COMSOL CONFERENCE 2014 CAMBRIDGE

COMSOL

Sound Field Analysis of Monumental Structures by Application of Diffusion Equation Model

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INTRODUCTION

- <u>Brief:</u>

  within the context of a larger research project
  the implementation of `diffusion equation modeling' in relation to architectural acoustics and acoustical coupling within spaces.

- <u>Research question:</u>

  whether single-volume systems with particular architectural compositions could provide the circumstances for the formation of non-exponential sound energy decays inside that specific enclosure.

- <u>Sample group:</u>

  multiple dome superstructures

Pilot Case: Süleymaniye Mosque in İstanbul (1550-1557)

- multiple-dome upper structure
- flying buttresses, arches and elephant feet
- central dome supported on two sides by semi domes
- side aisles sheltered by five smaller domes
- inner plan of the mosque: a rectangle measuring 63 by 69 m
- main dome diameter: 26 m
- height of the dome from the ground to the keystone: 48 m
- inner volume approx. 90,000 m^3
- basic interior materials:

  stone, plaster-paint, carpet

Previous Study Outcome

- particular sound field generated within this single, exceptionally large but physically fragmented volume is initially investigated by:

1. room-acoustic computer simulations
2. real-sized field measurements
3. data analysis - decay parameter estimations

Preliminary findings necessitated further scientific understanding of the non-exponential energy decay formation within such single-volume geometry.

At this point, diffusion equation modeling (DEM) is applied for visualizing the sound energy exchange mechanism within this superstructure.

Diffusion Equation Model (DEM)

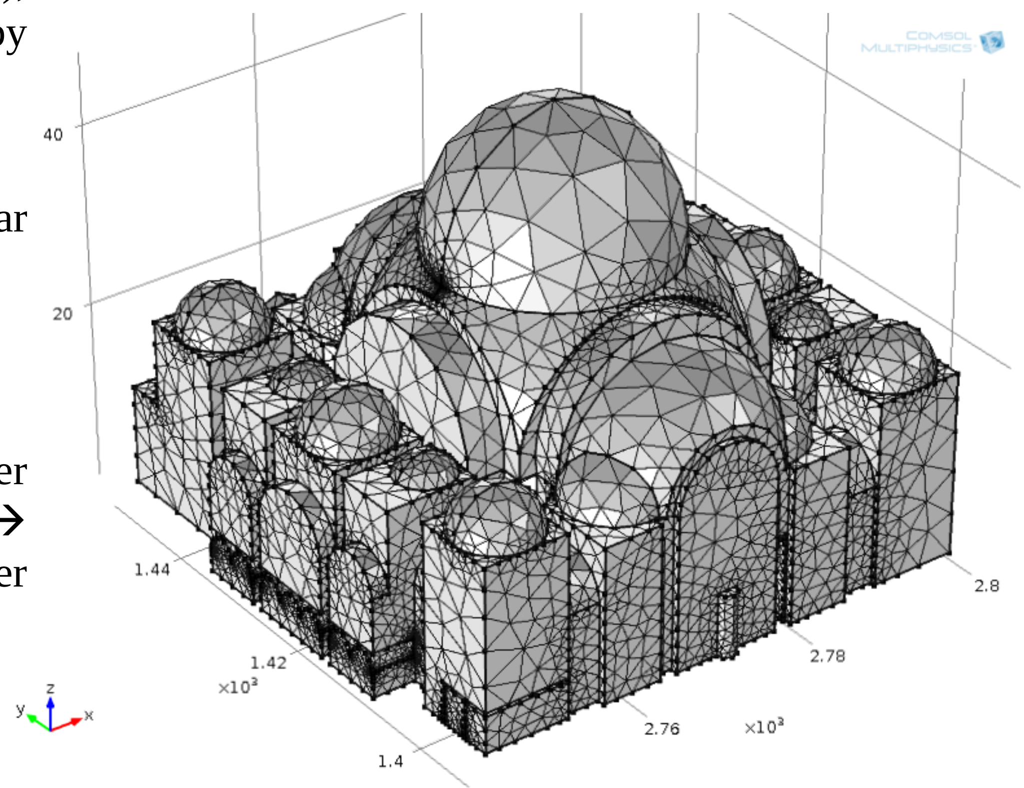
Why DEM in room acoustics?

