Finite Element Analysis of Ferrofluid Cooling of Heat Generating Devices

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Abstract: The viscous, two-dimensional, laminar and incompressible ferromagnetic fluid flow is considered in this paper. Flow takes place in channel between two parallel flat plates. There are rectangular blocks (heat-generating devices) below the upper wall.In this paper the considered ferrofluid flow is influenced by magnetic dipole. The magnetic dipole gives rise to a magnetic field. Ferrofluids have promising potential for heat transfer applications because a ferrofluid flow can be controlled by using an external magnetic field. A strong magnet placed near the device which produces heat will always attract colder ferrofluid towards it more than warmer.

Keywords: ferrofluid, Kelvin force, magnetic dipole, magnetic scalar.

1. Introduction

During the last decades, an extensive research work has been done on the fluids dynamics in the presence of magnetic field (magnetorheological fluids - MR, ferrofluids - FF, electrorheological fluids - ER and certain types of polymeric gels). The effect of magnetic field on fluids is worth investigating due to its innumerable applications in wide spectrum of fields. The study of interaction of the magnetic field or the electromagnetic field with fluids have been documented e.g. among nuclear fusion, chemical engineering, medicine, high speed noiseless printing and transformer cooling.

Ferrofluids are industrially prepared magnetic fluids which consist of stable colloidal suspensions of small single-domain ferromagnetic particles in suitable carrier liquids. Usually, these fluids do not conduct electric current and exhibit a nonlinear paramagnetic behavior. The variety of formulations available for ferrofluids permits a great number of applications, from medical to satellite and vacuum technologies [3-5].

An external magnetic field imposed on a ferrofluid with varying susceptibility, e.g., due to a temperature gradient, results in a nonuniform magnetic body force, which leads to a form of heat transfer called thermomagnetic convection. This form of heat transfer can be useful when conventional convection heat transfer is inadequate, e.g., in miniature microscale devices or under reduced gravity conditions.

A good understanding of the relationship between an imposed magnetic field, the resulting ferrofluid flow, and the temperature distribution is a prerequisite for the proper design and implementation of applications involving thermomagnetic convection.

2. Governing equations

In this paper the considered ferrofluid flow is influenced by magnetic dipole. We assumed that the magnetic dipole is located at distance $|b|$ below the sheet at point (a,b). The magnetic dipole gives rise to a magnetic field, sufficiently strong to saturate the fluid. In the magnetostatic case where there are no currents present, Maxwell-Ampere's law reduces to $\nabla\times\mathbf{H}=\mathbf{0}$. When this holds, it is also possible to define a magnetic scalar potential by the relation $\mathbf{H}=-\nabla V_m$ and its scalar potential for the magnetic dipole is given by [1]

$$V_m(\mathbf{x})=\frac{\gamma}{2\pi}\frac{x_1-a}{(x_1-a)^2+(x_2-b)^2} \qquad (1)$$

where γ is the magnetic field strength at the source (of the wire) and (a,b) is the position were the source is located.

The governing equations of the fluid flow under the action of the applied magnetic field and gravity field are: the mass conservation equation, the fluid momentum equation and the energy equation for temperature in the frame of Boussinesque approximation.

The mass conservation equation for an incompressible fluid is

$$\nabla\cdot\mathbf{v}=0\,. \qquad (2)$$

The momentum equation for magnetoconvective flow is modified from typical natural convection equation by addition of a magnetic term

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\eta\nabla^2\mathbf{v}+ \\ +\alpha\rho_0 g(T-T_0)\mathbf{k}+(\mathbf{M}\cdot\nabla)\mathbf{B} \quad (3)$$

where ρ_0 is the density, $\mathbf{v}$ is the velocity vector, p is the pressure, T is the temperature of the fluid, η is the viscosity, $\mathbf{k}$ is unit vector of gravity force and α is the thermal expansion coefficient of the fluid.

