Modelling of Viscoelastic Phenomena in Concrete Structures

A.A. Pomarico*[1], G. Roselli[1], D. Caltabiano[2]
[1]STMicroelectronics Lecce, [2]STMicroelectronics Agrate Brianza
*Corresponding author: anna.pomarico@st.com

Abstract: In this paper an analysis and discussion of COMSOL capabilities for the modelling of sensors embedded in concrete structures is presented. Concrete is a very complex material to be analysed, because of many inherent peculiarities, most of them time-dependent. In this work, COMSOL is used to model two of the most relevant phenomena that arise in concrete materials (viscoelastic creep and shrinkage) and their impact on the mechanical properties of a sensor embedded in it; in particular the simulation results of a prototypal pressure sensor embedded in concrete for Structural Health Monitoring (SHM) is presented.

Keywords: MEMS, sensors, Structural Health Monitoring (SHM)**,** equation based modelling, Kelvin chain, concrete modelling

1. Introduction

A growing demand is recently emerging of sensors suited to be integrated in the concrete in order to measure some critical parameters, such as pressure, humidity and temperature. For example, by monitoring the pressure in various strategic points of the structure and their evolutions over time it is possible to understand the health of the structure and the needs for maintenance intervention.

Concrete behaviour need special numerical tools to be properly modelled. For this reason, some Finite Element Method (FEM) modelling software have been specifically developed for the simulation of concrete structures. On the other side, an appropriate design of a silicon sensor asks for a detailed micro-electromechanical investigation of its behaviour when an external load is imposed; such an investigation is generally performed by means of dedicated simulation tools, with Multiphysics capabilities.

The design of a micromechanical sensor in concrete materials invokes for the simultaneous modelling of a concrete material, a high performance silicon microelectromechanical device and their interaction. This is a very challenging system, which we could successfully model thanks to the flexibility of the COMSOL software and its Equation-based modelling approach.

2. The problem of concrete modelling

Concrete structures are a very demanding benchmark for numerical modelling tools. In fact some of the most relevant mechanical properties (such as the Young modulus) of concrete are time-dependent, as a consequence of chemical/physical phenomena happening during the concrete curing (i.e., the hydration process that occurs after the concrete has been placed, which allows calcium-silicate hydrate (C-S-H) to form and then provides concrete optimal strength and hardness) and its following aging. Two of the most important inherent properties of concrete are creep and shrinkage.

Concrete is a viscoelastic material. As a consequence, when it is subjected to a step constant stress, it experiences a time-dependent increase in strain. This phenomenon is known as viscoelastic creep. Concrete that is subjected to long-duration forces is prone to creep.

The stress dependent strain $\varepsilon_{c\sigma}(t,t_0)$ at time t (in days) may be expressed as:

$$\varepsilon_{c\sigma}(t,t_0)=\sigma_c(t_0)\left[\frac{1}{E_{ci}(t_0)}+\frac{\varphi(t,t_0)}{E_{ci}}\right]= \\ =\sigma_c(t_0)J(t,t_0)$$

In this equation $J(t,t_0)$ is the creep function or creep compliance, representing the total stress dependent strain per unit stress; $E_{ci}(t_0)$ is the modulus of elasticity at the loading time t_0 (expressed in days); hence $1/E_{ci}(t_0)$ represents the initial strain per unit stress at loading. E_{ci} is the modulus of elasticity in MPa at an age of 28 days.

The Creep Function of concrete can be calculated according to the concrete equation theory, for given concrete class, sample size and humidity conditions, and after specifying the concrete age at the loading instant[1].

The second important phenomenon emerging in concrete structures is concrete volumetric change, even in steady temperature conditions. The change occurring without any thermal exchange with the environment is referred to as concrete shrinkage, and it is the global result of both autogenous deformation $\varepsilon_{cas}(t)$ and drying shrinkage $\varepsilon_{cds}(t)$:

$$\varepsilon_{cs}(t,t_s) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t)$$

The parameter t_s represents the concrete age at the beginning of drying (in days). Autogenous deformation (which can be an expansion or a shrinkage) takes place during concrete curing and it is caused by the chemical process of cement hydration. After curing, concrete shrinks as the water not consumed by cement hydration gradually leaves the system. This phenomenon is known as drying shrinkage. It generally represents the most relevant contribution to the total shrinkage.

