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Localization of Chemical Sources Using Stochastic Differential Equations in Realistic Environments

Author A. Mohammed[1], A. Jeremic[*,1]
[1]McMaster University
*Corresponding author: McMaster University, 1280 Main Street West, Hamilton, L8S4K1, Canada

Introduction

Signal processing algorithms for chemical sensing/monitoring have been subject of considerable research interest in the recent years mainly due to their diverse applicability. These algorithms consist mainly of two different parts: detection, in which presence of particular chemical agent is detected, and estimation, in which the chemical agents are localized and/or their dispersion is predicted.

In most cases detection is mainly focused on the development and design of chemical sensors with low sensitivity thresholds. On the other hand, estimation techniques are mainly based on diffusion/convection laws governed by Fick's law of diffusion. However in certain cases, especially when the concentration of chemical agent is small, the dispersion of particles is governed by stochastic differential equations describing more complex motion mechanisms such as Brownian motion.

In this paper we propose the computational framework for solving estimation problems using stochastic differential equations and finite-element analysis. In our previous work [1] we demonstrated the advantage of this approach in an infinite homogeneous environment and illustrated the advantages of the stochastic approach when the source intensity is small. In this paper we derive the corresponding estimators in the case of realistic geometry illustrated in Figure 1.

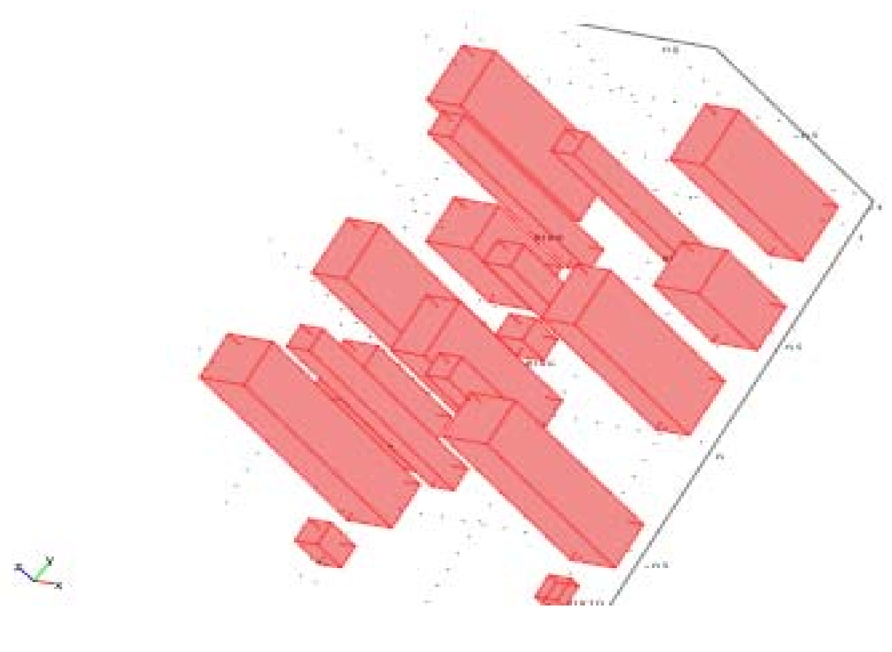

Figure 1: Arbitrary example of realistic geometry

Use of COMSOL Multiphysics

The SDE process for the transport of particle in an open environment is given by

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)W_t$$

where X_t is the location and W_t is a standard Wiener process. The function $\mu(X_t, t)$ is referred to as the drift coefficient while $\sigma(X_t, t)$ is called the diffusion coefficient such that in a small time interval of length dt the stochastic process X_t changes its value by an amount that is normally distributed with expectation $\mu(X_t, t)dt$ and variance $\sigma^2(X_t, t)W_t$ and is independent of the past behavior of the process.

It can be shown that the above SDE can be reformulated as an ordinary partial differential equation i.e., Fokker-Plank equation whose solution then represents the spatio-temporal probability density function of one arbitrary particle. The solution for n particles can then be obtained by obtaining the convolution of n spatio-temporal distributions. In this paper we derive computational framework for SDE equations based on solution of Fokker-Plank equation in realistic geometry illustrated in Figure 1. We solve the Fokker-Plank PDE using COMSOL Multiphysics and corresponding general PDE mode and convolve the solution using MATLAB. We then localize the chemical source of strength 1000 particles using non-linear least square optimization.

Conclusion

In this paper we illustrate the low-intensity chemical source estimation in realistic, arbitrarily shaped environments. We demonstrate that the location of the source can be estimated more accurately by accounting for the stochastic nature of dispersion which is unaccounted for in classically used techniques based on Fick's law. Although computationally intensive (modeling of 1000 particles required execution on a Blade server with 16 GB of RAM) the proposed technique is a promising tool for applications which do not require execution in real-time.

[1] A. Mohammed and A. Jeremic, "Localization of Chemical Sources Using Stochastic Differential Equations," ICASSP 2008, Las Vegas, April 2008.