The energy equation for an incompressible fluid which obeys the modified Fourier's law is

$$\rho_0 c\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k\nabla^2 T+ \\ +\eta\Phi-\mu_0 T\frac{\partial \mathbf{M}}{\partial T}\cdot((\mathbf{v}\cdot\nabla)\mathbf{H}) \quad (4)$$

where k is the thermal conductivity and $\eta\Phi$ is the viscous dissipation

$$\Phi=\left(2\left(\left(\frac{\partial v_1}{\partial x}\right)^2+\left(\frac{\partial v_2}{\partial y}\right)^2\right)+\left(\frac{\partial v_2}{\partial x}+\frac{\partial v_1}{\partial y}\right)^2\right) \quad (5)$$

The last term in the energy equation represents the thermal power per unit volume due to the magnetocaloric effects.

The last term in the momentum equation represents the Kelvin body force per unit volume

$$\mathbf{f}=(\mathbf{M}\cdot\nabla)\mathbf{B}\,, \quad (6)$$

which is the force that a magnetic fluid experiences in a spatially non-uniform magnetic field. We have established the relationship between the magnetization vector and magnetic field vector

$$\mathbf{M}=\chi_m\mathbf{H}\,. \quad (7)$$

Using the constitutive relation (relation between magnetic flux density and magnetic field vector) we can write the magnetic induction vector in the form

$$\mathbf{B}=\mu_0(1+\chi_m)\mathbf{H}\,. \quad (8)$$

The variation of the total magnetic susceptibility χ_m is treated solely as being dependent on temperature [2]

$$\chi_m=\chi_m(T)=\frac{\chi_0}{1+\alpha(T-T_0)}\,. \quad (9)$$

Finally, the Kelvin body force can be represented by

$$\mathbf{f}=\frac{1}{2}\,\mu_0\chi_m(1+\chi_m)\nabla(\mathbf{H}\cdot\mathbf{H})+ \\ +\mu_0\chi_m\mathbf{H}((\mathbf{H}\cdot\nabla)\chi_m)\,. \quad (10)$$

Using equations (7-9) we can write Eq. (3) and (4) in the form, respectively

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\eta\nabla^2\mathbf{v}+ \\ +\alpha\rho_0 g(T-T_0)\mathbf{k}+ \\ +\frac{1}{2}\,\mu_0\chi_m(1+\chi_m)\nabla(\mathbf{H}\cdot\mathbf{H})+ \\ +\mu_0\chi_m\mathbf{H}((\mathbf{H}\cdot\nabla)\chi_m) \quad (11)$$

and

$$\rho_0 c\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k\nabla^2 T \\ +\eta\Phi-\mu_0 T\frac{\partial(\chi_m\mathbf{H})}{\partial T}\cdot((\mathbf{v}\cdot\nabla)\mathbf{H})\,. \quad (12)$$

For simplicity the preferred work choice is to work in non-dimensional frame of reference [8]. Now some dimensionless variables will be introduced in order to make the system much easier to study. Moreover some of the dimensionless ratios can be replaced with well-known parameters: the Prandtl number Pr, the Rayleigh number Ra, the Eckert number Ec, the Reynolds number Re and the magnetic number Mn, respectively:

$$Pr=\frac{\eta_0}{\rho_0\kappa},\ Ra=\frac{\alpha\rho_0 g h^3\delta T}{\eta_0\kappa}, \\ Ec=\frac{v_r^2}{c\delta T}=\frac{\kappa^2}{c\delta T h^2},\ Re=\frac{h\rho_0 v_r}{\eta_0}=\frac{\rho_0\kappa}{\eta_0}, \\ Mn=\frac{\mu_0 H_r^2}{\rho_0 v_r^2}=\frac{\mu_0 H_r^2 h^2}{\rho_0\kappa^2}\,. \quad (13)$$

3. Numerical results

This case examines the temperature field in the ferrofluid and in the electronic component with heat source. The ferrofluid transports heat by convection and conduction. Finally, to approximates the electronic component that requires cooling, the model uses a rectangular blocks with a given volume heat source. The electronic components transports thermal energy by pure conduction.

