Particle Beam Tracking with COMSOL Multiphysics®

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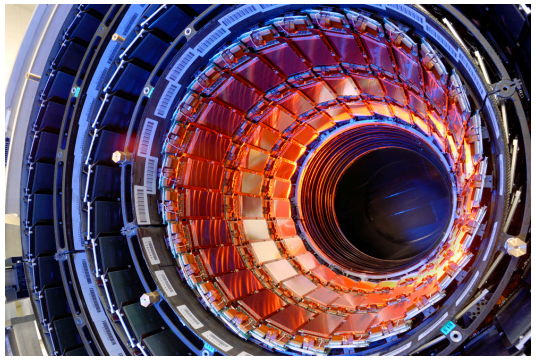
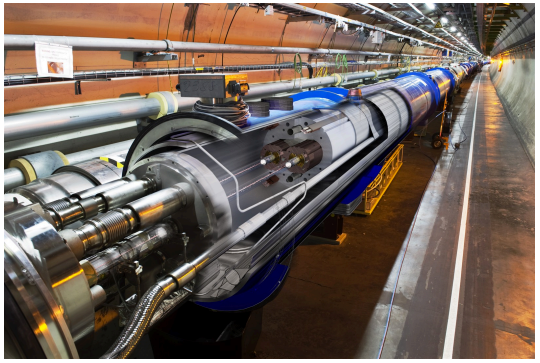
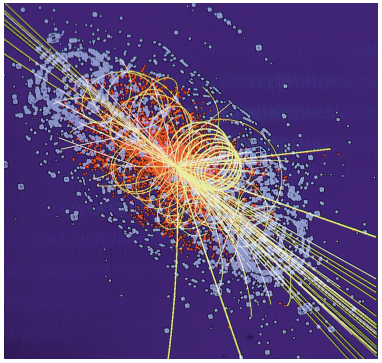
Accelerator physics



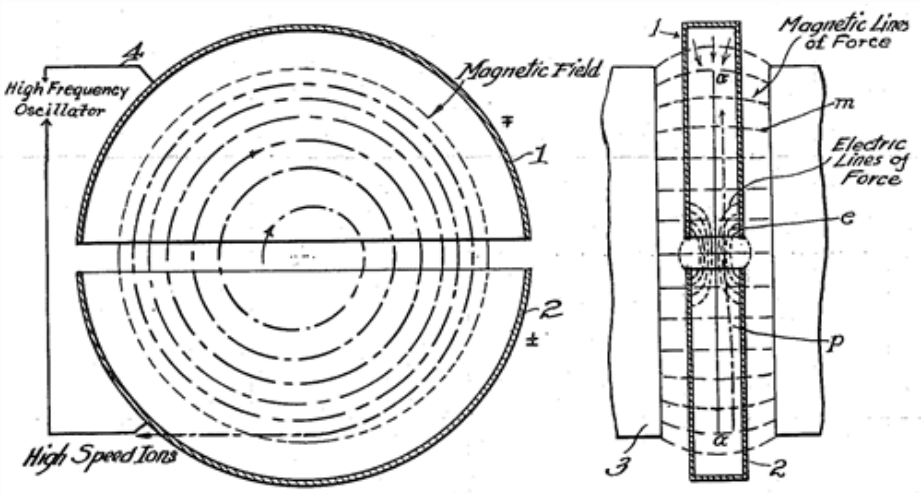




Accelerators + particle sources + detectors +….

- LINAC
- Cyclotron
- Synchrotron

Over 30 000 accelerators built in last 60 years

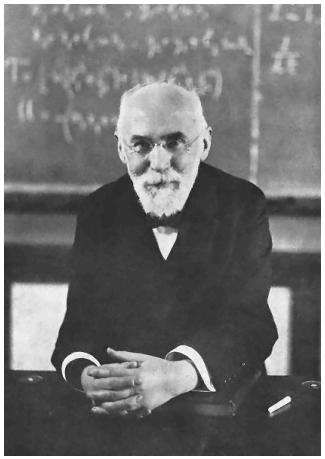


Energy range from sub eV's to TeV's

Different particles: electrons, positrons, protons, antiprotons, ions and more…

Physics

+



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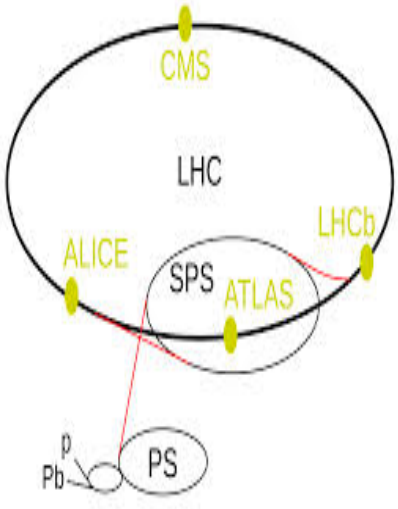


