A Mean Field Approach to Many-particle Effects in Dielectrophoresis

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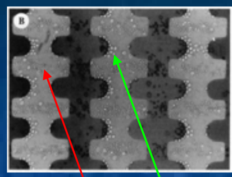
INTRODUCTION

In recent years dielectrophoresis (DEP) has emerged as an important technique for the manipulation of micro- and nano-sized particles suspended on a liquid medium [1,2]. Highly non-uniform electric field at a length scale comparable to cell size can be generated easily at low voltages. Since the relative dielectric responses (DEP spectrum) of the cells are dependent on the driving frequency of the applied electric field, an alternating electric field is usually applied to generate dielectrophoretic forces of different magnitudes and directions. Therefore, DEP devices may be easily employed for separating different cell types by simply modifying field frequency. In order to generate a spatially non-uniform electric field, essential ingredient for DEP separation, an array of metal electrodes is embedded inside a micro-channel network. Many physical parameters can affect dielectrophoresis and we refer the reader to the past literature [1-4] for a better introduction. Nevertheless, there is a strong evidence that many-particles effects, coming from high concentration of cells in the surroundings of electrodes, can be an important source of indetermination for the knowledge of separation (or trapping) efficiency. In the past decades different numerical approaches based on solving directly the equations of motion for a system of N-particles have been used to account for many-particles effects in dielectric suspensions [4]. Due to the limited number of particles (i.e. N ~ 100) considered, these techniques are no feasible in view of the simulation of real systems. In this letter we suggest a method to include many-particles effects in the calculation of DEP trapping by mean of the effective medium approximation (EMA) for electric parameters of the suspension [5], where the local value of the volume fraction of dispersed particles is ruled by a drift-diffusion dynamics. We will demonstrate the reliability of the method and the importance of the many-particle corrections discussing a simulation example in a realistic DEP device geometry.



Separation of cancer cells from blood ones

THE MODEL

Forces acting on a particle suspended in a fluid [2,3]:

$$\vec{F}_{drag} = -f\vec{u}_{DEP} \qquad f = \frac{8\pi\mu r_1}{2\ln\left(\frac{2r_1}{r_2}\right)+1}$$

Stokes formula

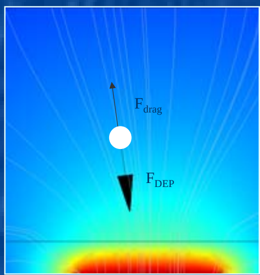
$$\vec{F}_{DEP} = \pi r_1 r_2^2 \varepsilon_m \,\mathrm{Re}[f_{CM}(\omega)]\nabla|\vec{E}|^2$$

Steady motion $\Rightarrow$

$$\vec{F}_{DEP} = -\vec{F}_{drag}$$

$$\vec{u}_{DEP} = \frac{r_2^2\varepsilon_m}{8\mu}\left(2\ln\left(\frac{2r_1}{r_2}\right)+1\right)\mathrm{Re}[f_{CM}(\omega)]\nabla|\vec{E}|^2$$

~5.0E-20 DEP mobility



Velocity induced by DEP force

Hydrodynamic part (first governing equation)[6]:

$$\rho(\vec{u}\cdot\nabla)\vec{u} = -\nabla p + \eta\nabla^2\vec{u} + \vec{F}$$
$$\nabla\cdot\vec{u} = 0$$

Navier-Stokes equation

Steady and uncompressible

Electric part:((second governing equation)[3]:

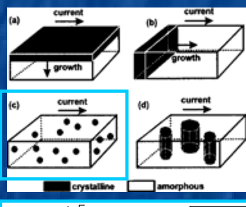
$$-\nabla\cdot\varepsilon\nabla V = 0$$
$$\vec{E} = -\nabla V$$

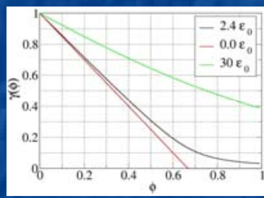
Laplace equation

$$\vec{u}_{tot} = \vec{u} + \vec{u}_{pDEP} + \vec{u}_{nDEP} \quad \Rightarrow \quad \begin{cases} y' = u_y^{tot} \\ x' = u_x^{tot} \end{cases}$$

Particle tracing

Effective medium approximation (EMA) [5].





$$\varepsilon(\phi) = \frac{1}{4}\left[2\varepsilon_p - \varepsilon_p' + \sqrt{(2\varepsilon_p - \varepsilon_p')^2 + 8\varepsilon_a\varepsilon_c}\right]$$
$$\varepsilon_p = (1-\phi)\varepsilon_a + \phi\varepsilon_c$$
$$\varepsilon_p' = \phi\varepsilon_a + (1-\phi)\varepsilon_c$$

$$\gamma(\phi) = \frac{\varepsilon}{\varepsilon_a}$$

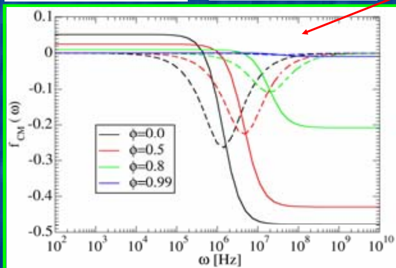
How does EMA affect DEP spectra? [7] (New!!!)