The viscous, two-dimensional, laminar and incompressible ferromagnetic fluid flow is considered in this paper. Flow takes place in channel between two parallel flat plates. There are rectangular blocks (heat-generating devices) below the upper wall. The length of the channel is $L = 10$ and distance between plates is $h = 1$. Outside the channel the magnetic dipole is located at point (a, b).

This magneto-thermo-mechanical problem considered in these examples is governed by following dimensionless equations:

- momentum equation for magnetoconvective flow (Navier-Stokes equation) with Kelvin force;
- mass conservation equation for an incompressible ferrofluid;
- thermal diffusion equation;
- heat transfer by conduction for solid domain is the heat equation.

The following boundary conditions for dimensionless variables are assumed:

- For upper wall: The velocity is 0 (no slip condition). Insulation condition for heat transfer by conduction (in solid domain) $\mathbf{n}\cdot\mathbf{q} = \mathbf{n}\cdot(-k_s\nabla T) = 0$ and for heat transfer by conduction and convection (in fluid domain) $\mathbf{n}\cdot\mathbf{q} = \mathbf{n}\cdot(-k_f\nabla T + \rho_f c_f T\mathbf{u}) = 0$ specifies where the domain is well insulated.
- For lower wall: The velocity is 0 (no slip condition). Insulation condition for heat transfer by conduction and convection (in fluid domain) $\mathbf{n}\cdot\mathbf{q} = \mathbf{n}\cdot(-k_f\nabla T + \rho_f c_f T\mathbf{u}) = 0$.
- For inlet (left wall): The temperature is $\frac{T_l}{\delta T}$ where $\delta T = |T_u - T_l|$. At the inlet boundary there is a parabolic laminar flow profile given by equation $u_{in} = -4\frac{u_0}{u_r}y(y-1)$ for $y \in \langle 0,1 \rangle$.
- For outlet (right wall): The convective flux is assumed for temperature, $\mathbf{n}\cdot(-k_s\nabla T) = 0$. Pressure outlet is also assumed, $(-p\mathbf{I} + \mathbf{S})\mathbf{n} = -p_0\mathbf{n}$, where p_0 is the dimensionless atmospheric pressure.
- For solid-fluid interface: The velocity is 0 (no slip condition). Continuity equation for heat transfer equation $\mathbf{n}\cdot(\mathbf{q}_s - \mathbf{q}_f) = 0$ where $\mathbf{q}_s = -k_s\nabla T$ and $\mathbf{q}_f = -k_f\nabla T + \rho_f c_f T\mathbf{u}$.

The following initial conditions for dimensionless variables are assumed: fluid is motionless (velocity is zero), pressure is zero and temperature is $\frac{T_l}{\delta T}$ for whole domain (with fluid and solid).

Table 1. Quantities of fluid

Quantity	Variable	Unit	Value
Density	ρ_0	$\left[\frac{kg}{m^3}\right]$	1180
Viscosity	η_0	$\left[\frac{kg}{m\cdot s}\right]$	0.08
Thermal conductivity	k	$\left[\frac{J}{m\cdot s\cdot K}\right]$	0.06
Heat capacity	c	$\left[\frac{J}{kg\cdot K}\right]$	4200
Thermal diffusivity (diffusion coefficient)	$\kappa = \frac{k}{\rho_0 c}$	$\left[\frac{m^2}{s}\right]$	1.21e-7
Thermal expansion coefficient	α		5.6e-3
Magnetic susceptibility	χ_0	-	6e-2

Table 2. Quantities of solid (heat generating devices)

Quantity	Variable	Unit	Value
Density	ρ_s	$\left[\frac{kg}{m^3}\right]$	8960
Thermal conductivity	k_s	$\left[\frac{J}{m\cdot s\cdot K}\right]$	401
Heat capacity	c_s	$\left[\frac{J}{kg\cdot K}\right]$	384
Thermal diffusivity (diffusion coefficient)	$\kappa_s = \frac{k_s}{\rho_s c_s}$	$\left[\frac{m^2}{s}\right]$	1.165e-4
Heat source	Q	$\left[\frac{J}{s\cdot m^3}\right]$	8.0e+8

