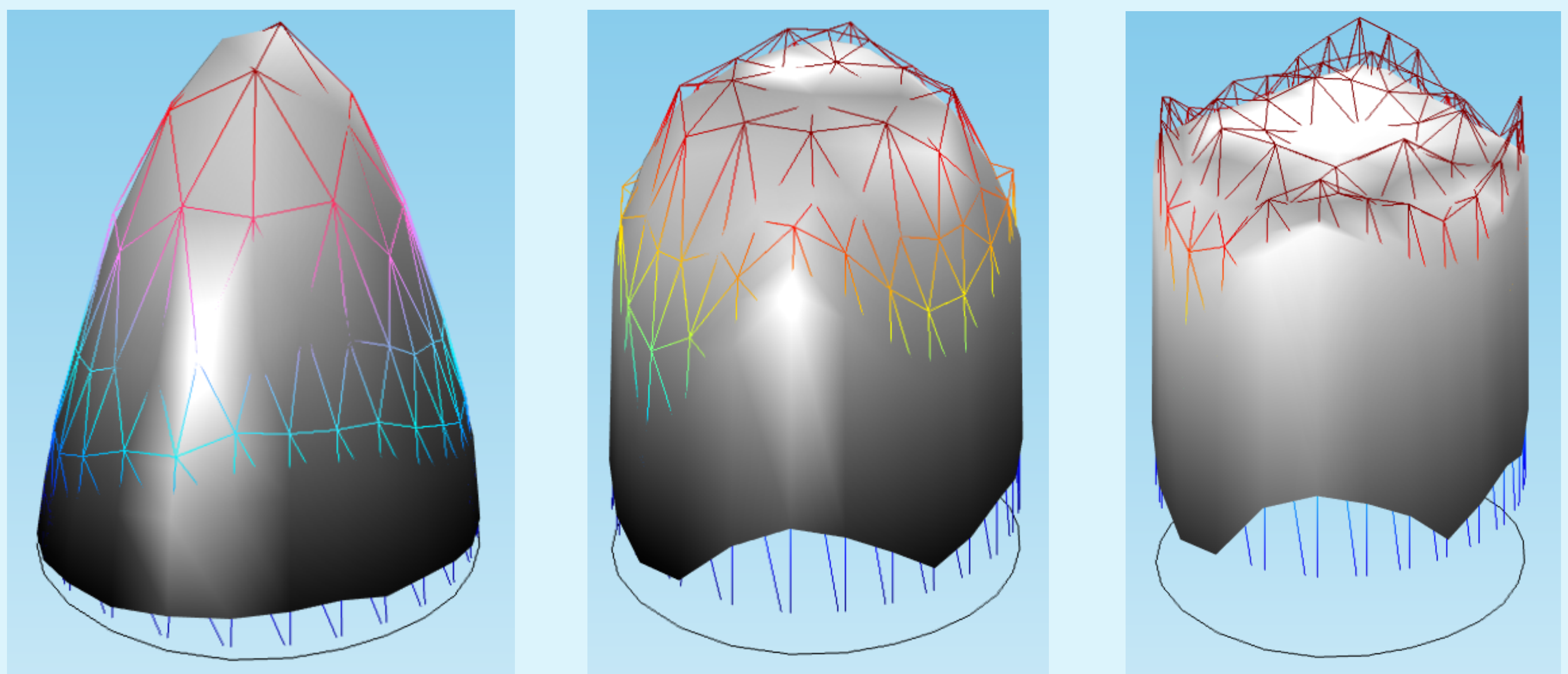

Figure 4. Control adjustment to the different inlet profiles, parabolic, semi-flat and flat.

Conclusions: The work here presented gives an automatic approach to obtain realistic blood flow simulations representing the reconstruction of the blood flow profile from partially available measurements. If successfully adapted to time dependent models, it may be a useful tool for predictions in medical practices. As for time dependent simulations the vessel wall can no longer be considered rigid, therefore, the next stage should include the fluid-structure interaction between the blood and the vessel.

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