Maxwell's equations

Lorentz transformation

1. $\nabla \cdot \mathbf{E} = \frac{\rho_V}{\varepsilon}$
2. $\nabla \cdot \mathbf{H} = 0$
3. $\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$
4. $\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E}$

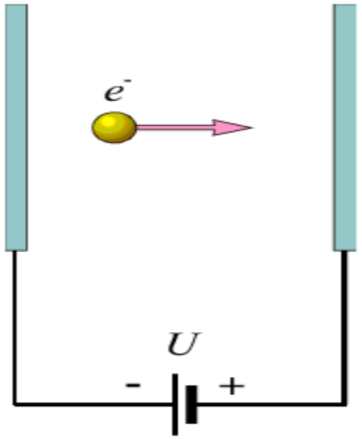
$$E'_x = E_x \qquad B'_x = B_x$$

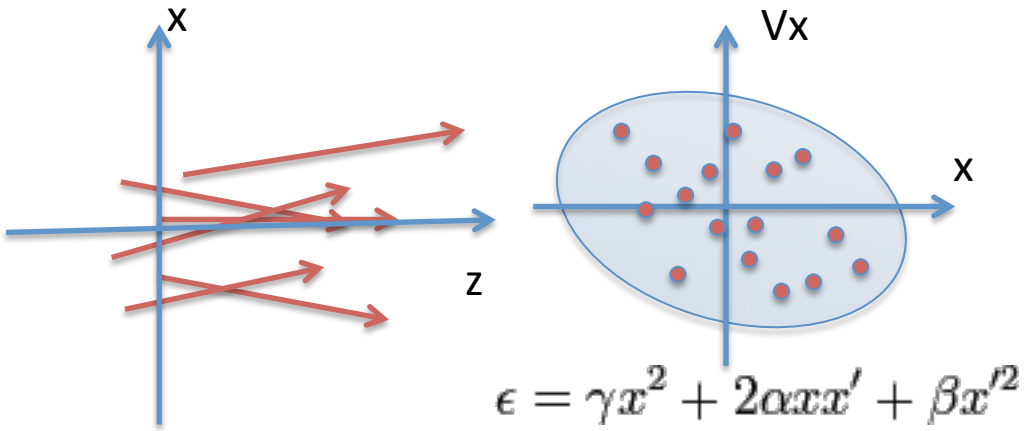
$$E'_y = \gamma (E_y - vB_z) \qquad B'_y = \gamma \left(B_y + \frac{v}{c^2} E_z \right)$$

$$E'_z = \gamma (E_z + vB_y) \qquad B'_z = \gamma \left(B_z - \frac{v}{c^2} E_y \right).$$

Equation of motion

$$\frac{\partial \vec{V}}{\partial t} = \frac{q}{m}\sqrt{1-\frac{V^2}{c^2}}\left\{\vec{E}+\left[\vec{V}\vec{B}\right]-\frac{1}{c^2}\vec{V}(\vec{V}\vec{E})\right\}$$