Table 3. Flow parameters

Quantity	Variable	Unit	Value
Velocity	u_0	$[m/s]$	5.0e-3
Characteristic velocity	v_r	$[m/s]$	1.210e-7
Magnetic permeability of a vacuum	$\mu_0 = 4\pi \cdot 10^{-7}$	$[N/A^2]$	1.256e-6
Difference of temperatures	δT	$[K]$	30
Temperature	T_0	$[K]$	300
Temp. of upper wall	$T_u = T_0 + \delta T$	$[K]$	330
Temp. of lower wall	$T_l = T_0$	$[K]$	300
High , length	h , L	$[m]$	1e-3, 1e-2
Centre of magnetic wire	(a,b)	$[m]$	(2e-3,-3e-3)
Magnetic field strength at the source	γ	$[A \cdot m]$	10
Ratio of thermal diffusivities	$\kappa_{ratio} = \frac{\kappa_s}{\kappa}$	-	962.687174

Table 4. Quantities of flow A and B

Quantity	Flow A	Flow B
a b	2 -3	2 -2.5
H_r	1.768388e+5	2.546479e+5
Pr	560	
Ra	200.7938	
Ec	1.16324e-13	
Re	0.001786	
Mn	2.272182e+9	4.711597e+9
Qn	64.019097	
V_{avg} (fluid)	292.980369	309.281869
T_{avg} (fluid)	101.330192	101.314674
T_{avg} (all)	118.521428	118.261882

Table 5. Quantities of flow C and D

Quantity	Flow C	Flow D
a b	2 -2	2 -1.5
H_r	3.978874e5	7.073553e+5
Pr	560	
Ra	200.7938	
Ec	1.16324e-13	
Re	0.001786	
Mn	1.150292e+10	3.635492e+10
Qn	64.019097	
V_{avg} (fluid)	331.905089	371.38104
T_{avg} (fluid)	101.096441	100.801837
T_{avg} (all)	117.760089	117.17941

In all tables in this chapter quantities V_{avg} , T_{avg} and T_{dom} are calculated using following formulas:

$$V_{\text{avg}} = \iint_{A_p} |\mathbf{u}| dA_p \,, \tag{14}$$

$$T_{\text{avg}} = \iint_{A_p} T dA_p \,, \tag{15}$$

$$T_{\text{dom}} = \iint_{A} T dA \,, \tag{16}$$

where $A_p = 8.8$ denotes area of ferrofluid and $A = 10$ denotes area of channel (fluid and solid).

In the flows A, B,C and D magnetic dipole is placed below the channel on the line perpendicular to the horizontal wall of channel in distance 2 from left wall. When the distance from source of magnetic dipole to bottom channel wall is decreases, it can be observed that, due to the value of the characteristic value of magnetic field H_r, the maximum value of: the magnitude of the velocity field of the flow increases and the temperature decreases (see Table 4 and 5).

