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Acoustics Session

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Improved Perfectly Matched Layers for Acoustic Radiation and Scattering Problems

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Overview

- The Sommerfeld Radiation Condition and **P**erfectly **M**atched **L**ayers (**PML**'s).
- Problems caused by the steep decay of evanescent waves at low frequencies.
- Improved PML formulation:
  (i) Improved accuracy in the presence of evanescent waves at low frequencies.
  (ii) Stability of the mesh with respect to frequency.
- Real-life application: loudspeaker design.
- Conclusions and further development.

The Sommerfeld Radiation Condition

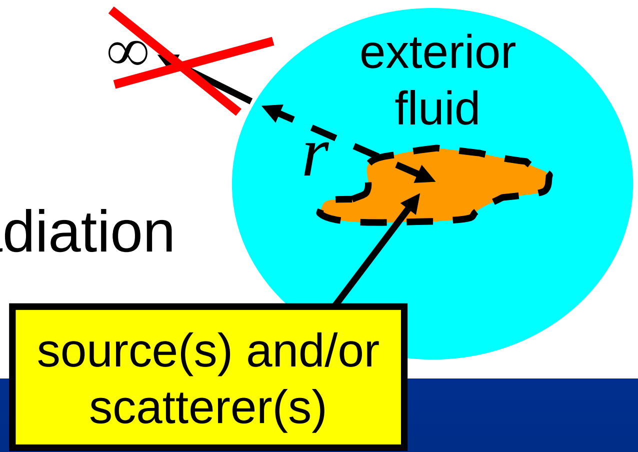
- In an unbounded medium [exp(+i ω t), k= ω /c] :

$$\nabla^2 p + k^2 p = 0$$

Helmholtz equation describing the acoustic pressure.

$$\frac{\partial p}{\partial r} + ik\,p = \mathrm{o}\left(\frac{1}{r}\right), \qquad r \to \infty$$

Sommerfeld radiation condition



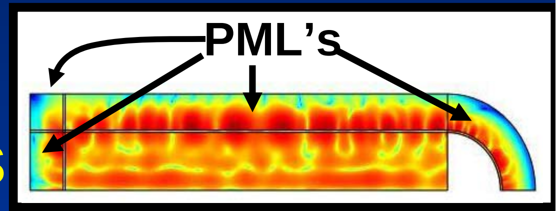
- In a computer model, r must be finite.

The Sommerfeld condition must be approximated numerically.

Perfectly Matched Layer (PML)

- An (efficient!) technique for approximating the Sommerfeld BC.

  J.-P. Bérenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *Journal of Computational Physics,* Vol. 114, pp. 185—200 (1994).

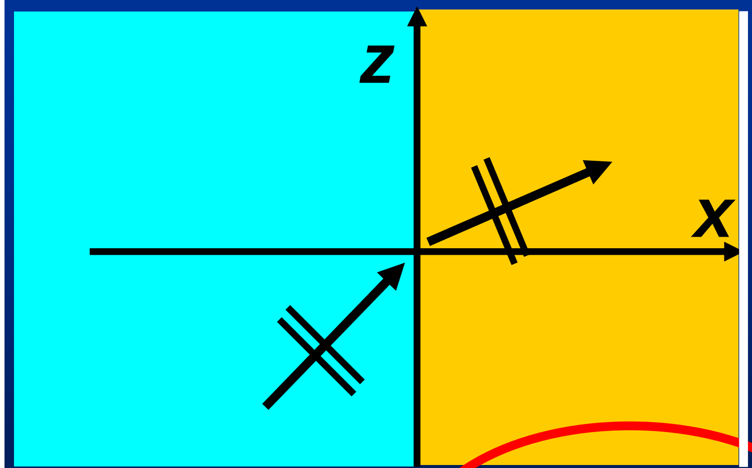


- Can be easily adapted to *CONVEX* geometries
- Straightforward implementation via complex coordinate scaling.

  F. Collino, P. Monk, “The perfectly matched layer in curvilinear coordinates,” *SIAM J. Sci. Comput.,* Vol. 19(6), pp. 2061 – 2090 (1998).

  F. Ihlenburg, Finite Element Analysis of Acoustic Scattering, Springer-Verlag (1998).

  M. Zampolli, A. Tesei, F.B. Jensen, N. Malm, J.B. Blottman, J.Acoust.Soc.Am. 122, 1472 – 1485 (2007).



Fluid Domain (physical) | PML (non-physical)

Reflections are negligible.

Perturbed Helmholtz Equation (PML):

$$\frac{i\omega}{i\omega+\sigma(x)}\frac{\partial}{\partial x}\left(\frac{i\omega}{i\omega+\sigma(x)}\cdot\frac{\partial p}{\partial x}\right)+\frac{\partial^2 p}{\partial z^2}+k^2 p=0$$

$$\frac{\partial^2 p}{\partial \tilde{x}^2}+\frac{\partial^2 p}{\partial z^2}+k^2 p=0$$

Complex coordinate
→ Damping only in the x-direction.

