Institut de Recherche Dupuy de Lôme

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Numerical model for predicting heat and mass transfer phenomena during cake baking

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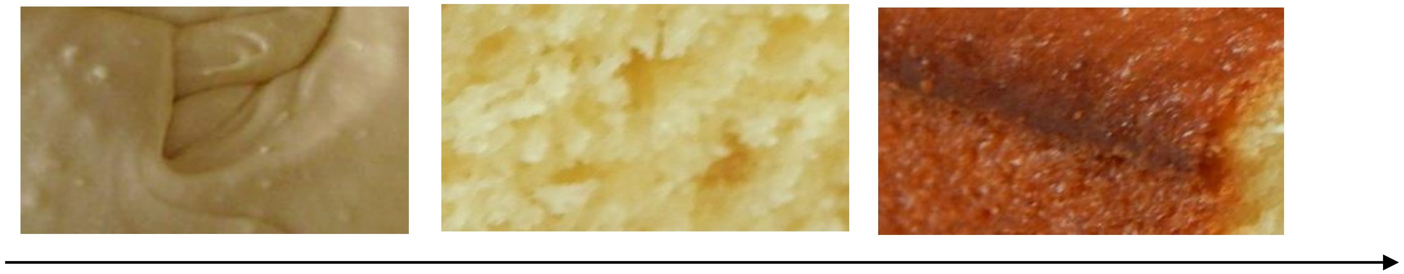
Objectives

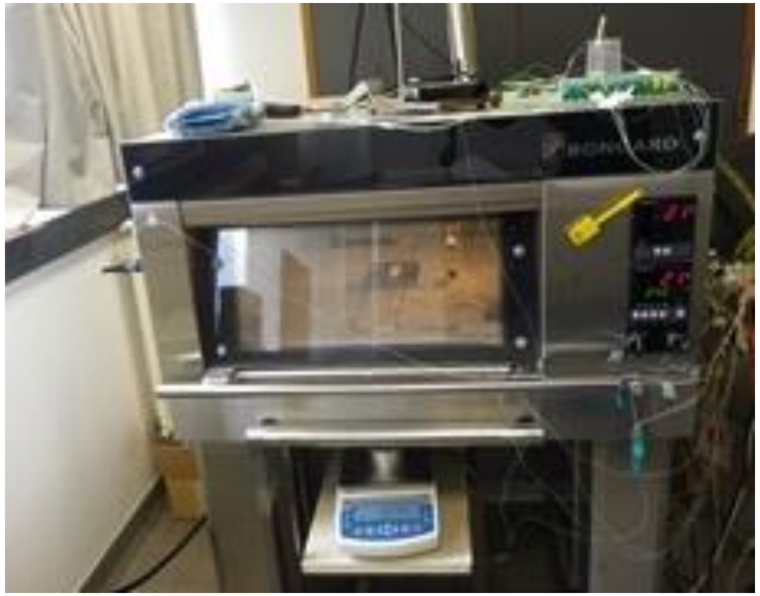
- Objectives
  - Implementation of a numerical model for predicting:
    - Temperature fields
    - Moisture content fields
    - Gas pressure fields
    - Swelling
  - Use this model and associated experiments for improving material and mechanisms understandings:
    - Water diffusion coefficients (liquid-vapour) in a porous deformable medium
    - Reaction kinetic of $C0_2$ production (caused by the leavening agents)
    - Evapo-condensation phenomena
    - …

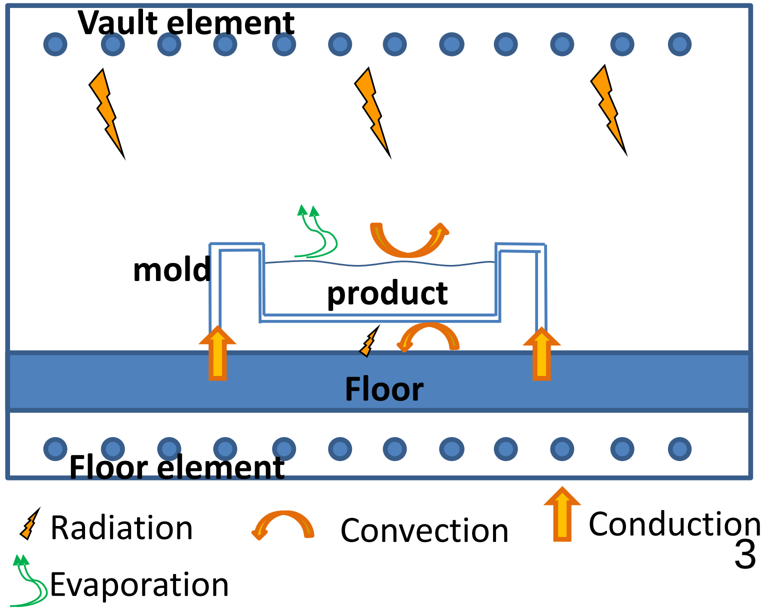
Cake baking

Composition :