1 eV = 1.602 × 10^{-19} J

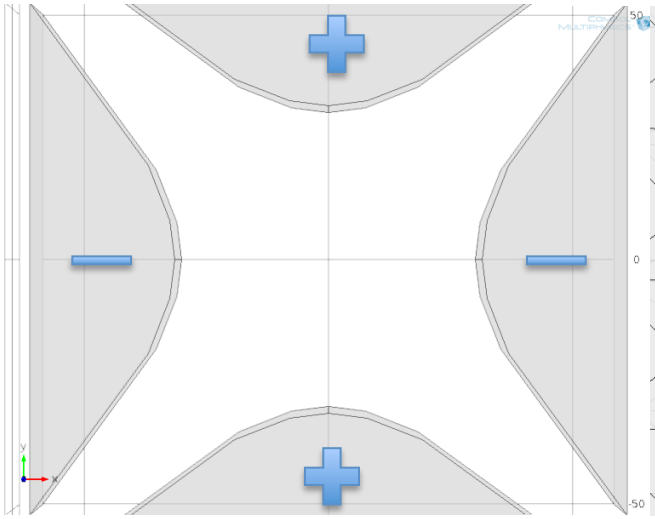
Space is constant! Liouville's theorem

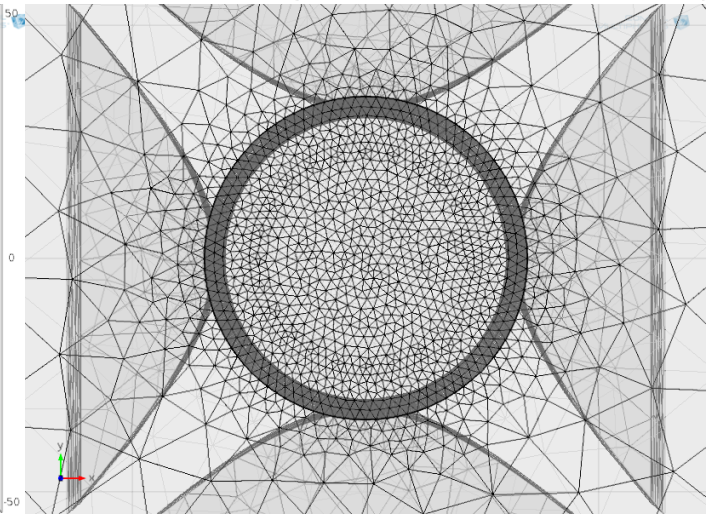
Challenges

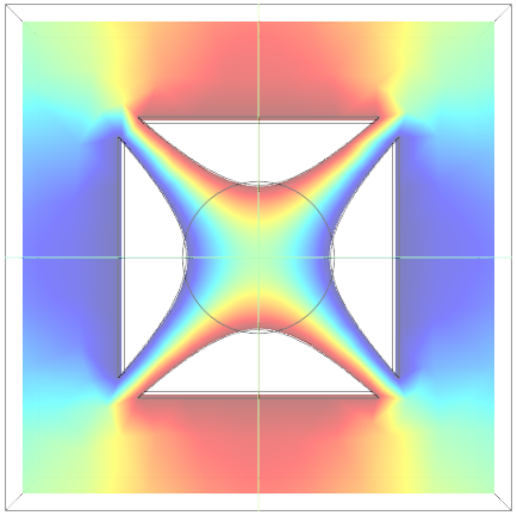
- Broad range of problems: different particles, different energies, different accelerators
- Generating the beam
- Errors estimation
- Post and during simulation data analysis
- Field maps from different sources
- Additional effect such as “Space charge” and interaction with residual gas

Particle tracking

$$\frac{\partial \vec{V}}{\partial t} = \frac{q}{m}\sqrt{1-\frac{V^2}{c^2}}\left\{\vec{E} + \left[\vec{V}\vec{B}\right] - \frac{1}{c^2}\vec{V}(\vec{V}\vec{E})\right\}$$