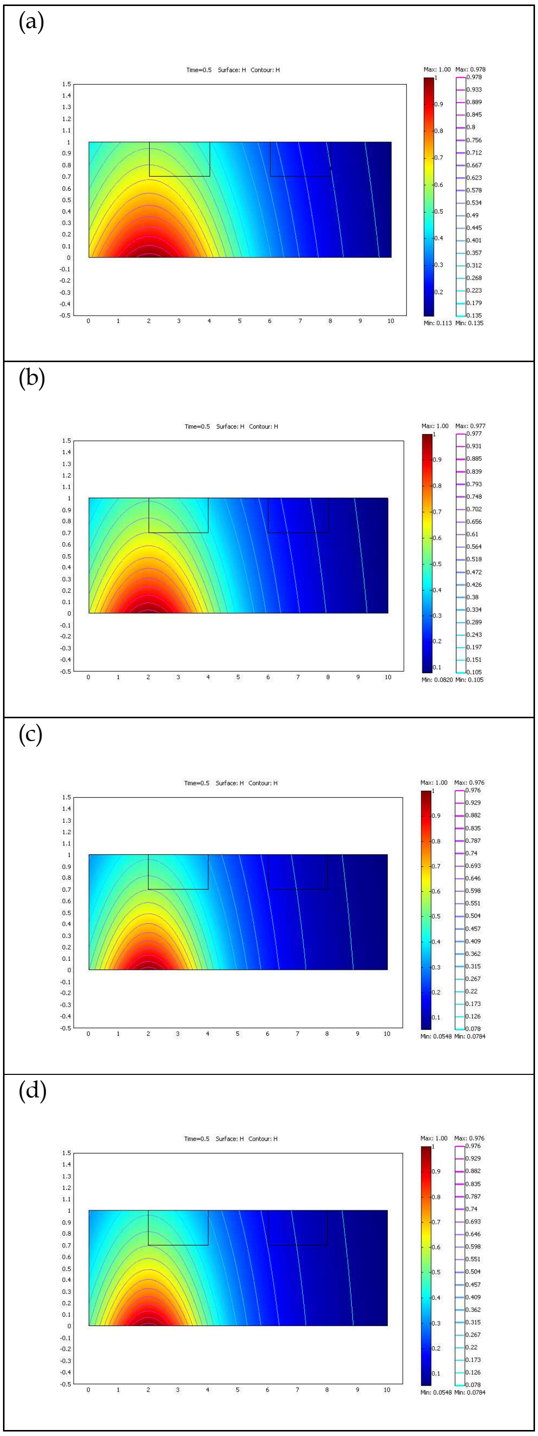

Figure 1. Comparison of intensity of magnetic field in channel: (a) flow A, (b) flow B, (c) flow C and (d) flow D for time t=0.5.