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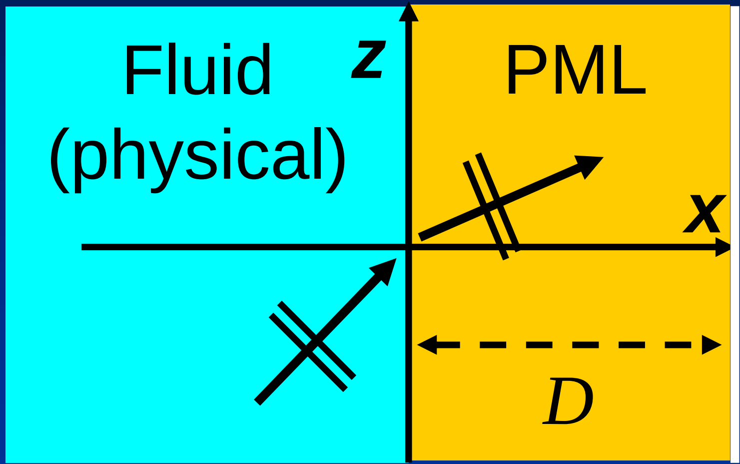
PML Formulation in the Current Implementation



PDE in the PML: $$\frac{\partial^2 p}{\partial \tilde{x}^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = 0$$

PML scaled coordinate: $$\tilde{x} = (1 - i)\lambda (x / D)^n$$

- Normalizing the scaled coordinate w.r.t. the wavelength λ
  → no need to adjust the mesh density in the PML
  in *x*-direction as the frequency varies → Mesh stability.

- The ***scaled coordinate*** is a ***polynomial***, with equal power n in the ***real part*** and in the ***imaginary part***.

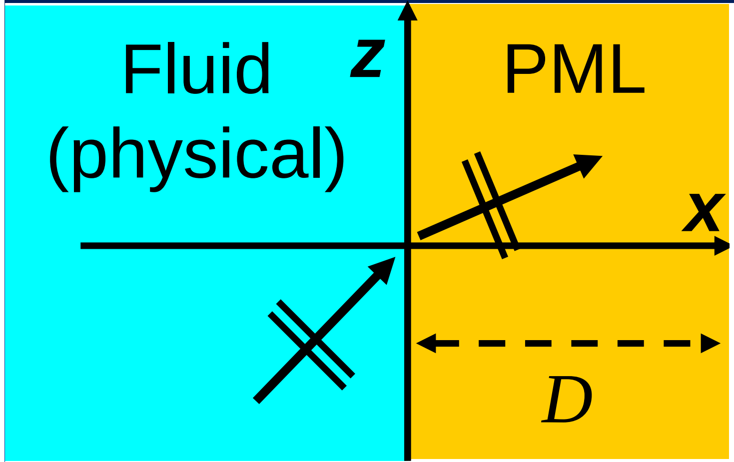
- The ***real*** and the ***imaginary part*** of the ***scaled coordinate*** each have different effects, depending on whether the ***incident wave*** is ***propagating or evanescent.***

Effect of the Real and Imaginary Parts of the Scaling

Wave Type	PML scaled coordinate: Real Part	PML scaled coordinate: Imaginary Part
Propagating	• Resolution of the oscillatory components in the PML, no damping.	• ***Damping*** in the PML. • Rapidly growing ***imaginary part*** → good damping.
Evanescent	• ***Damping (Decay)*** of the evanescent wave in the PML.	• Spurious anti-causal waves, which are absorbed by the PML.

Problems at ***very low frequencies***, where the evanescent field decays steeply: the PML ***"over-damps"*** an already steeply decaying wave → ***PML accuracy problems***.

Improved PML Scaling



$$\tilde{x} = A\lambda\left((x/D)^{n_r} + i\log_2\left(1-(x/D)^{n_i}\right)\right)$$

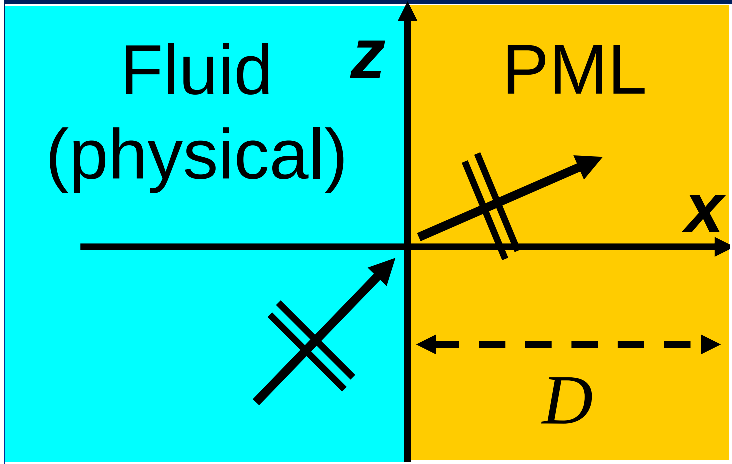
parameter ~0.25
sensitivity
→ convergence

$$n_i = \begin{cases} 1-\log_{10}\dfrac{ka}{10}, & ka<10 \\ 1, & ka\geq 10 \end{cases}$$

- At ***high frequencies*** $ka>10$: rapidly growing $\log(1-x/D)$ scaling of the imaginary part → ***good damping of propagating waves***.