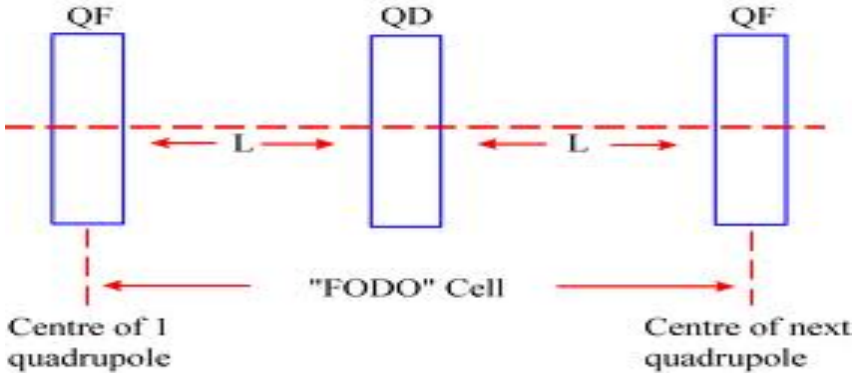
Geometry

Mesh

Result

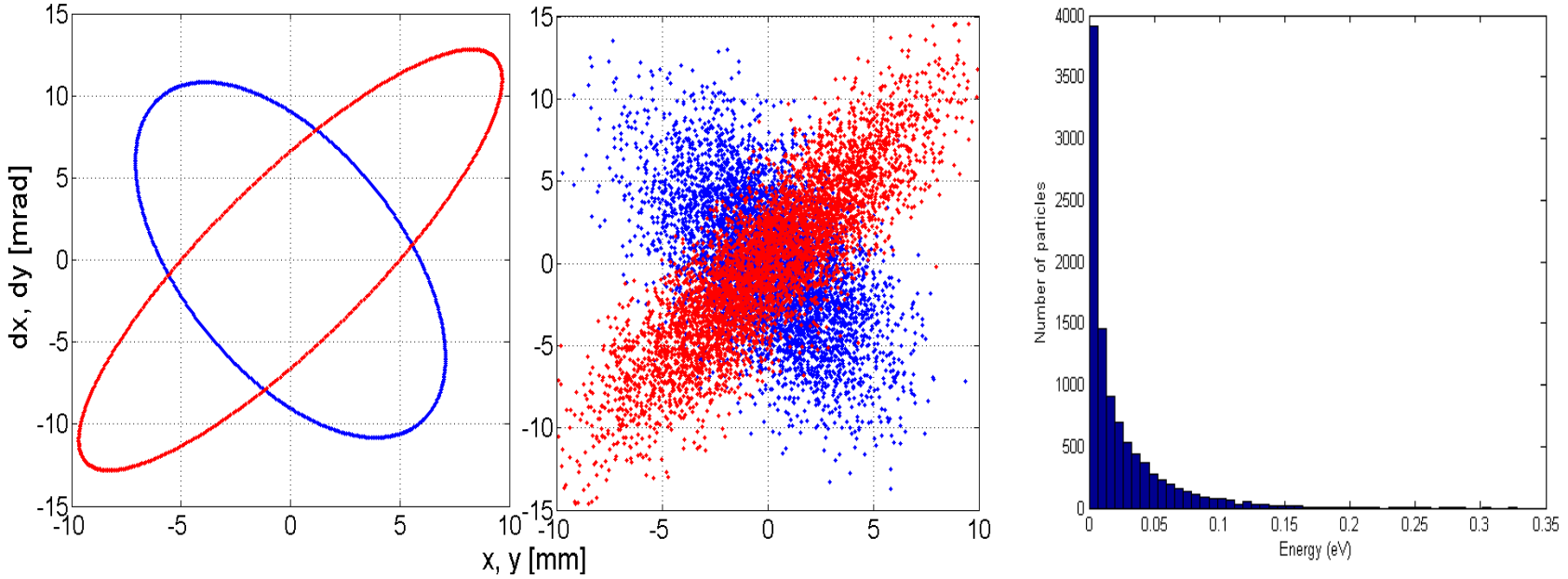
EXPORT DATA

Sometimes up to 15 GB of data…



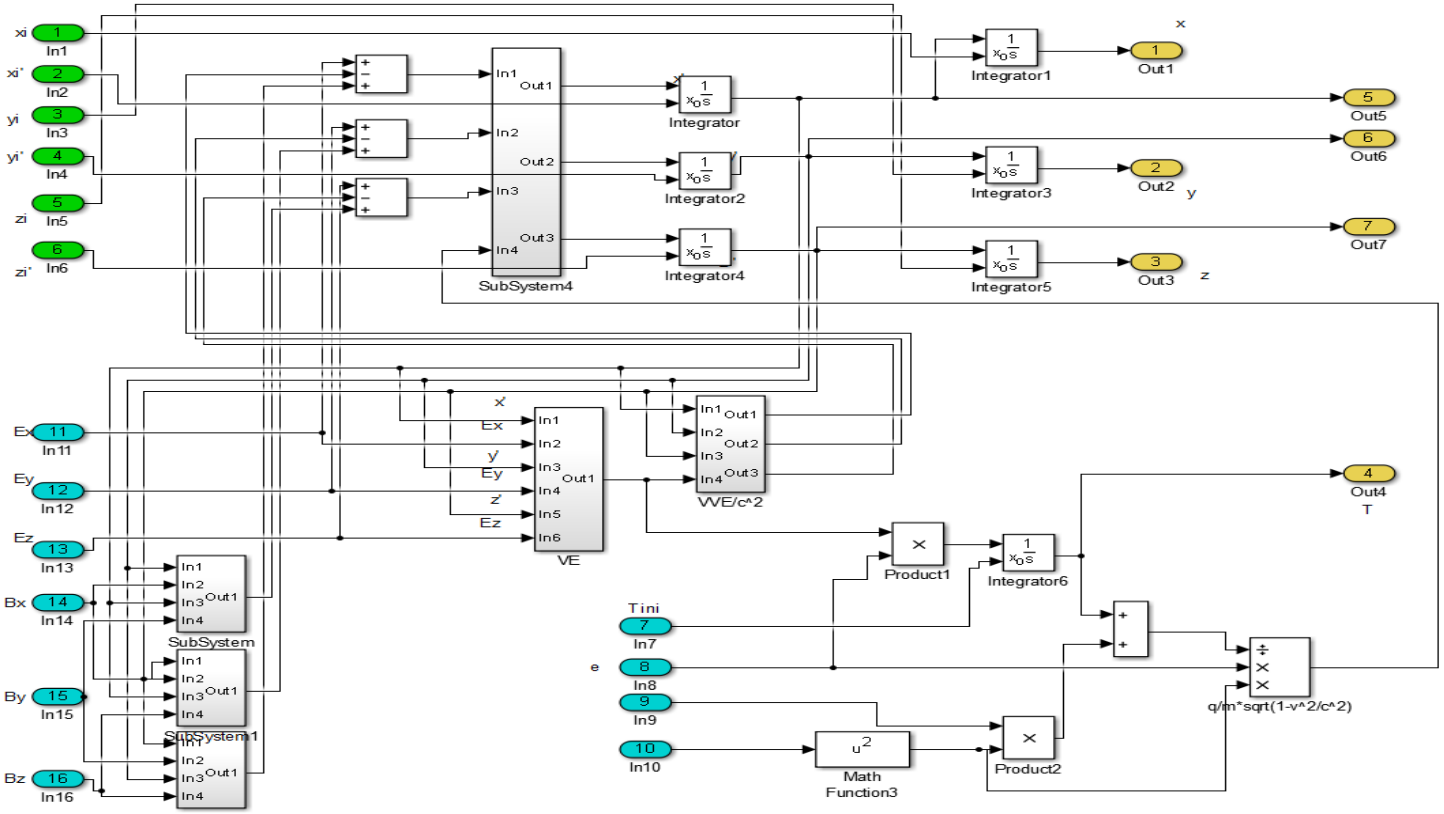
Generate the beam

$$\frac{\partial \vec{V}}{\partial t} = \frac{q}{m}\sqrt{1-\frac{V^2}{c^2}}\left\{\vec{E}+\left[\vec{V}\vec{B}\right]-\frac{1}{c^2}\vec{V}(\vec{V}\vec{E})\right\}$$