- Flour
- Eggs
- Fatty substance
- Sugar
- Chemical leavening







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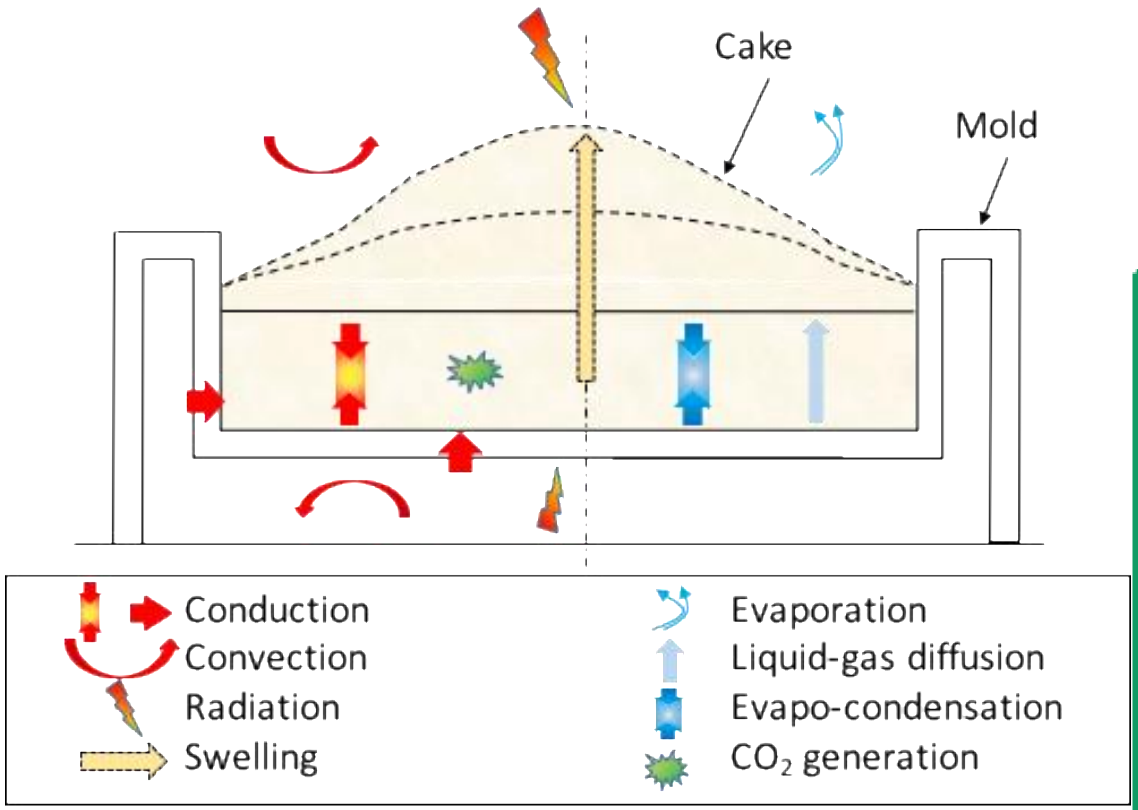
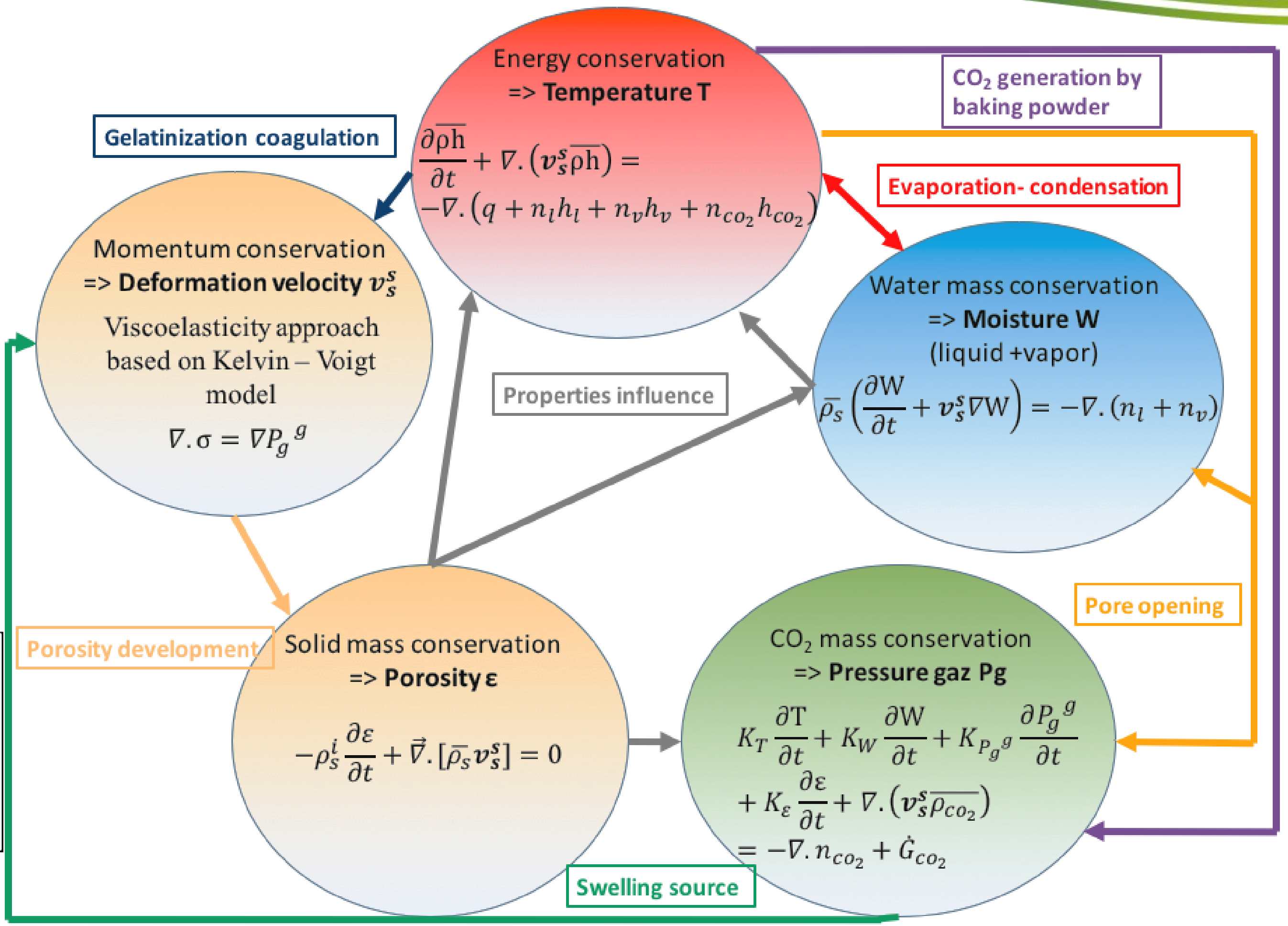
Coupled physical phenomena