Concrete total shrinkage can be expressed as a time-dependent function, assuming as equation parameters the concrete class, the sample size, the humidity conditions and the concrete age at the beginning of the drying process.

2.1. Numerical modelling of concrete viscoelasticity with COMSOL

The first phenomenon we tried to numerical model is concrete viscoelasticity. The modelling of viscoelasticity is one of the most relevant steps for a reliable analysis of a concrete structure and its embedded sensors.

Experimental viscoelastic properties can be described by different theoretical models, such as the Standard linear solid model, Kelvin-Voight model, Generalized Maxwell model, the Kelvin chain model. Each one of them has a specific field of applications. In particular, generalized Maxwell model and Kelvin chain model both consist of various "branches", composed of springs and dampers with different types of connections among them. These models with multiple branches (and therefore containing many parameters and many time constants) are often very useful in the modelling of a given viscoelastic behaviour over a long observation time.

Generalized Maxwell chain is particularly suitable to the description of stress Relaxation phenomena (stress decrease in viscoelastic material under constant strain) while Kelvin Chain is the preferred model for the description of Creep phenomena (strain increase under constant stress).[2] As in concrete creep phenomena are the most widely observed and investigated, in the classical theory of concrete the Kelvin chain approach has been adopted, and Kelvin chain parameters (obtained by a fitting procedure of theoretical creep function) for different types of concrete are also available. For these reasons, modelling concrete by means of Kelvin chain is a more convenient and recommended choice.

Kelvin chain is not a built-in model of COMSOL. Nevertheless, considering that it is the elective model for concrete viscoelasticity, we decided to build this new mathematical/material model, exploiting its Equation-based Modelling capability.

In a general multiaxial case, the generalized Kelvin model can be represented with an elastic spring (characterized with a shear modulus G_0) to provide the instantaneous stiffness, plus n Kelvin-Voigt branches connected in series, each one consisting of a spring (with modulus G_i) and a damper (with viscosity coefficient η_i). A sketch of the generalized Kelvin model is reported in Fig.1.

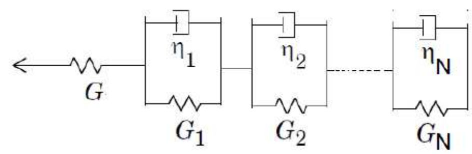

Figure 1 Sketch of the generalized Kelvin model

It can be demonstrated that, by using a generalized Kelvin model, the specific creep function is, for a loading instant t_0:

$$J(t-t_0) = \frac{1}{G_0} + \sum_{i=1}^{n} \frac{1}{G_i}\left[1 - e^{-\frac{t-t_0}{\tau_i}}\right]$$

$$\tau_i = \frac{\eta_i}{G_i}$$

The parameter τ_i is the retardation time per branch, i.e. an estimate of the time required for the creep process to approach completion, with reference to the *i-th* branch. The creep function of the Kelvin model can be easily adjusted to model real systems behaviour, thanks to its *2n+1* parameters, which can act as fitting parameters in case experimental data for the creep are available.

In such a model, the stress at each unit is the same external stress, whereas the total strain is the sum of the elastic strain of the first spring plus the strains of the branches:

$$\varepsilon = \varepsilon_{el} + \sum_{i=1}^{n} \varepsilon_i$$

In particular, in order to model the concrete viscoelasticity of a realistic concrete specimen, with given class, size, and humidity conditions, we used 8 Kelvin branches. It can be demonstrated[2] that each Kelvin-Voight branch obeys a differential equation relating the stress to the strain and its derivative; in the multiaxial stress case, the differential equation has the following expression:

$$\tau_i \dot{\varepsilon}_i + \varepsilon_i = \frac{1}{2G_i} \sigma_{dev}$$

In this equation, σ_{dev} is the deviatoric stress tensor.