- a new/recent method in room acoustics
- the theory that relies on the diffusion of particles by scatterers in previous research proved further advantageous in compare to statistical theory (Sabine - Eyring), wave theory and geometrical acoustics approaches (Valeau et al., 2006; Jing & Xiang, 2008; Xiang et al., 2009)

- a COMSOL based DEM solution is superior in respect to its high computational speed, and additional outputs (visualization tools) as of spatial energy and flow vector analysis

Implementation of DEM via COMSOL

- the solid geometry of the superstructure (interior), relying on the latest field drawings is generated by AutoCAD solid tools
- model is imported in COMSOL Multi-physics
- geometry is fine meshed → total of 124,788 linear Lagrange-type mesh elements are used
- basic coefficient form PDE module is used
- point source is defined at imam at mihrab position
- sound absorption coefficient information for the upper shell structure (stone-paint) and the floor (carpet) → implemented in boundary/ absorption term under flux/source, for manually selected boundaries
- mean free path of the room:18.26 m
- mean free time of the room: 0.053 s



∇^2

Interior Diffusion Equation

in the presence of an omni-directional sound source within a room or region/domain (V) with time-dependent energy density , **the particle density or the acoustic energy density** (w) at a position (r) and time (t) is;

Domain $$\frac{\partial w(r,t)}{\partial t} - D\nabla^2 w(r,t) + cmw(r,t) = q(r,t)\,, \quad \in V$$

where, ∇^2 is the Laplace operator,
D is the 'diffusion coefficient',
c is the speed of sound,
m is the coefficient of air attenuation.

In a time-dependent solution, resulting $w(r,t)$'s after relevant logarithmic scaling, are used for spatial sound energy density distribution and sound energy flow vector analysis.

Interior Diffusion Equation

Diffusion coefficient takes into account the room morphology through its mean free path which is;

$$D = \frac{\lambda c}{3} = \frac{4Vc}{3S}$$

where, λ is the mean free path,
c is the speed of sound,
V is the volume of the room,
S is the total surface area of the room.

In a room-acoustics problem, for time-dependent solutions a point source with an arbitrary acoustic power can be modeled as follows;

Source $$q(r_s, t) = P(t)\delta(r - r_s)$$

where, $P(t)$ is the acoustic power,
r_s is the location of source,
δ is the Dirac-delta function.

step function is multiplied by t (time) in the point source term for providing gradual/continuous decrease on the source power from initiation to cut off within a very short time span (0.1 s).

Boundary Equations

$$J(r,t) \cdot n = -Dw(r,t) \cdot n = A_X c w(r,t)$$

where, c is the speed of sound, A_x is an exchange coefficient or the so called ***absorption factor***.

for the case Mosque, the room has an absorptive carpet floor -for specific octave bands- versus a low absorptive/reflective upper shell;
the boundary condition fits best to the modified mixed boundary model.

For the mixed boundary conditions the absorption factor is defined as follows;