Almost ALWAYS programming required!

Run the simulation

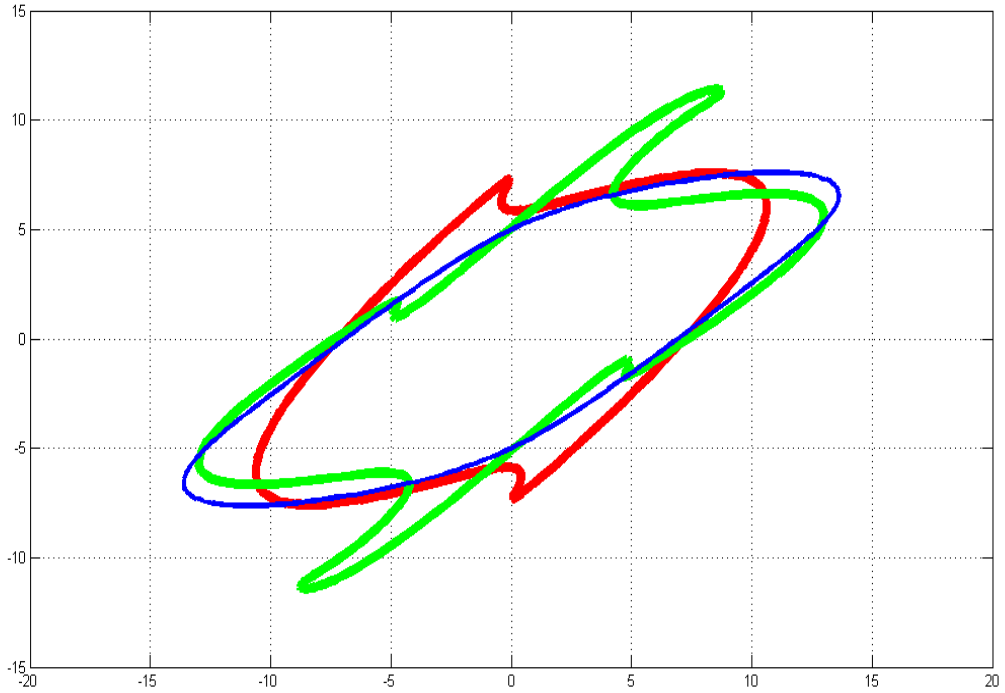


$$V_{i+1} = V_i + a\,\partial t$$

$$x_{i+1} = x_i + Vx_i\,\partial t$$

$$\frac{\partial \vec{V}}{\partial t} = \frac{q}{m}\sqrt{1-\frac{V^2}{c^2}}\left\{\vec{E} + \left[\vec{V}\vec{B}\right] - \frac{1}{c^2}\vec{V}\left(\vec{V}\vec{E}\right)\right\}$$

