Dynamic Simulation of Electromagnets

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Abstract: The main design goals for electromagnetic actuators are force, switching speed, packaging, and energy efficiency. In the development of magnetic valves for electronic brake systems at Continental Automotive Systems, these criteria are assessed using COMSOL Multiphysics. From the point of view of modelling, the dynamic behaviour is most interesting. For instance, the response to a voltage step involves the calculation of the time-dependent magnetic field, self-inductance, eddy currents, and the movement of magnetic parts. We achieve this by coupling the equations of motion with the electromagnetic equations by means of a moving mesh. Finally, we show how to approximate the model by a small system of ordinary differential equations.

Keywords: ALE mesh deformation, electromagnetism, equation of motion, magnetic induction, reduced dynamic model

1. Main results

In a typical valve design, mesh variation indicates that the numerical error in the calculation of electromagnetic forces is less than 1 %. The movement of the magnetic armature is described by a mesh deformation with the following settings. The armature is surrounded by a rigid nonmagnetic cylindrical hull. This is surrounded by a second cylindrical hull. Between these two cylinders, the mesh deformation is prescribed explicitly. For the time-dependent solver algorithm, a rather fine tolerance is recommended.

The response of an electromagnet to a voltage step is calculated in terms of armature movement, solenoid current and eddy current powers. The influence of eddy currents on a voltage step response is small. Calculation time decreases when the solenoid is replaced with a circular line current source. However, this also leads to a significant change in the result.

When eddy currents are neglected, the dynamic system behaviour is determined by the stationary behaviour. The latter can be described by look-up tables. This approach leads to an efficient reduced dynamic model. The effect of eddy currents is re-introduced by a linear model extension.

2. Model description

The valves used in hydraulic anti-lock brake systems are actuated by electromagnets, which usually are axially symmetric. The magnetic armature is surrounded by brake fluid. The magnetic circuit consists of a core, a yoke around the solenoid, a radial gap filled with non-magnetic steel, the armature which moves in axial direction, and an axial gap filled with break fluid. For the purpose of this paper, the simplified geometry shown in Figure 1 was used. The total calculation volume is a sufficiently large ball centred at the origin. We choose a radius of 5 cm. As solenoid parameters, we assume a winding number of w = 500 and a resistance of R = 5 Ω.

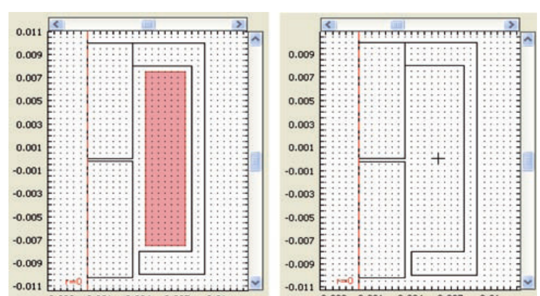

Figure 1. Typical designs of magnetic circuits

In the left-hand part of Figure 1, the section area of the solenoid is shaded in red. In the right-hand part, a point current source, which represents a circular current in the centre of the solenoid, is used instead. This reduces the number of mesh elements in the calculation. However, we will see that it also changes the calculation results to some extent.

3. Maxwell's equation

The problem is axially symmetric with currents in the angular direction only. COMSOL describes such a problem by Maxwell's equation

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \mathrm{rot}\left(\frac{1}{\mu_0\, \mu_r} \,\mathrm{rot}\, \mathbf{A} \right) - \sigma\, \mathbf{v} \times \mathrm{rot}\, \mathbf{A} = \mathbf{J}_{\mathrm{ext}},$$