$$A_X = A_M = \frac{\alpha}{2(2-\alpha)}$$

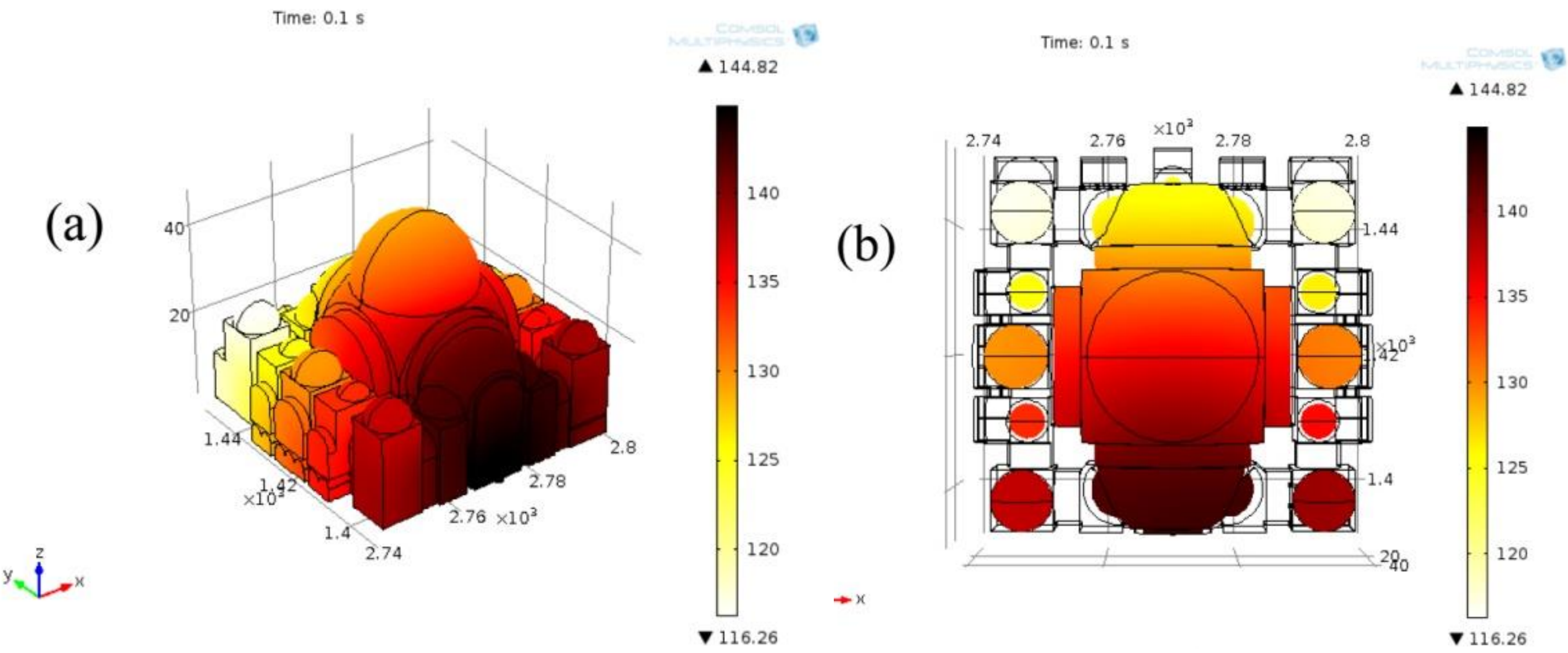
where, α is sound absorption coefficient of specific boundary surface for specific octave band

the resulting system of the boundary equation, as follows;

$$-D\frac{\partial w(r,t)}{\partial n} = \frac{c\alpha}{2(2-\alpha)} w(r,t), \qquad on\ S$$