$$f_{CM}(\omega) = \frac{\tilde{\varepsilon}^{(p)} - \tilde{\varepsilon}^{(m)}}{\tilde{\varepsilon}^{(p)} + 2\tilde{\varepsilon}^{(m)}}$$

With:

$$\tilde{\varepsilon}^{(j)} = \varepsilon_r^{(j)}\varepsilon_0 - i\sigma^{(j)}/\omega$$

$$\varepsilon^{(m)} \to \varepsilon(\phi)$$
$$\sigma^{(m)} \to \sigma(\phi)$$



DEP spectrum of latex micro-spheres obtained from data given in [4]. Straight and dashed lines refer respectively to the real and imaginary parts of the Clausius-Mossotti factor. Colors refer to different values of particle volume fraction (see legend). The effect of EMA in the DEP spectrum is a gradually flattening to zero of both real and imaginary part of Clausius-Mossotti factor as the volume fraction approaches one. For dielectrophoresis induced by electrical field gradients only the real part of the CM-factor is relevant and we deal with it through out this poster.

Drift-diffusion part (third governing equation)[7]: (New!!!!)

$$\frac{\partial\phi}{\partial t} = -\vec{\nabla}\bullet\vec{J}$$

where

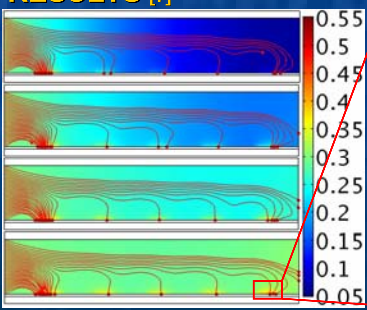
$$\vec{J} = \underbrace{-D(\phi)\nabla\phi}_{\text{Diffusion term}} + \underbrace{\vec{u}_{tot}\phi}_{\text{Drift term}}$$

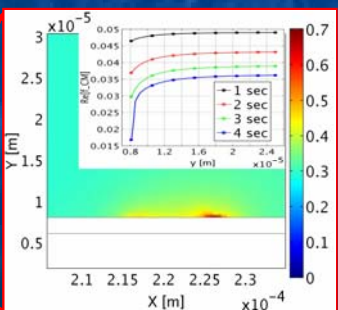
$$D(\phi) = \frac{9.6\cdot 10^{-9}}{\sqrt{1-\phi/0.74}}$$

Diffusion enhanced by particle-particle collision

Packing fraction

RESULTS [7]





Left panel: Snapshots taken at times 1, 2, 3, 4 seconds (from upper to lower) regarding the time evolution of particle volume fraction ϕ (color field) and particle trajectories (red lines). The color bar on the right gives the correspondence between colors and values of ϕ. Particles and fluid enter from the left (initial fluid velocity of 15 micron) where at the boundary ϕ is fixed to 0.3, in the other boundaries we employ the condition of external vanishing flux. Electrodes (in number of 10) are separated from the fluid by a silicon layer 3 micron thick and they are taken at a voltage of 0 or +5 Volt in a alternate sequence (see [3]). **Right panel:** An enlarged portion of snapshot at T=4 sec. The color field represents the particle volume fraction ϕ at the surroundings of an electrode where it reaches its maximum value. Abscissa and ordinate are spatial coordinates. In the inner panel the cross-section, taken just above the red region at X=2.26 micron, of the Clausius-Mossotti factor as a function of the ordinate is shown, colours of the curves refer to the same times of the snapshots (see legend).

CONCLUSIONS

The simulation results here discussed show the importance of the inclusion of the many particle corrections for a reliable prediction of the trapping efficiency in devices aimed at the manipulation of micro- and nano-sized particles. The formalism should be applied in all the case where the diluted limit locally fails. This situation is rather common (not only for devices working in trapping configuration) since high particle density can be achieved in device geometry where tight regions are built to suitably tailor the electric field. The combination of EMA and drift-diffusion opens the possibility to study carefully how many-particles effects can influence the features dielectrophoresis based devices. The formalism here presented represents only a first step toward a complete description of the possible complex particle kinetics in these devices and advances are still needed. Indeed, the diffusion formalism can be easily generalized to a reaction-diffusion one in order to considered particle's stitching and clustering. Work in this direction is in progress.

REFERENCES

[1] M.P. *Electrophoresis* **23**, 2569 (2002).
[2] T.B. Jones, "Electromechanics of particles", Cambridge University Press, Cambridge 1995.
[3] F. Aldaeus, Y. Lin, J. Roeraade, G. Amberg, *Electrophoresis* **26**, 4252 (2005).
[4] A.T.J. Kadaksham, P. Singh, N. Aubry, *Electrophoresis* **25**, 3625 (2004).
[5] R. Landauer, *J. Appl. Phys*. **23**, 779 (1952); A.N. Norris, P. Sheng, A.J. Callegari, *J. Appl. Phys*. **57**, 1990 (1985).
[6] L.D. Landau and E.M. Lifshitz, "Fluid Mechanics", Butterworth-Heinemann, Oxford 1987.
[7] O.E. Nicotra, A. La Magna, S. Coffa *Appl. Phys. Lett.* (in press)

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