Post processing, data analysis

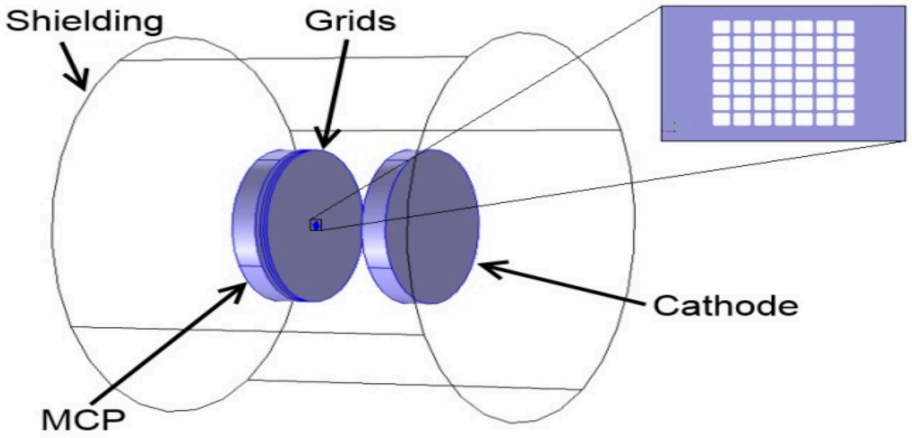


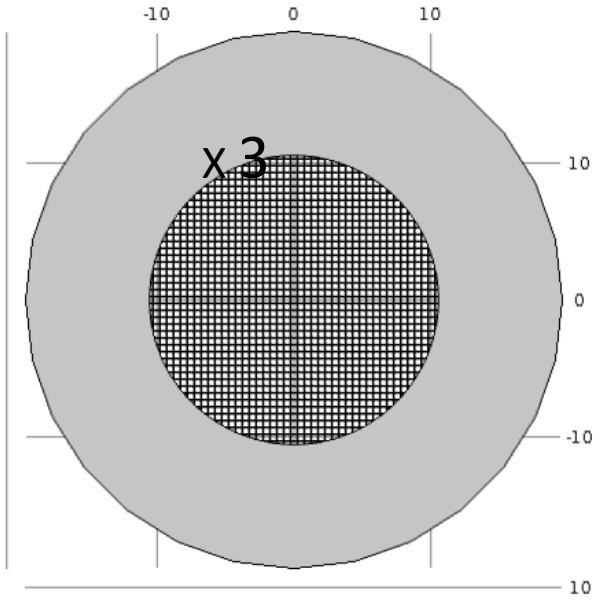
Some of our models return up to 50GB of data,

Optimization of data output is crusial.

Numerical error or physical effect?