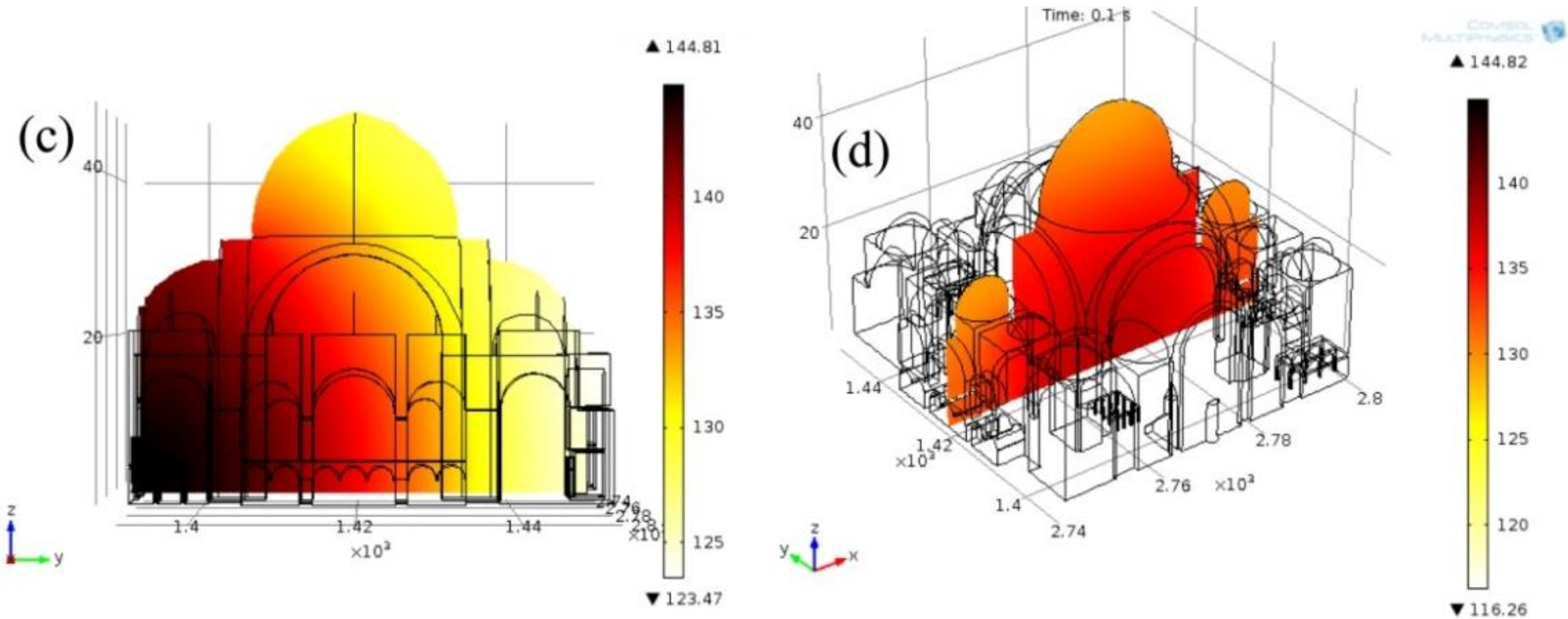
RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 0.1 s. a) axonometric view, b) plan view



RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 0.1 s