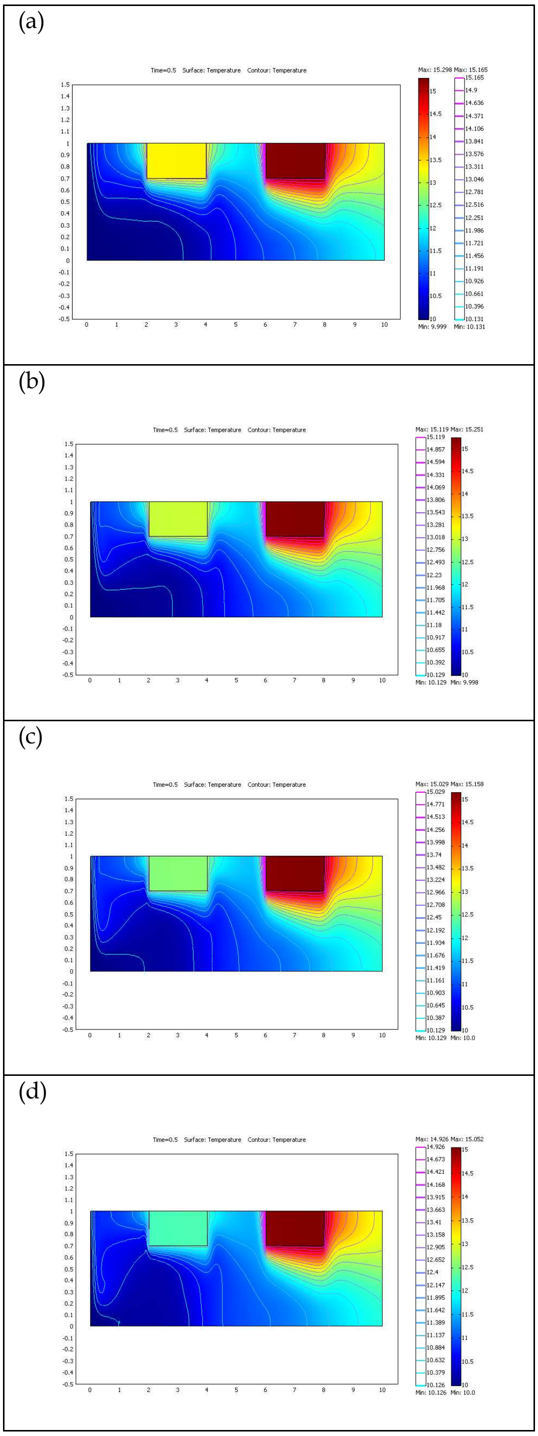

Figure 2. Comparison of temperature in channel and heat generating devices for the different flows in channel: (a) flow A, (b) flow B, (c) flow C and (d) flow D for time t=0.5.

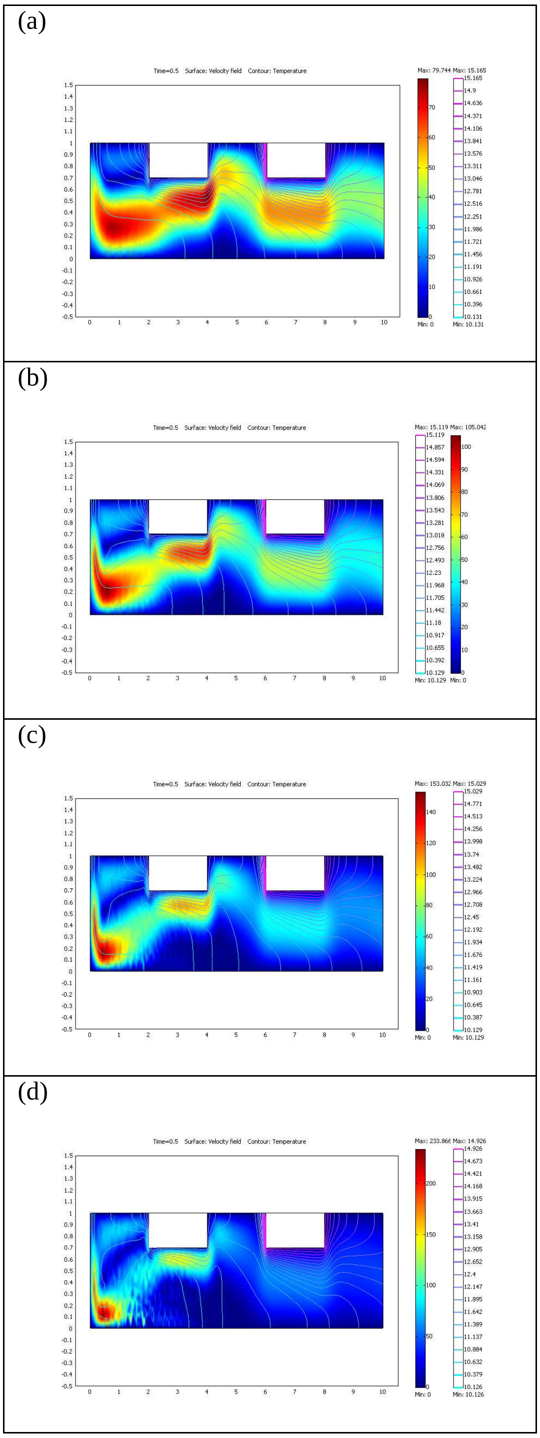

Figure 3. Comparison of velocity field in ferrofluid for the different flows in channel: (a) flow A, (b) flow B, (c) flow C and (d) flow D for time t=0.5.