- ***At low frequencies***: $\log(1-(x/D)^{n_i})$ with exponent of n_i >1 in the imaginary part
  → imaginary coordinate grows more slowly near the interface with the physical domain,
  → ***element size compressed near the interface***,
  → ***good resolution*** of the ***phase*** of the **decaying wave**.

Improved PML Scaling



$$\tilde{x} = A\lambda \left((x/D)^{n_\mathrm{r}} + i \log_2 \left(1 - (x/D)^{n_\mathrm{i}} \right) \right)$$

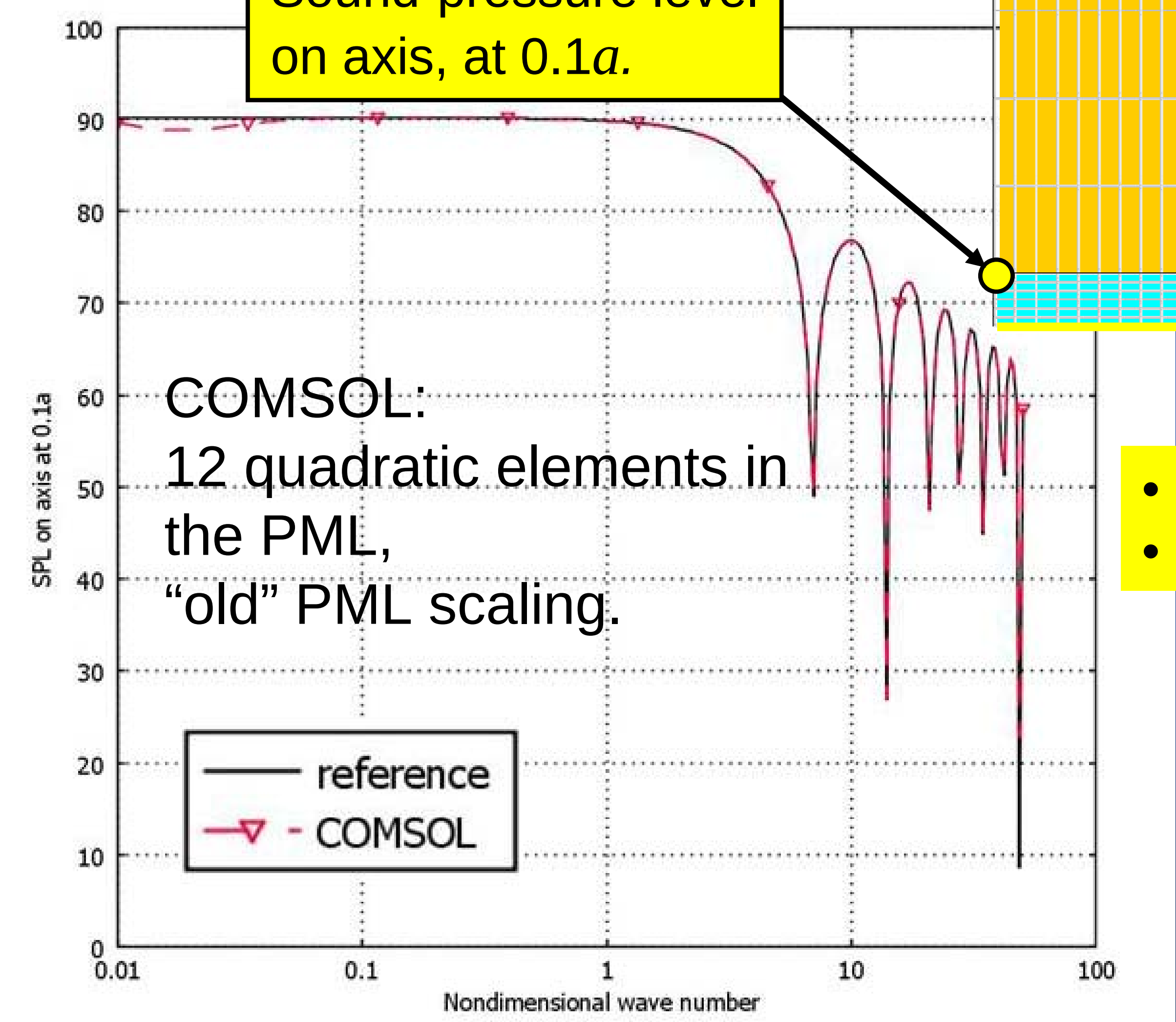
$$n_\mathrm{r} = \begin{cases} 1 - \log_{10} ka, & ka < 1 \\ 1, & ka \geq 1 \end{cases}$$