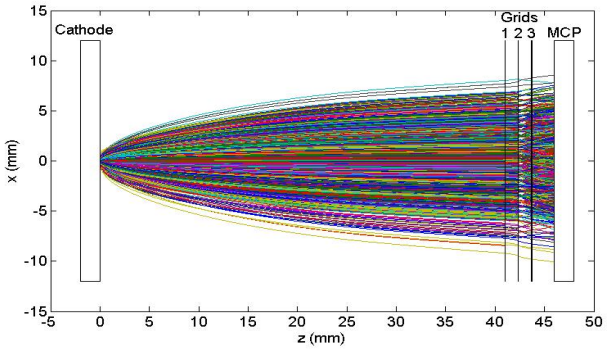
Manipulating field maps





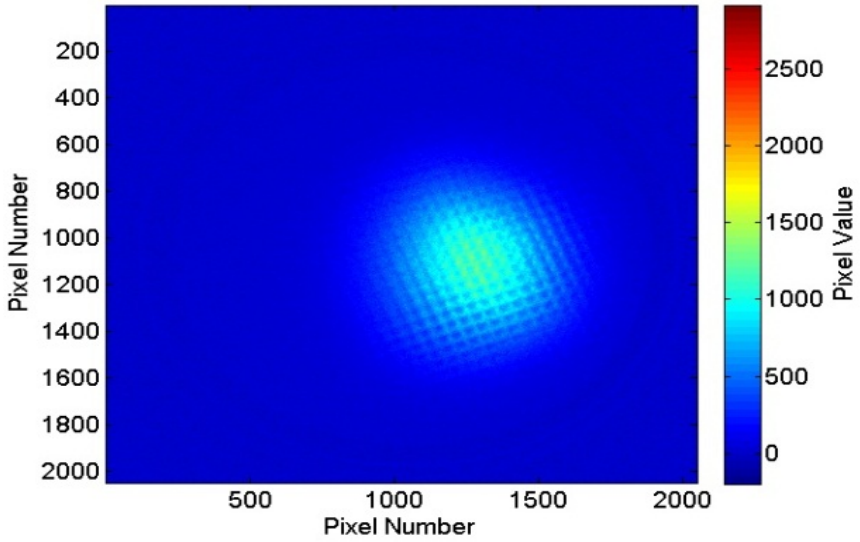
Combining COMSOL field map

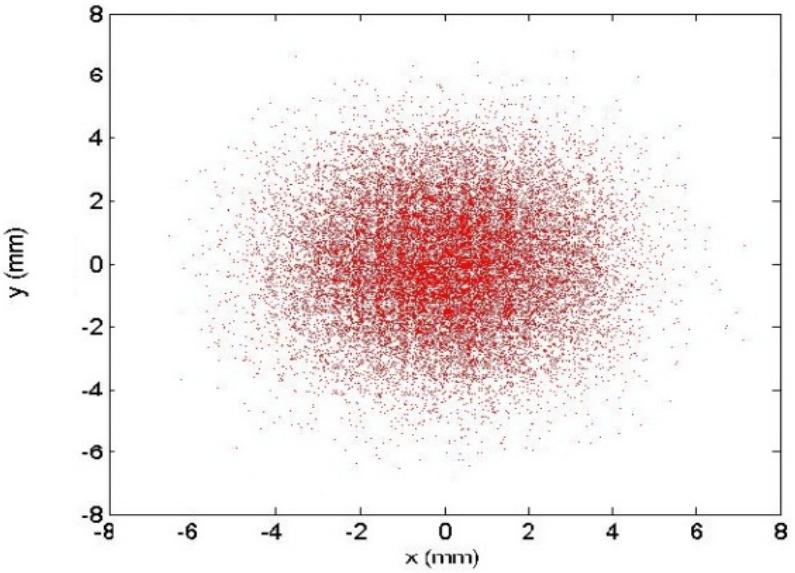
Additional conditions



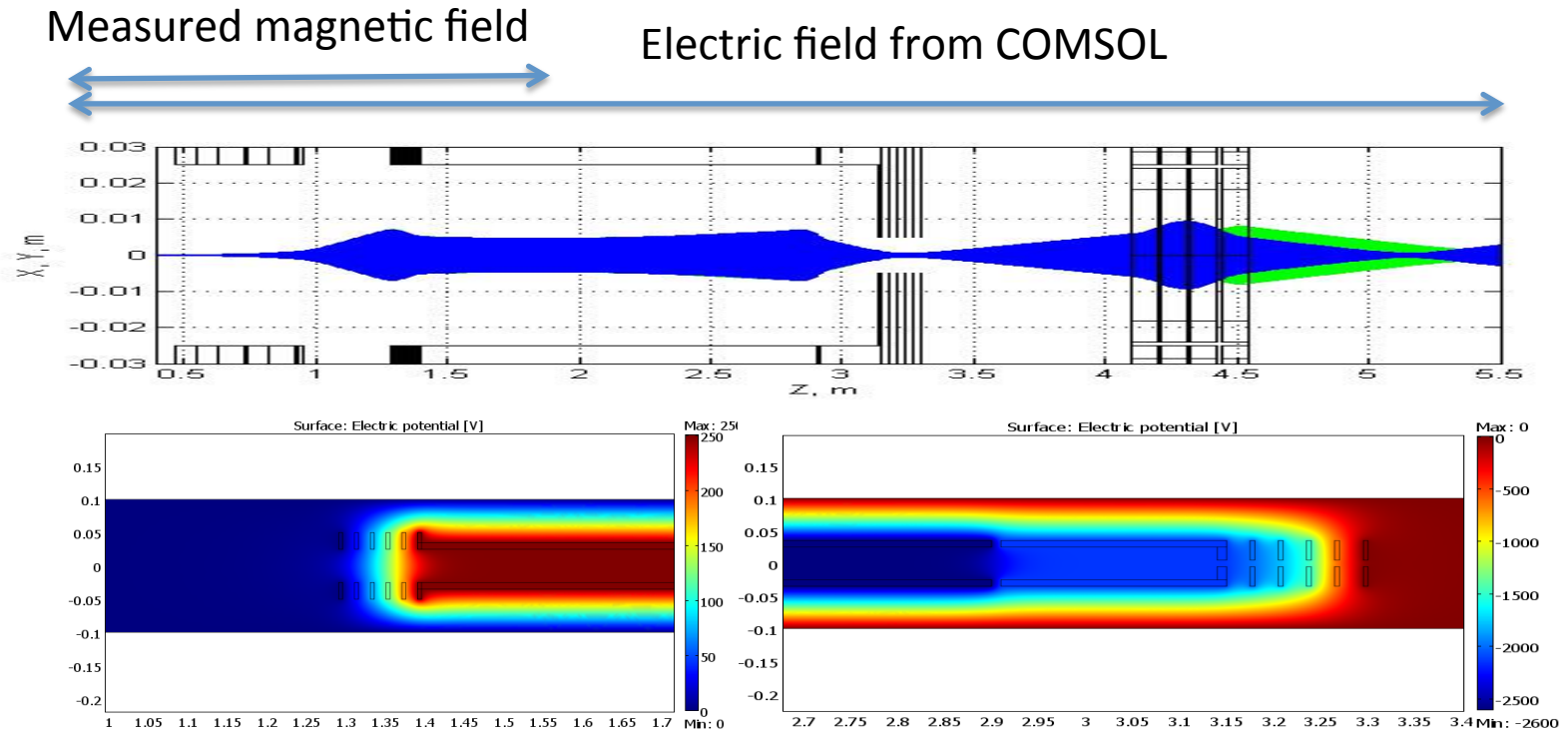
Loss condition: some particles are lost on the grids