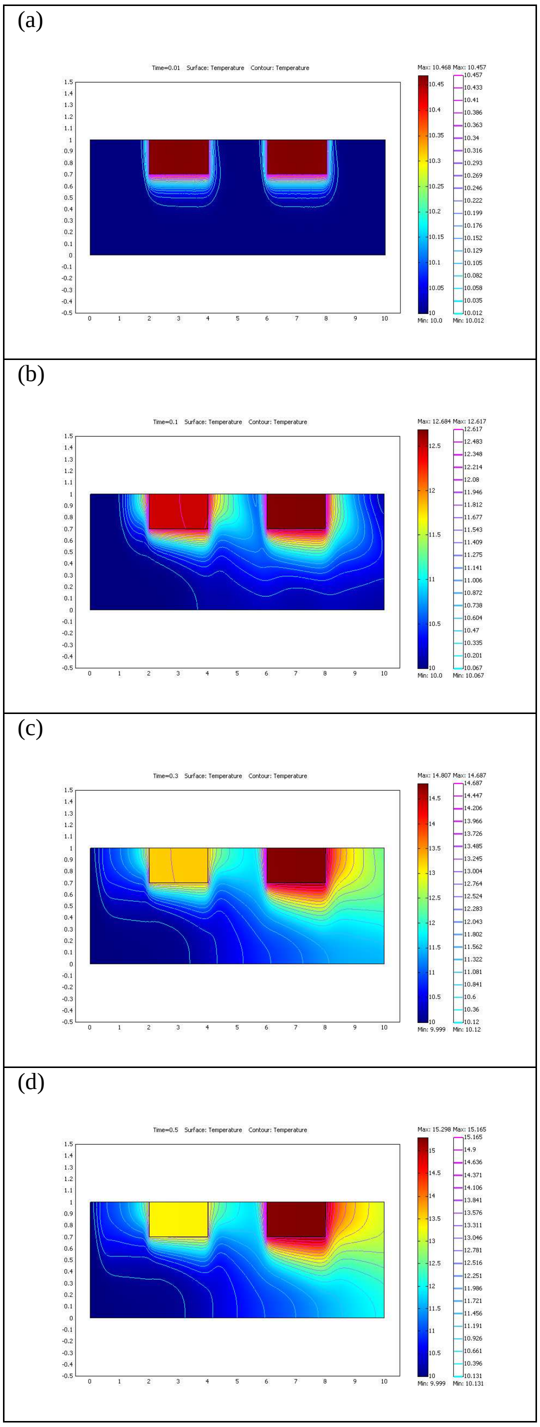

Figure 4. Temperature evolution in channel in flow A for time: (a) 0.01, (b) 0.1, (c) 0.3 (d) 0.5.

Table 6. Quantities of flow E and F

Quantity	Flow E	Flow F
a	4	4
b	-3	-2.5
H_r	1.768388e+5	2.546479e+5
Pr	560	
Ra	200.7938	
Ec	1.16324e-13	
Re	0.001786	
Mn	2.272182e+9	4.711597e+9
Qn	64.019097	
V_{avg} (fluid)	300.949913	331.078973
T_{avg} (fluid)	101.964051	102.076129
T_{avg} (all)	119.429217	119.129908

Table 7. Quantities of flow G and H

Quantity	Flow G	Flow H
a	4	4
b	-2	-1.5
H_r	3.978874e5	7.073553e+5
Pr	560	
Ra	200.7938	
Ec	1.16324e-13	
Re	0.001786	
Mn	1.150292e+10	3.635492e+10
Qn	64.019097	
V_{avg} (fluid)	375.238349	445.972581
T_{avg} (fluid)	102.080147	101.443362
T_{avg} (all)	118.70548	117.611141

In the flows E, F, G and H magnetic dipole is placed below the channel with ferrofluid on the line perpendicular to the bottom wall of channel at distance 4 from left wall. Again we observed that, due to the value of the characteristic value of magnetic field, the maximum value of: the magnitude of the velocity field of the flow increases and the temperature decreases (see Table 6 and 7).