Conservation equations

Assumptions:

- 3 phases :solid (s), liquid (l), gas (g)
- 2 species in gaseous phase : water (v) et CO_2
- Local thermodynamic equilibrium
- Gaseous phase: ideal gas mixture

Masse conservation:

Solid : $$\frac{\partial \overline{\rho_s}}{\partial t} + \nabla.(\boldsymbol{v_s^s}\overline{\rho_s}) = 0$$

Liquid (water) : $$\frac{\partial \overline{\rho_l}}{\partial t} + \nabla.(\boldsymbol{v_s^s}\overline{\rho_l}) = -\nabla.n_l - \dot{G}_W$$

Vapour (water) : $$\frac{\partial \overline{\rho_v}}{\partial t} + \nabla.(\boldsymbol{v_s^s}\overline{\rho_v}) = -\nabla.n_v + \dot{G}_W$$

$$\overline{\rho_s}\left(\frac{\partial \mathrm{W}}{\partial t} + \nabla.(\boldsymbol{v_s^s}\mathrm{W})\right) = -\nabla.(n_l + n_v)$$

CO_2 : $$\frac{\partial \overline{\rho_{co_2}}}{\partial t} + \nabla.\left(\boldsymbol{v_s^s}\overline{\rho_{co_2}}\right) = -\nabla.n_{co_2} + \dot{G}_{co_2}$$

$\overline{\rho_i}$	Mass concentration (kg.m^{-3})
$\boldsymbol{v_s^s}$	Deformation velocity (m.s^{-1})
$\dot{G}$	Species Production / consumption (kg.m^{-3}.s^{-1})
n	Species flow(kg.m^{-2}.s^{-1})
W	Moisture content (kg.kg$_{DB}^{-1}$)

Conservation equations

Assumptions:

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Masse conservation:

Liquid water transport (Fick's law) :

$$n_l = -\mathrm{D}_{eff,l} \nabla \rho_l = \underline{D_l^T \nabla T + D_l^W \nabla W + D_l^{P_g} \nabla {P_g}^g}$$

Vapour water transport (Darcy and Fick laws) :

$$n_v = -{\rho_g}^g \mathrm{D}_{eff,v} \nabla \omega_v \; - {\rho_v}^g \frac{k_{rg} k_g}{\mu_g} \nabla {P_g}^g = \underline{D_v^T \nabla T + D_v^W \nabla W + D_v^{P_g} \nabla {P_g}^g}$$

CO_2 transport (Darcy and Fick laws) :

$$n_{CO_2} = -{\rho_g}^g \mathrm{D}_{eff,CO_2} \nabla \omega_{CO_2} \; - {\rho_{CO_2}}^g \frac{k_{rg} k_g}{\mu_g} \nabla {P_g}^g = \underline{D_{CO_2}^T \nabla T + D_{CO_2}^W \nabla W + D_{CO_2}^{P_g} \nabla {P_g}^g}$$

$k_r k$	Permeability (m^2)
ε	Porosity
P	Pressure (Pa)
D_{eff}	Diffusion coefficient (m^2.s^{-1})
ω	Mass fraction
μ	Dynamic viscosity (Pa.s)

Conservation equations

Assumptions:

- 3 phases :solid (s), liquid (l), gas (g)
- 2 species in gaseous phase : water (v) et CO_2
- Local thermodynamic equilibrium
- Gaseous phase: ideal gas mixture

Masse conservation:

$$\bar{\rho_s}\left(\frac{\partial \mathrm{W}}{\partial t} + \nabla.(\boldsymbol{v}_{\boldsymbol{s}}^{\boldsymbol{s}}\mathrm{W})\right) = -\nabla.\left(\left(D_l^W + D_v^W\right)\nabla W + D_v^T \nabla T + D_v^{P_g} \nabla {P_g}^g\right)$$

Moisture content *W*

$$K_T \frac{\partial \mathrm{T}}{\partial t} + K_W \frac{\partial \mathrm{W}}{\partial t} + K_{{P_g}^g} \frac{\partial {P_g}^g}{\partial t} + K_\varepsilon \frac{\partial \varepsilon}{\partial t} + \nabla.\left(\boldsymbol{v}_{\boldsymbol{s}}^{\boldsymbol{s}} \overline{\rho_{co_2}}\right) = -\nabla.\left(D_{CO_2}^T \nabla T + D_{CO_2}^W \nabla W + D_{CO_2}^{P_g} \nabla {P_g}^g\right) + \dot{G}_{co_2}$$

Gas Pressure *Pg*

$$-\rho_s^s \frac{\partial \varepsilon}{\partial t} + \nabla.\left(\boldsymbol{v}_{\boldsymbol{s}}^{\boldsymbol{s}}(1-\varepsilon)\rho_s^s\right) = 0$$

Porosity *ε*

Conservation equations

Assumptions:

- 3 phases :solid (s), liquid (l), gas (g)
- 2 species in gaseous phase : water (v) et CO_2
- Local thermodynamic equilibrium
- Gaseous phase: ideal gas mixture

Energy conservation (product):

Enthalpy formulation:

$$\frac{\partial \overline{\rho h}}{\partial t} + \nabla.\left(\boldsymbol{v_s^s}\overline{\rho h}\right) = -\nabla.\left(q + n_l h_l + n_v h_v + n_{co_2} h_{co_2}\right)$$