c) section through the mihrab wall, central axis, d) section parallel to the mihrab wall, central axis



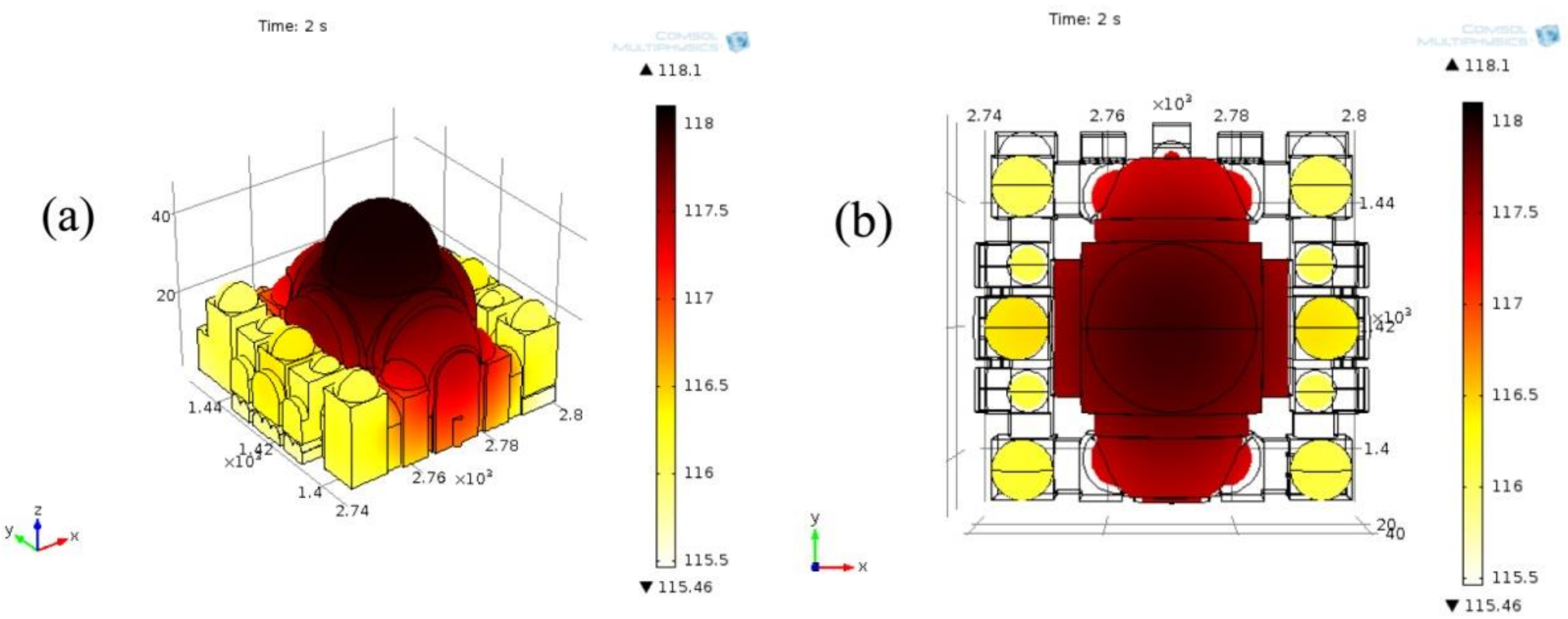
RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 0.1 s