L.J.Devlin et al., "Measurements of the Longitudinal Energy Distribution ofLow Energy Electrons", Proc. IPAC14, MOPRI051, Dresden, Germany(2014)





Combining field maps from different sources

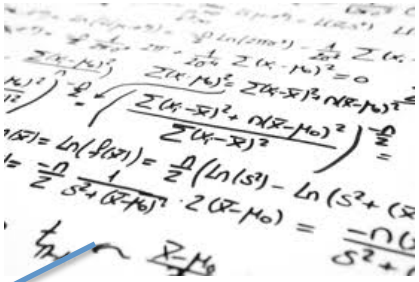


O Karamyshev et all, "Design of the injector for a small recycling ring on the CERN-AD", Nucl. Instr. Meth. A, 700 (2013) 182–187

Combining field maps from different sources

COMSOL MULTIPHYSICS

measurements



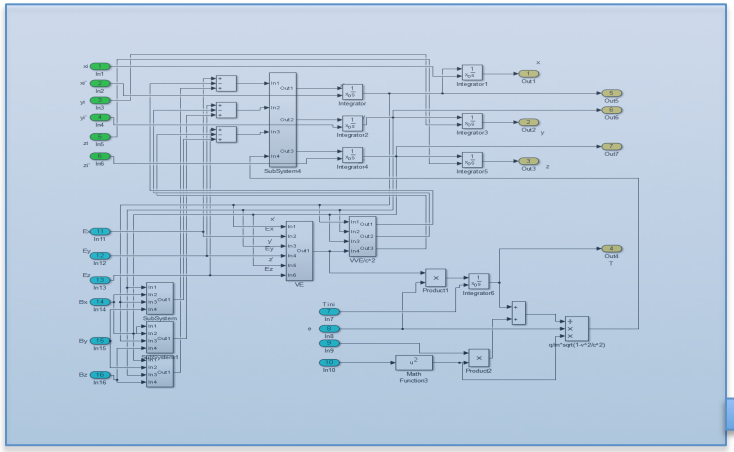
Analytical solution

Initial beam:

x y z

Vx Vy Vz

Energy



Integration block

"Add-ons"

RESULTS

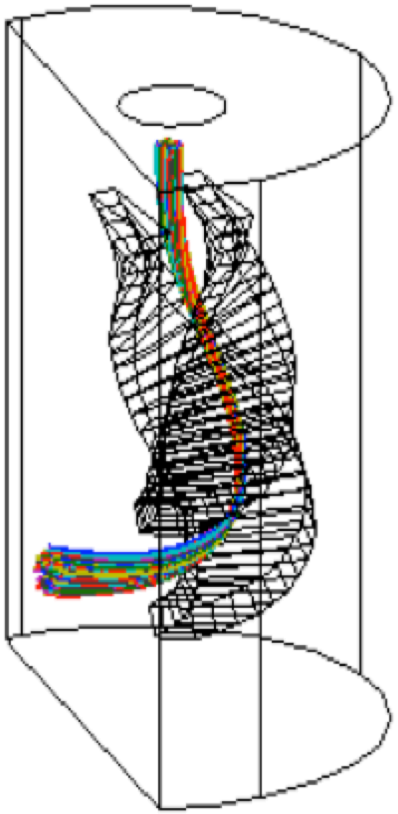
Conclusion

We have developed a code that allows us to solve broad range of particles tracking problems.

We use COMSOL Multiphysics® to simulate electro-magnetic fields of different elements of accelerators and experimental setups.

Our code allows us to use fields solutions in our simulation in combinations with fields from other sources and manipulate them.

Matlab environment provides us with powerful tools for data analysis, code development and imaging.



Thank You!