- At ***low frequencies*** : real part grows slowly near the boundary
  → element size compressed near the interface,
  → rapidly decaying evanescent waves are better resolved
  by the PML near the interface with the physical domain,

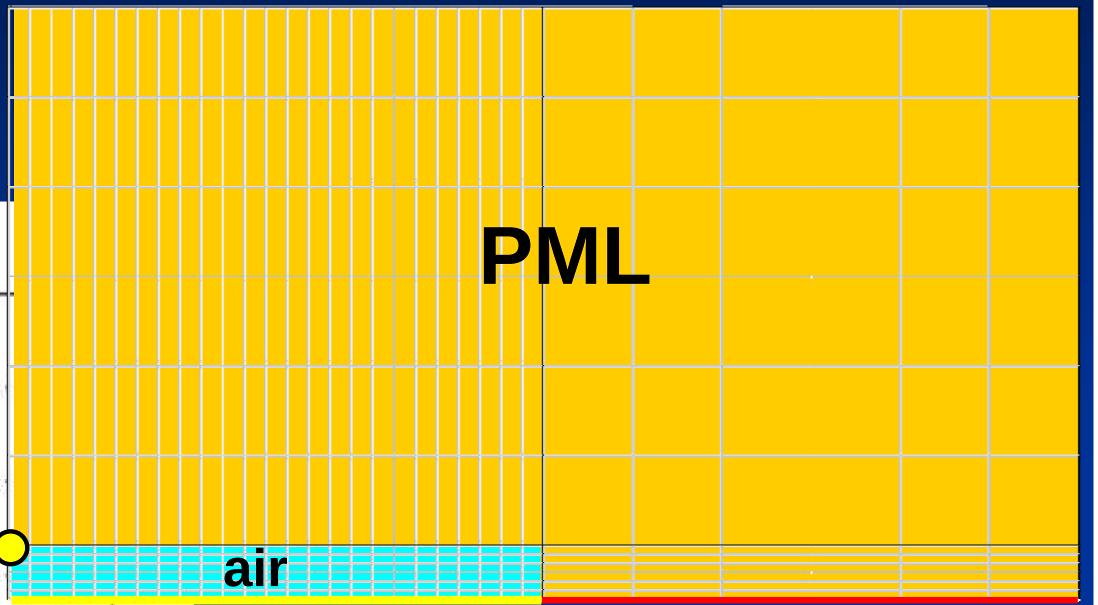
  → ***improved accuracy***.

Example:
Circular Piston Radiation

Sound-pressure level on axis, at 0.1*a*.



COMSOL:
12 quadratic elements in the PML,
“old” PML scaling.