The COMSOL model we built was able to accurately compute viscoelastic effect by solving the strains in the viscoelastic branches with the *Domain ODEs* (Ordinary Differential Equations) *and DAEs* (Differential Algebraic Equations) *interface*.

In building the COMSOL model of the concrete sample, no defined material was assigned to the domain, and eight stiffness moduli and eight retardation times (one for each branch of the chain) were fed into the model as user-defined variables. The procedure consisted, then, in setting up and solving, for each Kelvin-Voigt branch, a first-order Ordinary Differential Equation (ODE) for the auxiliary strains ε_i. In the *Domain ODEs and DAEs* interface, a differential equation having the following expression can be directly implemented:

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} = f$$

In the Kelvin model case, for each branch of the chain such an equation has to be solved, with σ_{dev}-$2G_i\varepsilon_i$ playing the role of the source term f, $2G_i\tau_i$ representing the damping coefficient d_a, whereas the mass coefficient e_a is zero.

As a first step, in order to validate our model, we selected a very simple system, a concrete cylinder under uniaxial pressure, whose exact mathematical solution can be calculated from theory.[1] In particular, we computed the creep function for a concrete with a f_{ck} (specified characteristic cylinder compressive strength) of 48MPa, considering a specimen in the shape of a cylinder 7cm height and 7 cm diameter, and assuming 50% of environmental relative humidity. We then also built a COMSOL model for the same system, and we compared simulation output for creep with the trend obtained from theoretical model. The comparison showed a perfect agreement (Fig.2).

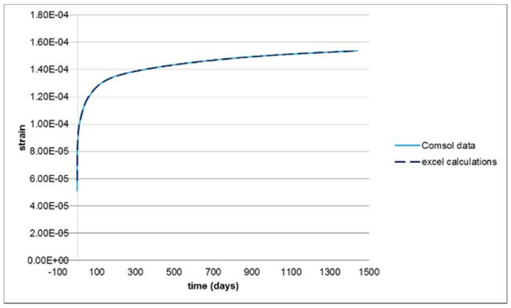

Figure 2 Comparison between COMSOL simulations of a creep experiment, based on a Kelvin-chain modelling of the material viscoelasticity, and the theoretical results computed according to the ModelCode equations. In the simulations, a constant load of 1MPa is applied.

Table 1 Kelvin chain parameters using for the modelling

Branch number	Shear modulus G_i [MPa]	Retardation time τ_i [s]
0	7254	
1	25388	5E+02
2	36268	5E+03
3	29015	5E+04
4	18134	5E+05
5	10881	5E+06
6	14507	5E+07
7	21761	5E+08
8	21761	5E+09

2.2. Numerical modelling of shrinkage with COMSOL

Time-dependent concrete shrinkage can be easily modelled in COMSOL by adopting a strategy based on the analogy with thermal phenomena. Once the shrinkage profile of concrete has been calculated using suitable equations[1], we conferred a unitary thermal coefficient to the material, then we input into the model a temperature profile T that was exactly retracing the calculated shrinkage trend *versus* time. The resulting "thermal" strain, calculated in COMSOL by means of the following equation, was supposed to exactly imitate the actual shrinkage strain.

$$\varepsilon_{th} = \alpha\left(T - T_{ref}\right)$$

In this equation, α is the thermal expansion coefficient (that we fixed as having a unitary value) and T_{ref} is a conventional reference temperature for the thermal simulation.

First of all we applied our method on the simple benchmark structure already used for testing the creep model (that is a concrete having f_{ck} of 48MPa, considering a specimen in the shape of a cylinder 7cm height and 7 cm diameter, and assuming 50% of environmental relative humidity). We exactly computed and modelled the shrinkage strain, obtaining a perfect agreement, as shown in Fig.3.