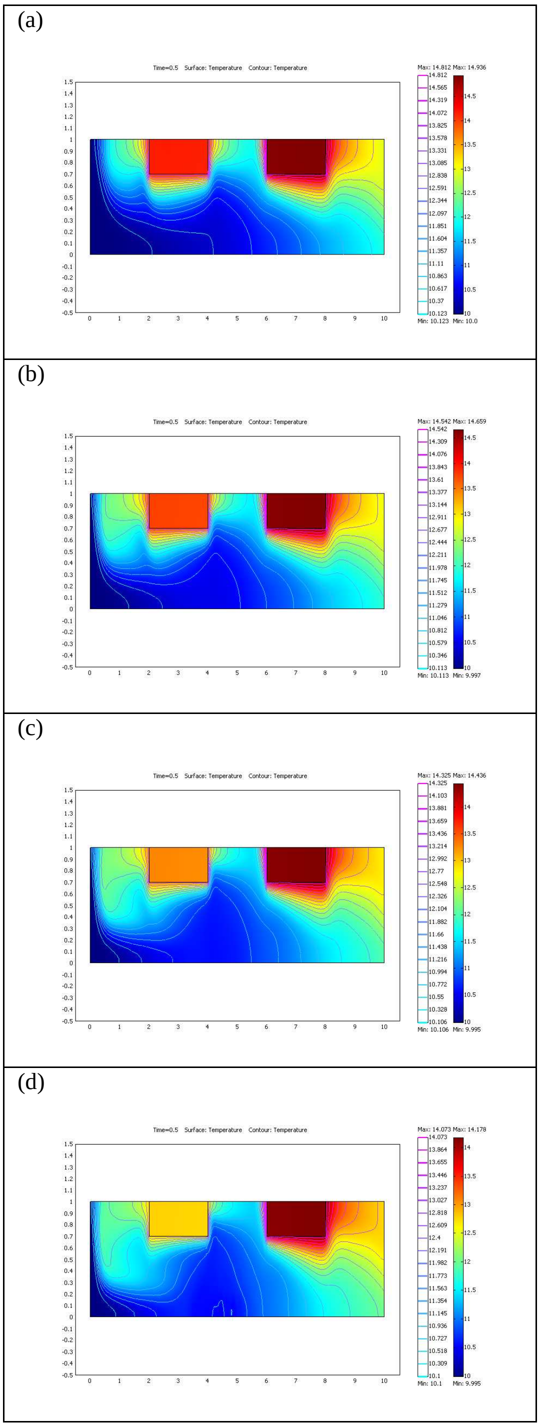

Figure 5. Comparison of temperature in channel and heat generating devices for the different flows in channel: (a) flow E, (b) flow F, (c) flow G and (d) flow H for time t=0.5.

4. Conclusion

We have simulated two-dimensional heat transfer in ferrofluid channel flow under the influence of the magnetic field created by magnetic dipole using computational fluid dynamics code COMSOL based on finite element method. At the left end of rectangular channel there was assumed a parabolic laminar flow profile. The upper plate was kept at constant temperature T_u and the lower at T_l .

The flow was relatively uninfluenced by the magnetic field until its strength was large enough for the Kelvin body force to overcome the viscous force. The magnetoconvection was induced by the presence of magnetic field gradient.

We observed that the cooler ferrofluid flows in the direction of the magnetic field gradient and displaced hotter ferrofluid. Ferrofluids have promising potential for heat transfer applications because a ferrofluid flow can be controlled by using an external magnetic field [6-7].

The Kelvin body force arises from the interaction between the local magnetic field within the ferrofluid and the molecular magnetic moments characterized by the magnetization. An imposed thermal gradient produces a spatial variation in the magnetization through the temperature-dependent magnetic susceptibility for ferrofluids and therefore renders the Kelvin body force non-uniform spatially. This thermal gradient induced inhomogeneous magnetic body force can promote or inhibit convection in a manner similar to the gravitational body force.

A strong magnet placed near the device which produces heat will always attract colder ferrofluid towards it more than warmer ferrofluid thus forcing the heated ferrofluid away, towards the heat sink. This is an efficient cooling method which requires no additional energy input.

5. References

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