- the concentration of sound energy density is at the front part of the mihrab wall, where the point source is defined,
- energy starts to flow from mihrab wall towards the back of prayer hall,
- at this point the central dome and back wall aisles has not been completely filled with the sound energy,
- inverse section (Figure d) indicates a more even distribution and an average sound level,
- zones closer to the floor (receiver/prayer heights) underneath the central dome at that direct sound period get more energy in compare to prayer locations in front of the back wall and get least underneath back wall corner domes.

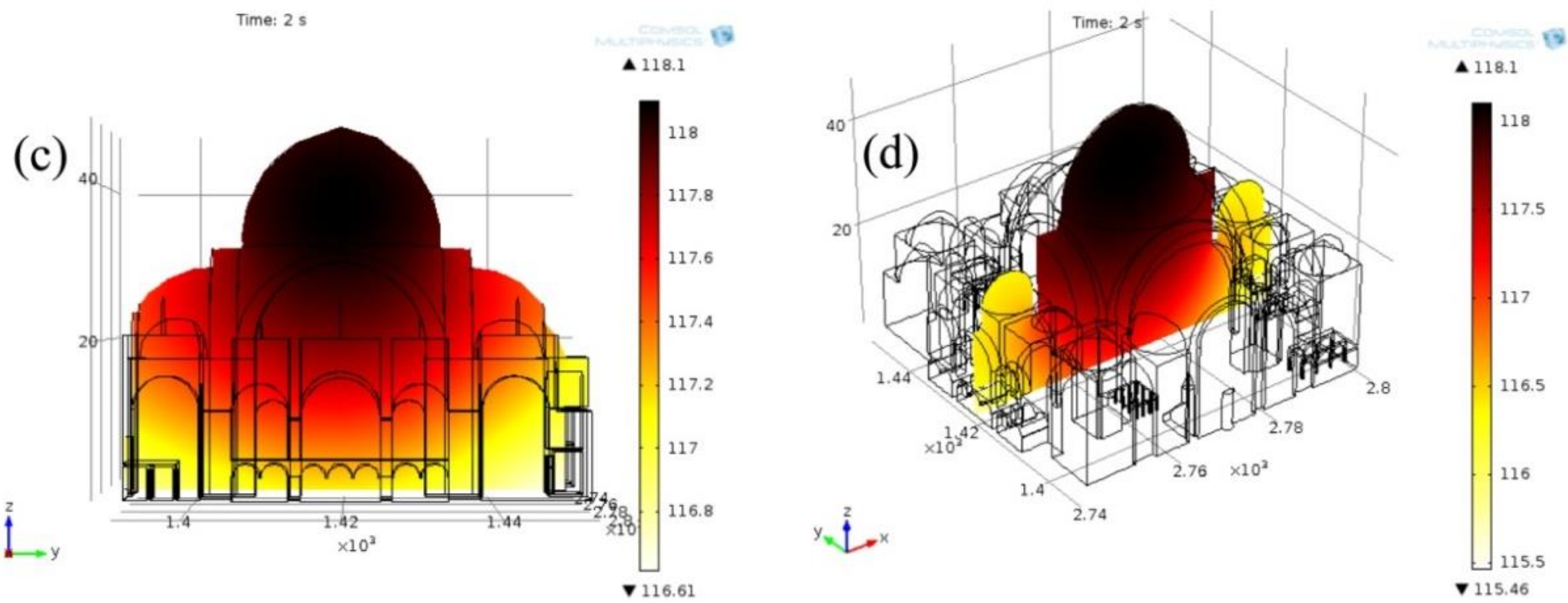
RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 2 s a) axonometric view, b) plan view



RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 2 s c) section through the mihrab wall, central axis, d) section parallel to the mihrab wall, central axis



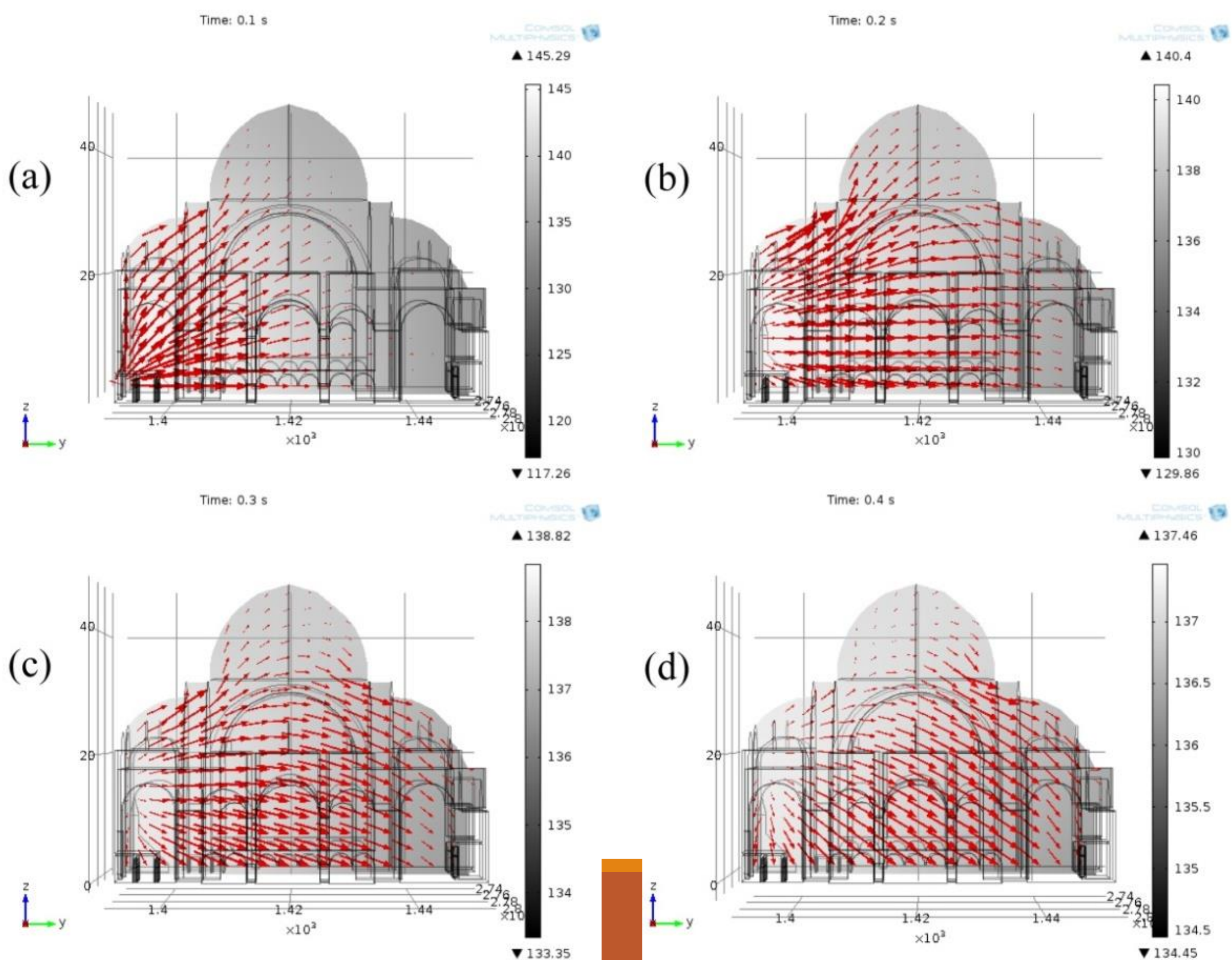
RESULTS & DISCUSSIONS

Spatial sound energy level (dB) distribution; volume and slice plots of Süleymaniye Mosque DEM solution, for 1 kHz, time: 2 s

- at this state the sound energy is concentrated at the central axis underneath the main dome and the semi-domes,
- from this point out, the energy center is the central dome, with its comparatively reflective surfaces and focusing geometry,
- this energy accumulation center keeps feeding energy back to the floor area,
- the side aisles underneath the secondary domes gets substantially less energy compared to the mihrab wall section (Figure c),
- this energy fragmentation indicates the zone underneath the main and the secondary domes to work as of a reverberation chamber, while the aisles are fairly dead areas that get later energy feedback from the central zone.