Enthalpy equation *h*

Temperature *T*

With:

$$\overline{\rho h} = \overline{\rho c_p}\left(T - T_{ref}\right) + \overline{\rho_v} L_v\left(T_{ref}\right),$$
$$h_l = c_{p,l}\left(T - T_{ref}\right),$$
$$h_v = c_{p,v}\left(T - T_{ref}\right) + L_v\left(T_{ref}\right) = h_l + L_v(T),$$
$$h_{co_2} = c_{p,co_2}\left(T - T_{ref}\right),$$
$$q = -k_{eff}\nabla T.$$

h	Enthalpy (J.kg^{-1})
c_p	Specific heat (J.kg^{-1}.K^{-1})
Lv	Latent heat (J.kg^{-1})
T	Temperature (K)
q	Heat flux (W.m^{-2})
k_{eff}	Effective thermal conductivity (W.m^{-1}.K^{-1})

Momentum conservation:

Solid approach (Viscous behaviour) :

$$\nabla.\sigma = \nabla {P_g}^g$$

Deformation velocity v_s^s

with : $\sigma = 2\mu\dot{\epsilon}$

$$\dot{\epsilon} = \frac{1}{2}\left[(\nabla \boldsymbol{v_s^s})^T + \nabla \boldsymbol{v_s^s}\right]$$

σ	Stress tensor(Pa)
$\dot{\epsilon}$	Strain rate tensor (s^{-1})
μ	Dynamic viscosity (Pa.s)

Conservation equations

Boundary conditions :

Air / product interface:

$$-n(n_l + n_v) = \frac{k_m M}{R}\left(\frac{P_v}{T} - \frac{P_{v,\infty}}{T_\infty}\right) = \dot{m}_w$$ **W**

$$-n(q) = h(T_\infty - T) + \varepsilon_p \sigma\left({T_{oven}}^4 - T^4\right) - n_l L_v$$ **T**

$${P_g}^g = P_{atm}$$ **Pg**

$$\sigma = 0 \qquad -n(\boldsymbol{v_s^s}\bar{\rho}_s) = 0$$ **Deformation velocity**

Mold / product interface:

$$-n(n_l + n_v) = 0$$ **W**

$$-n\left(-k_{eff}\nabla T\right) = -n(-k_{mold}\nabla T_{mold})$$
$$T = T_{mold}$$ **T**

$$-n\left(n_{co_2}\right) = 0$$ **Pg**

$$-n(\boldsymbol{v_s^s}) = 0$$ **Deformation velocity**

Mold / air interface:

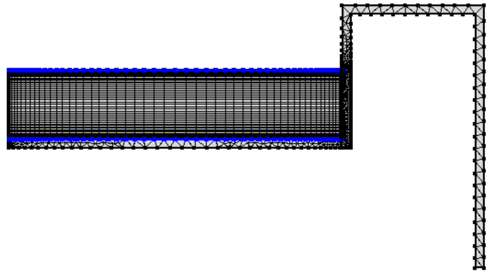
$$-n(q) = h(T_\infty - T_{mold}) + \varepsilon_m \sigma\left({T_{oven}}^4 - {T_{mold}}^4\right)$$ **T**

Symmetry axis : zero fluxes conditions

- P_v Vapour Pressure (Pa)
- k_m Mass transfer coefficient ($m.s^{-1}$)
- $\dot{m}_w$ Water evaporation rate ($kg.m^{-2}.s^{-1}$)
- h Heat transfer coefficient ($W.m^{-2}.K^{-1}$)
- ε_p Emissivity

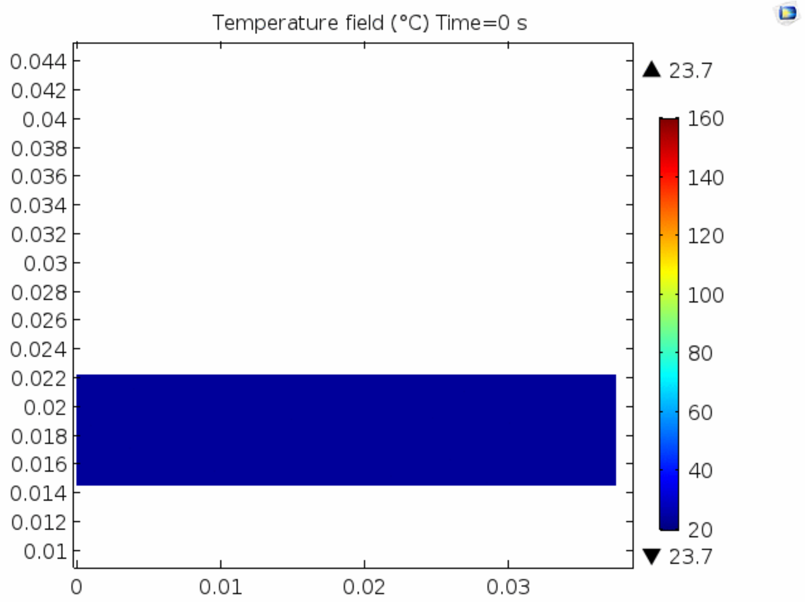
Implementation of governing equations:

- Comsol Multiphysics 5.2®
  - Transient 2D axisymmetric model
  - 4 equations in PDE formulations in general form (*W, T, Pg, ε*)
  - Structural Mechanics Module for $\boldsymbol{v_s^s}$
  - ALE formulation (Arbitrary Lagrangian Eulerian) => mobile meshing
- Meshing
  - 715 mapped and triangular elements

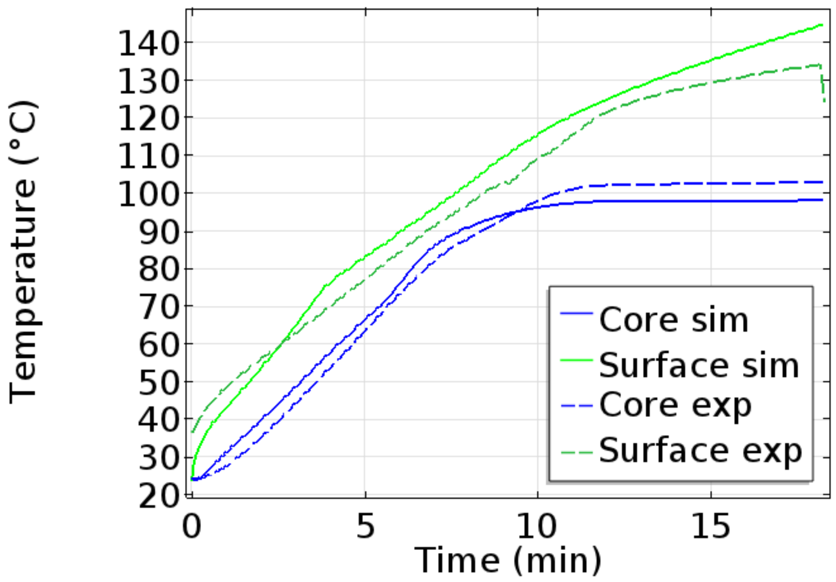


- Compute time
  - 20 min (CPU Intel Xeon 2,66 GHz (6 cores), Ram 24 GO)