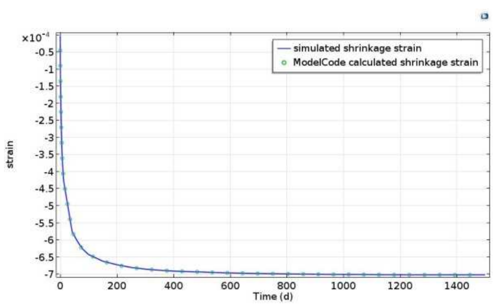

Figure 3 Comparison between the computed shrinkage strain of the concrete sample (according to the theoretical model of concrete in ModelCode) and the simulated strain. The simulated time was about 1500 days.

3. Analysis of a membrane pressure sensor in a concrete material

Once our model for concrete had been validated, both as for the viscoelastic behaviour and the shrinkage, it could be profitably used to describe concrete in more complex simulations. In particular, we were interested in the analysis of the system consisting of a concrete structure and a silicon sensor embedded in it.

We can expect that concrete viscoelasticity and shrinkage would impact on any sensor embedded in a concrete structure, anyway the role of these phenomena acquires a pivotal importance when pressure sensors for concrete are concerned. We therefore decided to use our COMSOL model for concrete to study the effect of concrete on an embedded sample of silicon, with a membrane (Fig.4), which could represent a simplified sensor structure for pressure measurements, provided that stress-sensing elements, for example four piezoresistors connected in a Wheatstone bridge,[3] are conveniently fabricated on it.

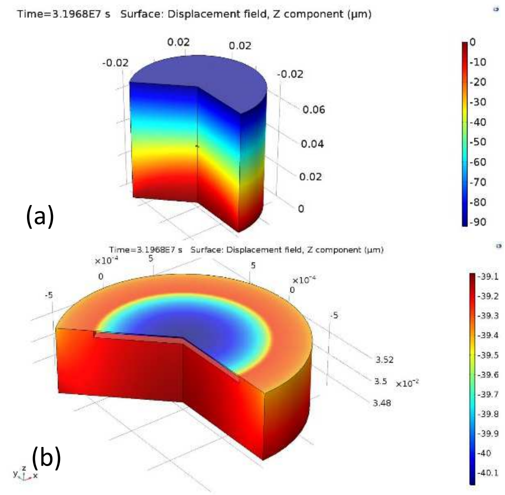

Figure 4 (a) Geometry of the simulated system, representing a sensor structure embedded in a viscoelastic concrete cylinder. (b) Zoom of the previous picture, highlighting the axial displacement in the sensor.

The sensor prototype has a cylindrical shape, with 2mm diameter, 600µm height. The sensing part of the structure is the membrane, which is 10µm thick and 700µm radius. Internal cavity is 50µm deep. A constant pressure load of 10MPa is applied on top of the concrete cylinder. We used the Structural Mechanics module to develop our model.

In a first step, we applied only creep equations to our model, excluding shrinkage strain. We observed that concrete creep strongly modifies the membrane displacement over time, even if a constant load is applied. In particular, we can see that the difference between the displacement at the membrane centre and the one at its edge (Fig. 5), which is a measure of the membrane actual deformation, changes from the initial value of about -0.42µm to the final value (after 370 days of constant load on the concrete sample) of about -0.87µm, then more than doubling its value.

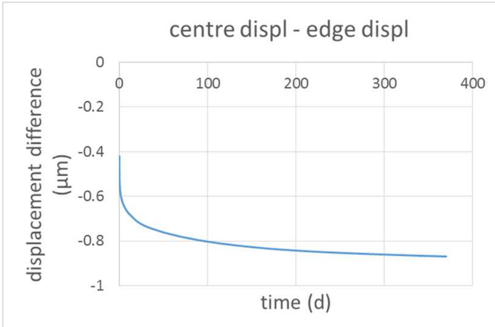

Figure 5 Difference between the vertical displacement at the membrane centre and the one at its edge, for a constant load of 10MPa on top of the concrete cylinder.