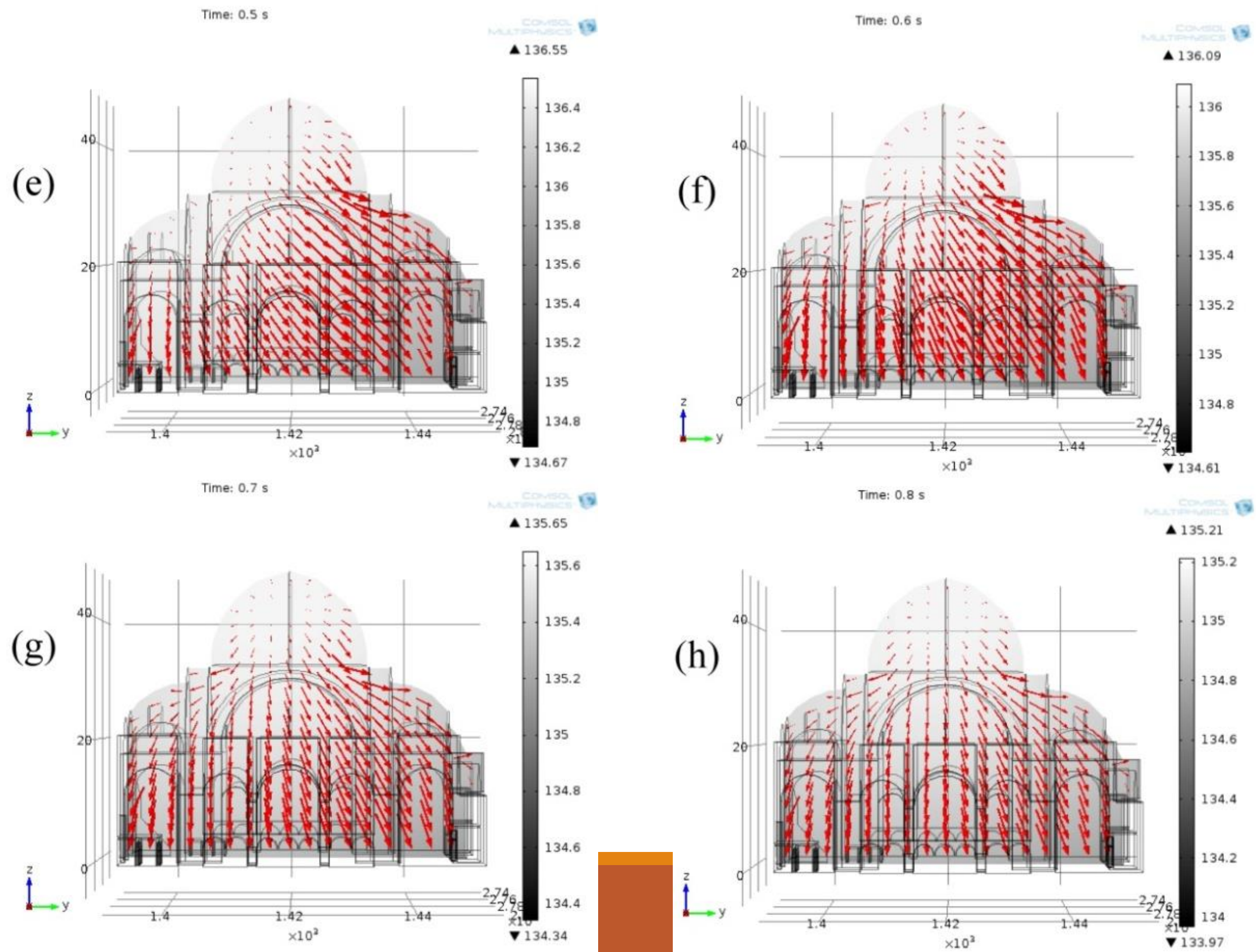
RESULTS & DISCUSSION

Impulse-response derived sound-energy flow vectors (arrow volume plots) of Süleymaniye Mosque DEM solution for the time dependent solution, for 250 Hz a) time: 0.1 s, b) time: 0.2 s, c) time: 0.3 s, d) time: 0.4 s



RESULTS & DISCUSSIONS

Impulse-response derived sound-energy flow vectors (arrow volume plots) of Süleymaniye Mosque DEM solution for the time dependent solution, for 250 Hz e) time: 0.5 s, f) time: 0.6 s, g) time: 0.7 s, h) time: 0.8 s



RESULTS & DISCUSSIONS

- according to the arrow vector analysis the flow return is around 0.7 s,
- after 1 s, the energy is stabilized at the central upper zone (main dome), and starts feeding back the rest of the mosque's prayer zones,
- frequency is selective in the boundary definition; the absorptive carpet and reflective upper shell structure -including walls-, together with the dominant geometrical features cause the main energy flow characteristics,
- initial part of the decay after the shut off time of the sound source; from 0.1 s to 0.8 s in this case, is trivial in terms of many acoustical parameters such as of EDT, C80, D50, LF and ITDG.
- flow patterns include clues on the acoustical coupling trends.

CONCLUSION

- the multi-slope decay formation is estimated to be primarily the result of the energy division by the upper central zone of the mosque:

  in PLAN between the four elephant feet, and the side aisles underneath the secondary dome structures

  in SECTION divided into two main zones below-above pendententives of the main dome

- another reason for the energy divergence within the space is the absorptive and reflective sound area break-up in between carpet and stone and/or plastered brick upper shell and wall surfaces

CONCLUSION

- Exercised over an existing monumental structure within the context of this study, the DEM solution findings are significant in revealing more specific causes of multi-slope decay formation within single volume structures with specific geometric attributes,

- DEM application by finite element modeling is a practical and scientific method of room acoustics predictions, particularly for in-depth sound field analysis,

- COMSOL as a finite element solver could find many grounds in room acoustics applications as in this case, can be utilized over existing structures or over virtual spaces as an acoustical analysis or acoustical design tool..

THANK YOU FOR YOUR TIME AND ATTENTION…