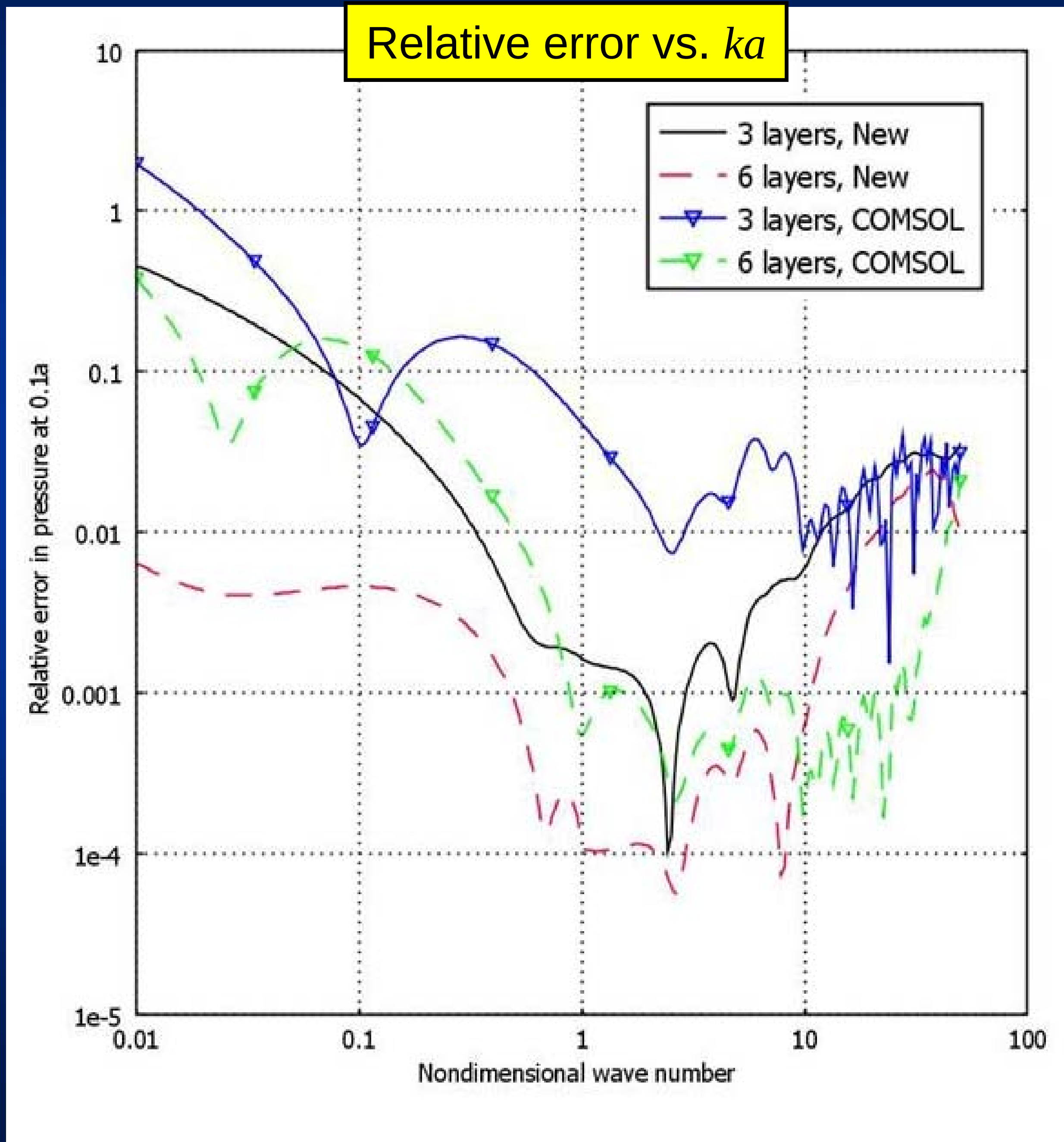


rigid baffle

- prescribed constant normal velocity.
- Piston radius = *a*.

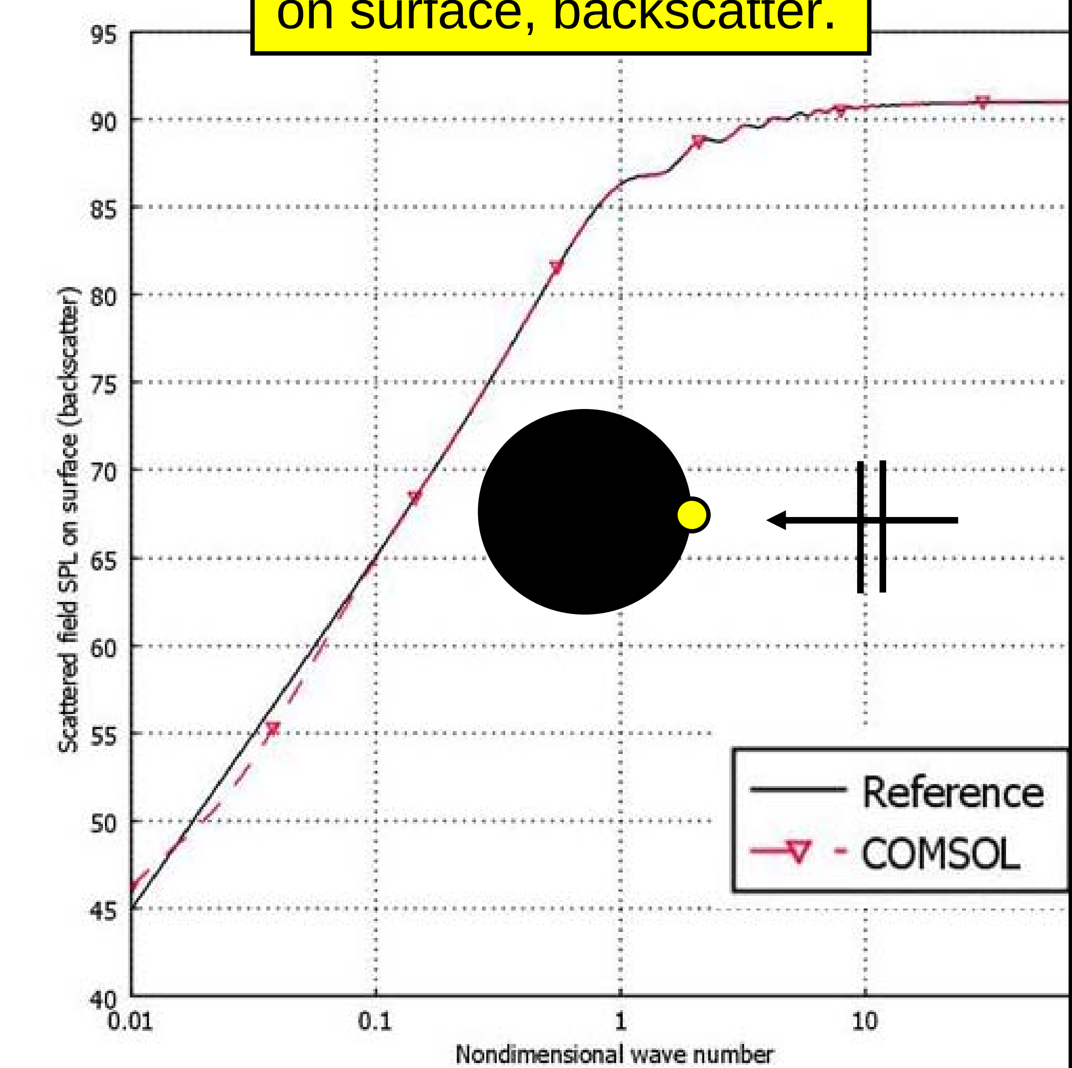
Errors at low frequency in the solution with the “old” PML scaling.