We decided to investigate the possible correlation with a creep-induced change in the stress state of the membrane. In fact, we could observe that the stress increases over time. This is shown, for example, in the 3D plots (at t=0 and at t=370days) of the radial component of the stress (Fig. 6, where the same scale range is used for comparison).

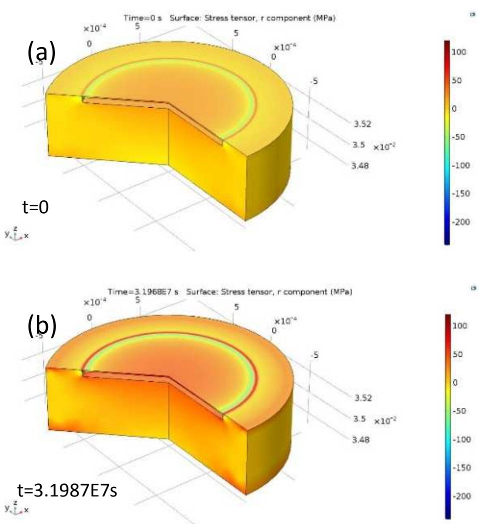

Figure 6 Radial stress distribution on the sensor inside concrete, (a) at the beginning of the time span, when there is only the instantaneous effect of the external load, and (b) at the end, when creep has modified the stress distribution

In more detail, the line plots of Fig. 7 show the radial and the angular components of the stress along a radius of the membrane (from x=0, centre of the membrane, to x=0.7mm, edge of the membrane), at the beginning and at the end of the creep process. Both components vary during time, because of the creep effect. Assuming that piezoresistors are fabricated on the membrane, we will then observe a time dependent, creep-induced variation of the sensor output. The relative weight of the two stress component variations on the sensor output will depend on the actual position of piezoresistors. For example, very close to the membrane edge the radial stress would give the major contribution.

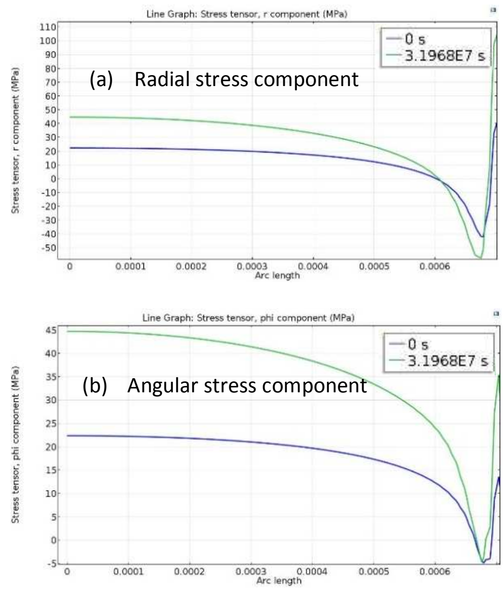

Figure 7 (a) Plots at t=0 and at t=370 days of the radial stress along a radius (from x=0, centre of the membrane, to x=0.7mm, edge of the membrane). The variation is only due to the creep effect (as a constant load is applied). (b) The same for the angular stress.

The membrane behaviour is also affected by the other typical concrete phenomenon considered in this study, which is its shrinkage. In our simulation conditions, when shrinkage is added in the previous model (which was taking into account only the creep), there is only a small modification of the deformation of the membrane at its centre, in fact the difference between the centre and the edge displacements reaches the final value of about -0.90 µm (very close to the value we obtained considering only creep). The shrinkage has a more relevant impact on the stress distribution. It can be seen by comparing, at the end of the time range, the two (radial and angular) component stress distributions for the two cases (with or without shrinkage), as shown in Fig. 8.