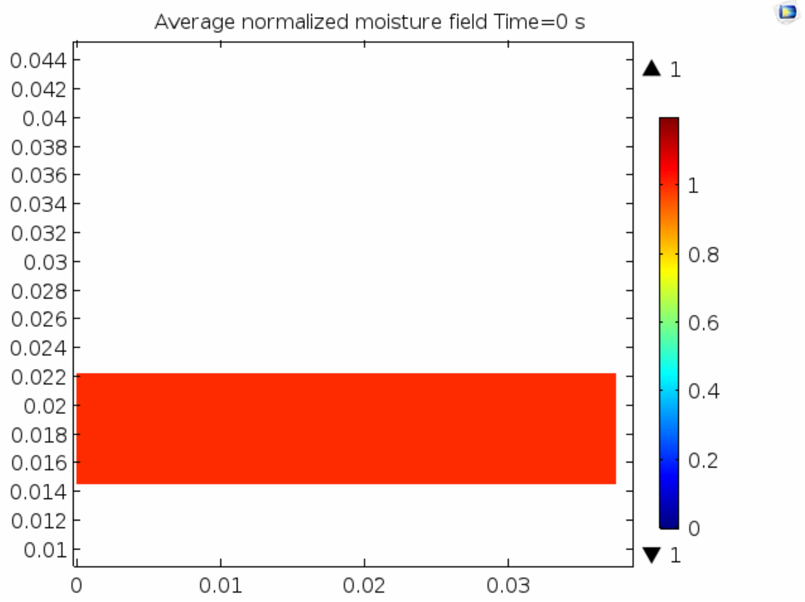
Results



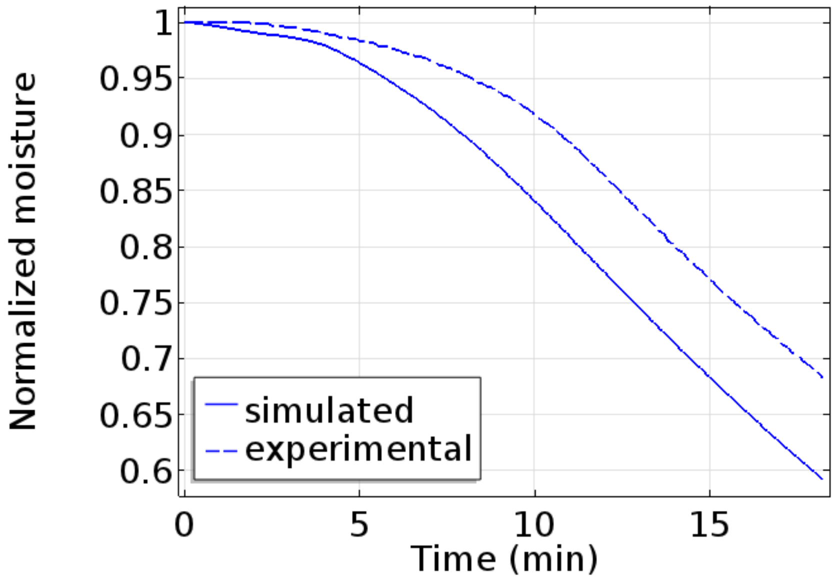
Temperature field (°C) Time=0 s
0.044
0.042
0.04
0.038
0.036
0.034
0.032
0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
0
0.01
0.02
0.03
▲ 23.7
160
140
120
100
80
60
40
20
▼ 23.7