Comparison: "old" vs Improved Scaling

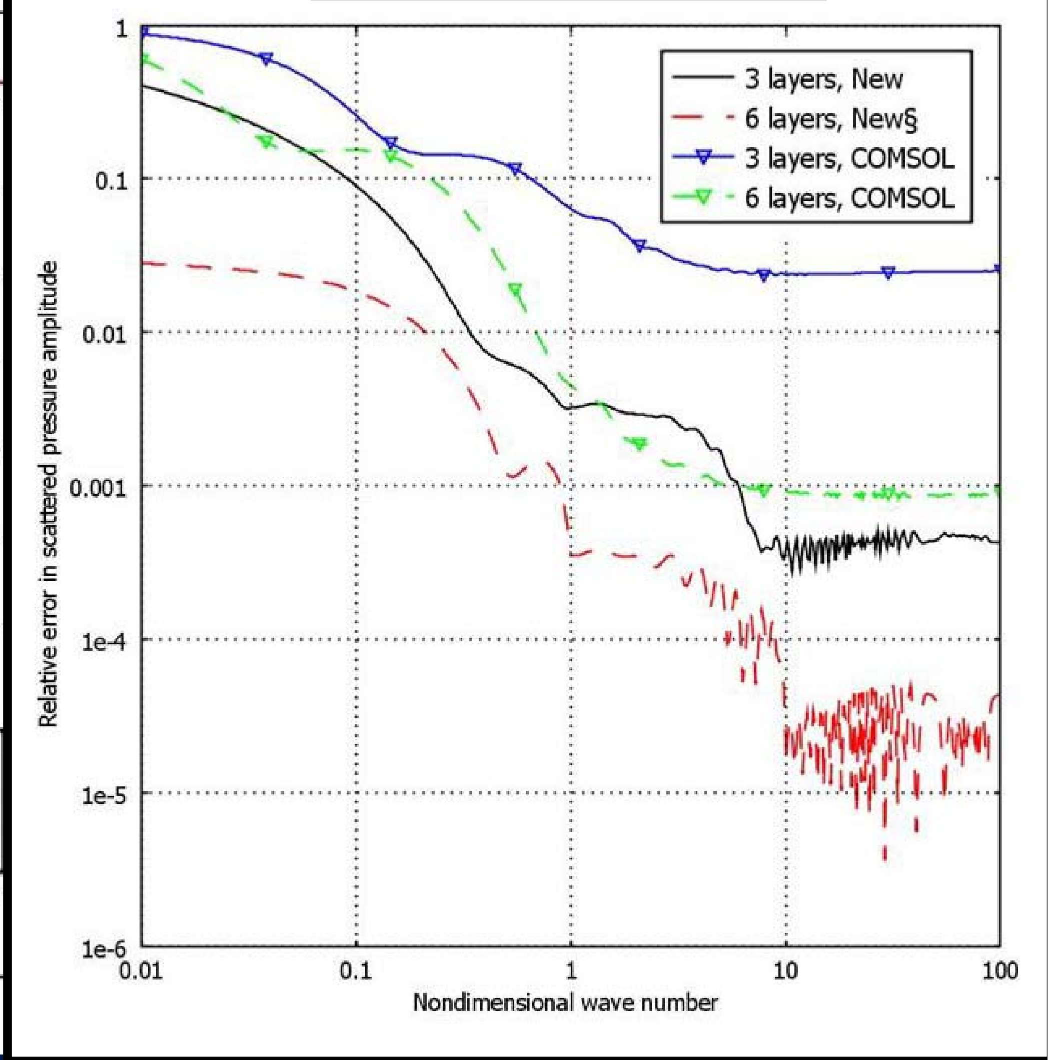


Plane-wave Scattering from a Hard Sphere

Scattered pressure level on surface, backscatter.