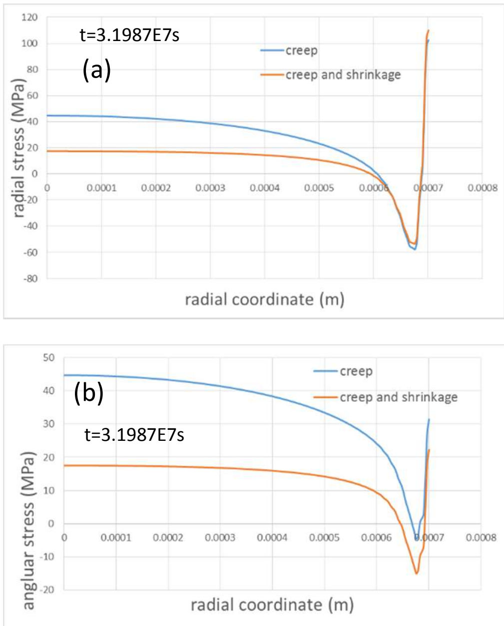

Figure 8 (a) Comparison of the radial stress along a radius of the membrane, at the end of the time span, when only creep is considered or when also shrinkage is included. (b) The same for the angular stress.

We observe that, with respect to the case in which only creep is considered, both radial and angular stress have a modified distribution on the membrane. So, a shrinkage-induced impact on the sensor output is expected too. Also in this case, piezoresistors exact locations will determine the effective contribution of the two stress components to the shrinkage-induced output variation. Anyway, angular stress is everywhere more affected by the shrinkage effect than the radial stress.

As a whole, both creep-induced and shrinkage-induced changes in the membrane deformation and stress are observed. They would necessarily impact on the output of the membrane-based pressure sensor, modifying the output voltage of a Wheatstone bridge of piezoresistors, located on the membrane itself. A quantitative evaluation of the viscoelastic and shrinkage effects on the sensor, as COMSOL enabled us to do, is then critical for a reliable design of a pressure sensor.

4. Conclusions

In our study, we tried to implement a COMSOL model for an analysis of the main concrete properties. COMSOL, thanks to its multiphysics capabilities, allows the modelling different kinds of sensor functionality; being able to use COMSOL also to model concrete peculiarities would provide a complete tool for simulating various types of sensors in concrete structures.

We were able to build COMSOL models for two fundamental concrete properties, in particular viscoelasticity and total shrinkage. Simulations on a concrete sample show similar results to the theoretical model. They allowed a preliminary evaluation of the effects of concrete properties on the functionality of a MEMS sensor, which is an essential step for a reliable design of the sensor itself.

Our next step will be trying to implement in COMSOL the superposition principle, in order to model the case of a time-dependent applied load and, also, in order to effectively take into account time-dependent rearrangements of the internal stresses which can take place even in case of constant load. The superposition principle is generally assumed to be valid for concrete, in fact concrete can be considered as an aging linear viscoelastic material for many practical applications. On the basis of this assumption, the general constitutive equation for the stress dependent strain $\varepsilon_{c\sigma}(t,t_0)$ in concrete may be written, according to the superposition principle, as:[1]

$$\varepsilon_{c\sigma}(t,t_0)=\sigma_c(t_0)J(t,t_0)+\int_{t_0}^{t} J(t,\tau)\frac{\partial\sigma_c(\tau)}{\partial\tau}d\tau$$

We will investigate the possibility of modelling such a constitutive equation with COMSOL.

5. References

1. “Model Code 2010, Final Draft, Volume 1” and “Model Code 2010, Final Draft, Volume 2”, fib Bulletin No. 65

2. “Computational Viscoelasticity”, S. Marques and G. Creus, Springer 2012

3. “Package Design of Pressure Sensors for High Volume Consumer Applications”, M. Shaw, F. Ziglioli, C.Combi, L. Baldo, 2008 Electronic Components and Technology Conference (IEEE)

6. Acknowledgements

Finally, we would like to thanks, for their important contribution, STMicroelectronics, Prof. G. Bertagnoli (Politecnico di Torino) and COMSOL support.