Temperature (°C)
Time (min)
Core sim
Surface sim
Core exp
Surface exp

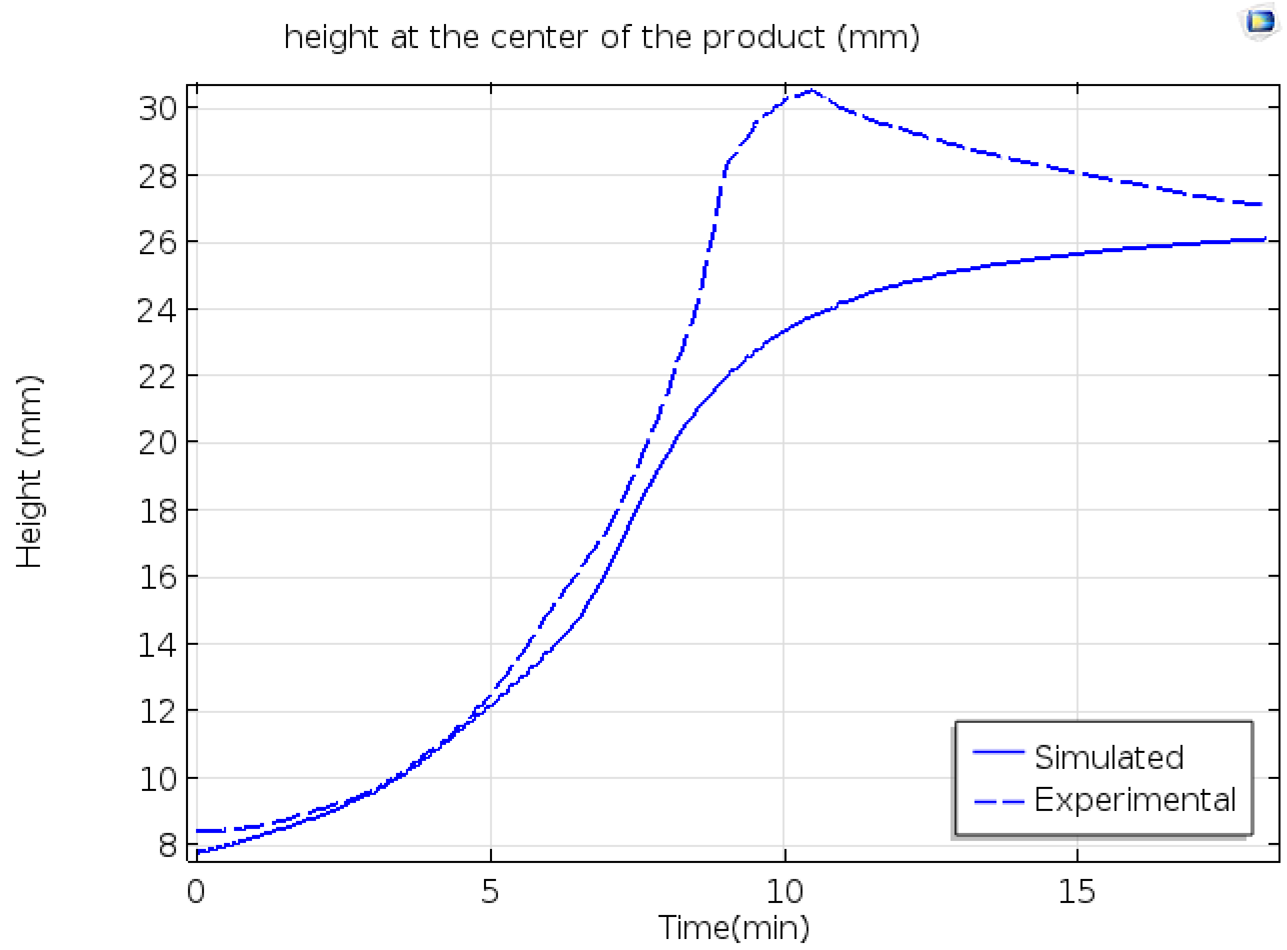


Average normalized moisture field Time=0 s
▲ 1
1
0.8
0.6
0.4
0.2
0
▼ 1

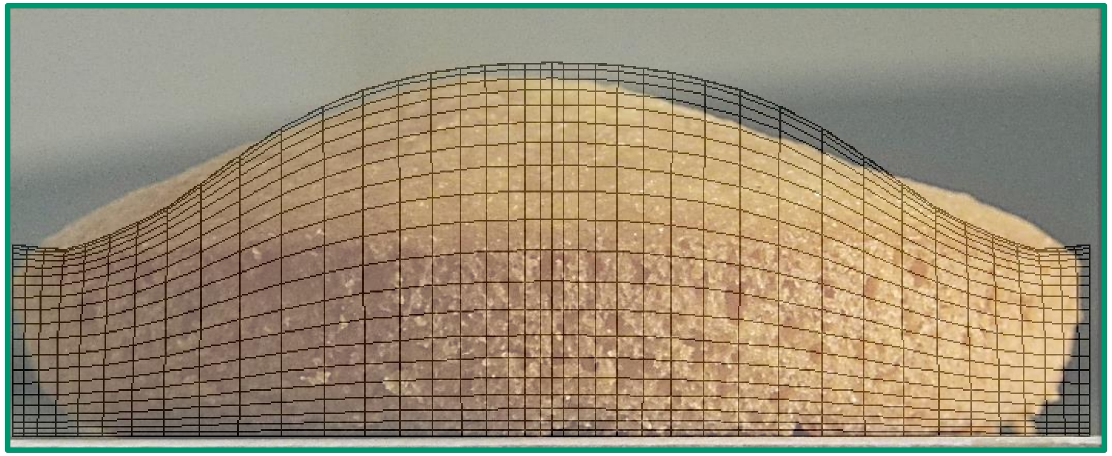


Normalized moisture
Time (min)
simulated
experimental

Results



height at the center of the product (mm)
Height (mm)
Time(min)
30
28
26
24
22
20
18
16
14
12
10
8
0
5
10
15
Simulated
Experimental



Conclusion / Perspective

- Conclusion
  - A numerical model was implemented for describing heat an mass transfer into a deformable porous medium.
  - This model predicts: temperature, moisture content, gas pressure, porosity and swelling.
  - A correct agreement with experimental data for temperatures, mass losses and global deformation is noted. Nevertheless, results could be improved.
  - Provide better knowledge about product and the mechanisms.
- Perspectives
  - Improving the model
    - Gas phase with 3 species : water, CO_2 and air
    - Colouring (brownness) prediction
    - Adding mechanical laws and reaction kinetics (coagulation, gelatinization, CO_2 production) more realistic.

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