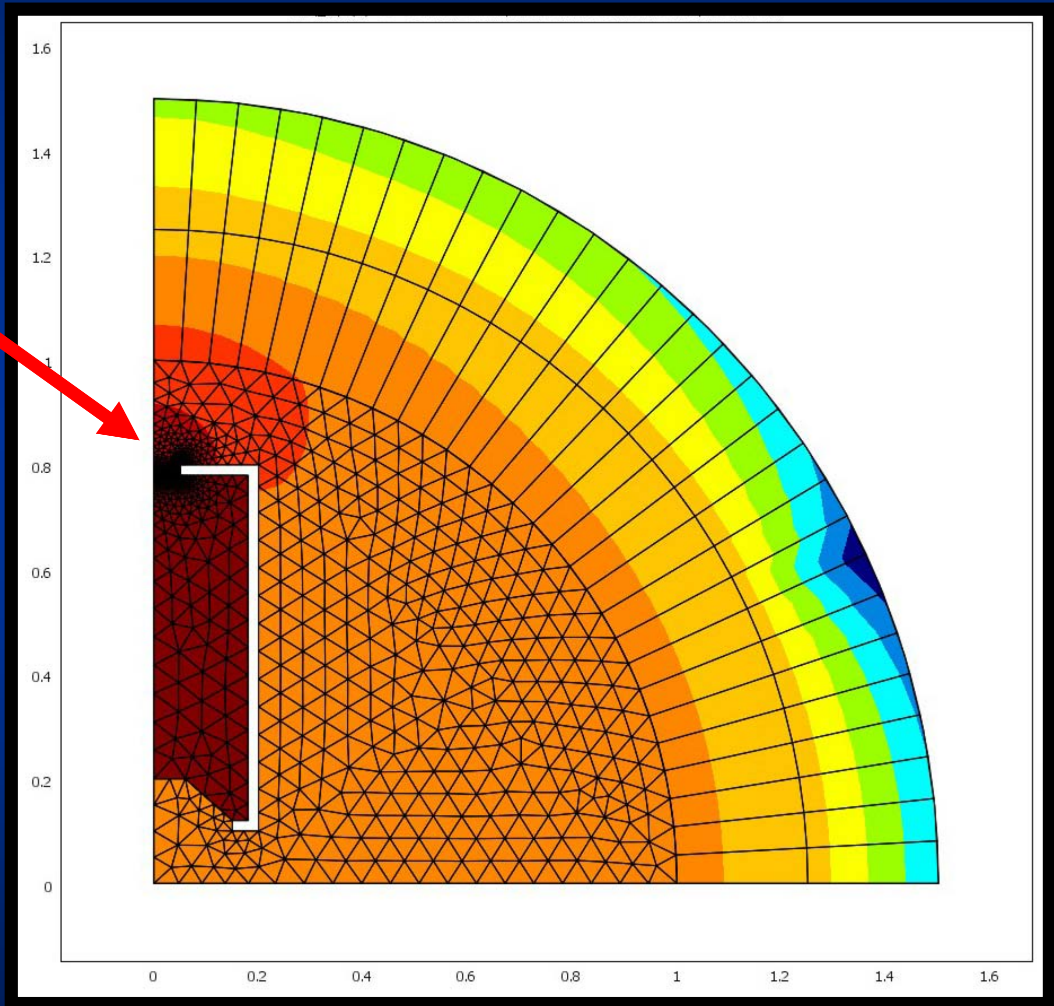
Relative error vs. *ka*



Performance of the Modified PML

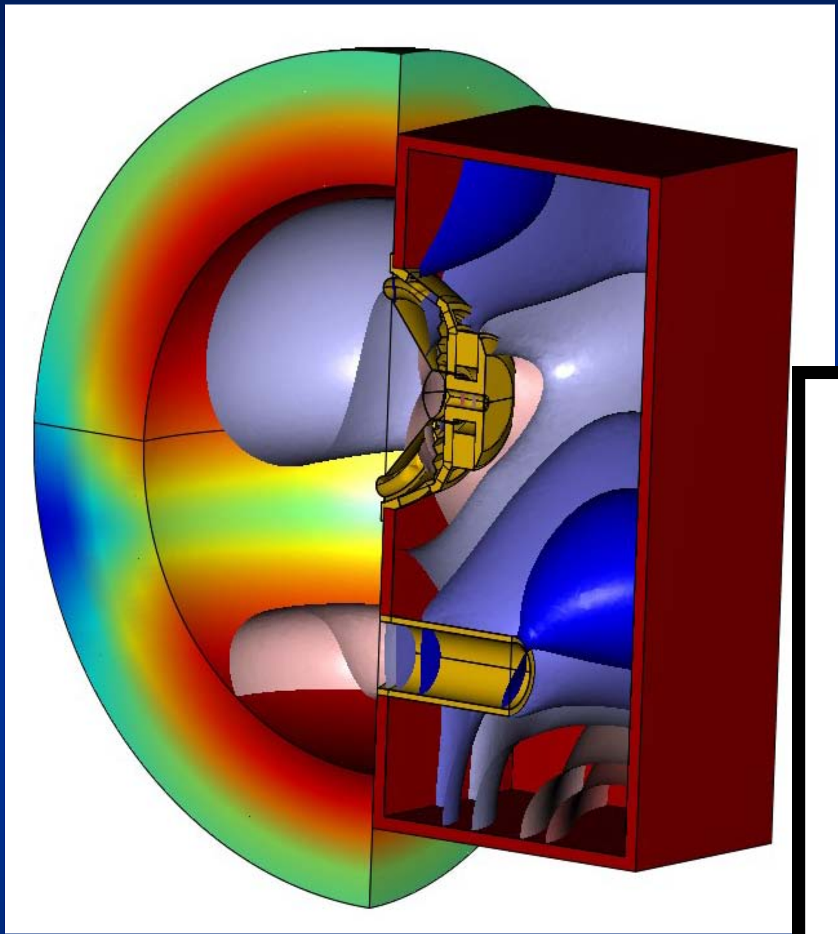
Cylindrical Sub-woofer

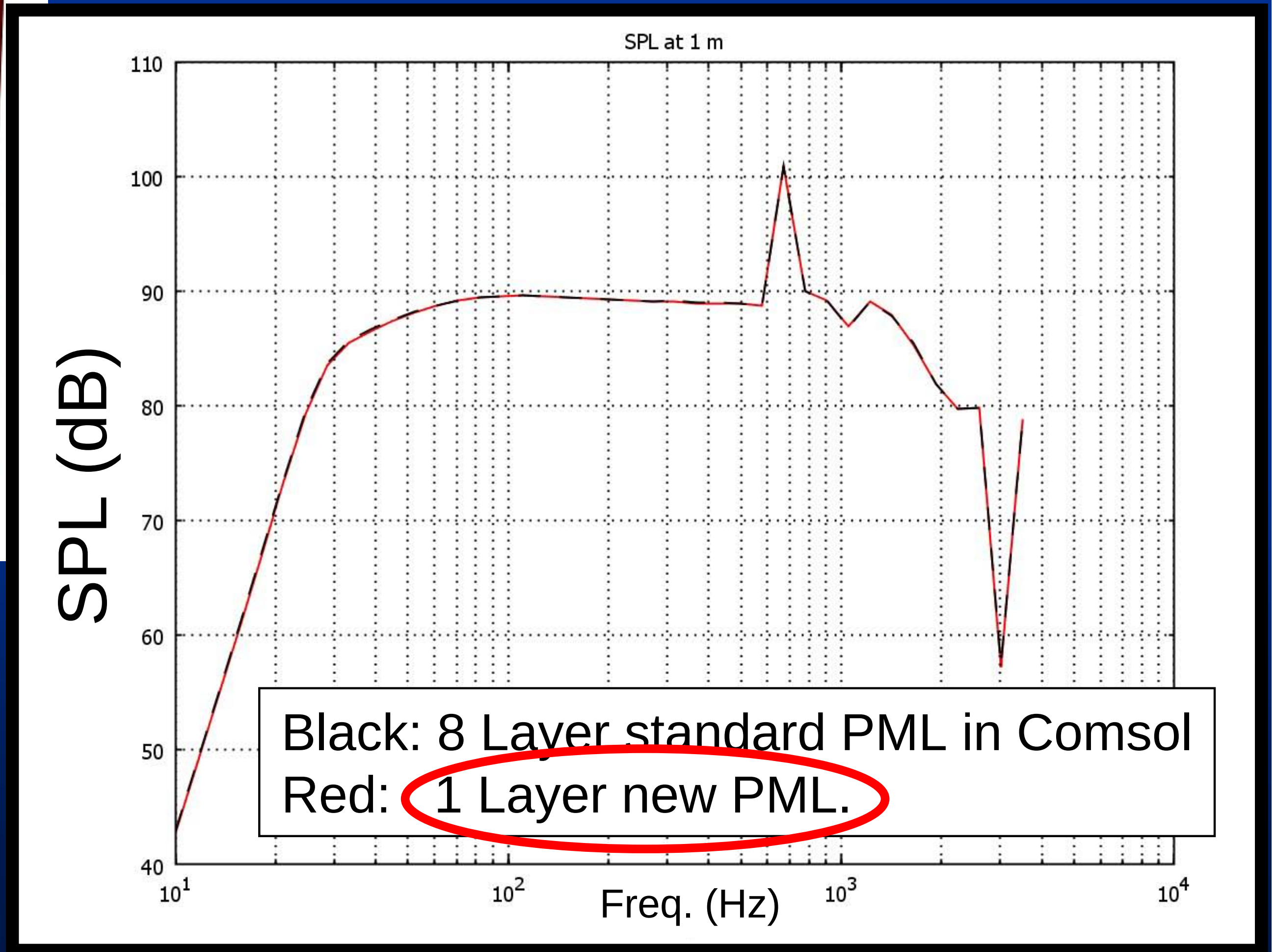
Helmholtz resonator
→ Evanescent field at the opening.



Accurate solution with 4-layer modified PML, compared to 16-layer standard PML.

Audio Loudspeaker Design





Conclusions and Further Development

- The real and imaginary parts of the PML coordinate scaling affect evanescent and propagating wave components in different ways.

- Modified scaling strategy proposed:
  (i) improves the performance at low frequencies,
  where evanescent waves dominate.
  (ii) mesh stability with respect to frequency,
  from ***ka*** = 1/100 to ***ka*** = 100.

- Increasing error at high frequencies observed in radiation problems → To be addressed.

- What is *a* in *ka*? → Problem dependence?
  → A measure of the smallest
  wave scales of the problem?