where **A** is the magnetic vector potential, and the magnetic flux density is given by $\mathbf{B} = \mathrm{rot}\, \mathbf{A}$. When we write this equation in cylindrical coordinates, the symmetry assumptions imply that both the external current density $\mathbf{J}_{\mathrm{ext}}$ and the vector potential **A** have only an angular component. Moreover, the derivative with respect to the angle vanishes. Rewriting the rotation operator in cylindrical coordinates, we obtain Maxwell's equation for the angular component A_φ of the vector potential **A**. This contains the term A_φ / r. It can be shown that this term approaches ½ B_z as r tends to 0. In particular, we may define a new dependent variable $u = A_\varphi / r$. Writing $\nabla = (\partial_r, \partial_z)$, we obtain the form

$$\sigma\, r \frac{\partial u}{\partial t} - \nabla \cdot \left(\frac{r}{\mu_0\, \mu_r} \nabla u \right) - \frac{\partial}{\partial r}\left(\frac{2}{\mu_0\, \mu_r} u \right) + \sigma\left(r\, \mathbf{v} \cdot \nabla u + 2 v_r u \right) = J_\varphi$$

of Maxwell's equation which is used by COMSOL.

4. Magnetization

The magnetic flux density **B** is related to the magnetic field **H** by the equation $\mathbf{B} = \mu_0\, \mu_r\, \mathbf{H}$. The relative permeability μ_r is very close to 1 for non-ferromagnetic materials, but it depends on the magnetization in a ferromagnetic material. Therefore, the problem is non-linear. We use the relation between **B** and **H** which is described by Figure 2. Thus we simplify the situation by neglecting magnetic hysteresis and by assuming that μ_r is a scalar.

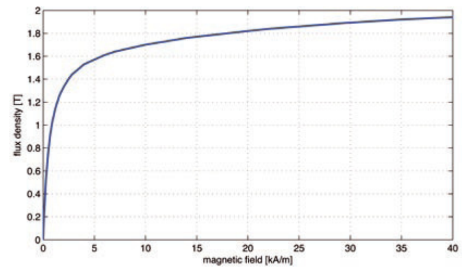

Figure 2: magnetisation of machining steel

In the present form of Maxwell's equation, we must use this **B**-**H**-curve in order to express μ_r (or, since the introduction of COMSOL version 3.4, the norm of **H**) as a function of **B**. The table of values of this function is saved in a file and used by a COMSOL interpolation function `mur`. We choose piecewise cubic interpolation with constant extrapolation. In the COMSOL subdomain settings tab, we enter `mur(normB_emqa)` in the field for the isotropic relative permeability. Finally, we must avoid the singularity in the derivative of this expression at 0 by providing a non-constant initial value or by redefining `normB_emqa` as

```
sqrt(eps + abs(Br_emqa)^2
          + abs(Bz_emqa)^2).
```

5. Convergence of force

For the comparison of the different mesh deformation techniques which COMSOL provides, we would like to have a reference value for the magnetic force. Therefore, we study how the calculated force converges over regular mesh refinement steps. The working gap is fixed to 0.25 mm, and the excitation is $I\ w$ = 500 A. We start with different pre-defined mesh sizes. Since the two non-magnetic gaps in the ferromagnetic circuit are particularly interesting, we also set the "resolution of narrow regions" mesh parameter. One of the initial meshes is mixed, with a mapped mesh in and around the armature.

Figure 3 contains the forces which we calculate in this way. The three (indistinguishable) horizontal lines at 19.88 N indicate the result of the COMSOL adaptive solver (refinement methods: longest edge / regular / remesh). The forces show a spread which decreases over mesh refinement steps. The uncertainty of the force calculation with a sufficiently fine mesh is below 1 %. The result obtained with a mixed triangular-quadrilateral mesh is more stable. However, this advantage seems to be too small to justify the extra modelling effort in general.

When we compare the calculations for a volume current source and for a line current source (Figure 4), we find that the spread of the results seems to be somewhat larger for a line current source. This is probably due to the fact that the line current source is a singularity of the magnetic field, which cannot be properly resolved by the mesh. However, it is much more important to observe that the force value changes by more than 10 %. In a quantitative investigation, it is therefore important to keep the current in its proper position, even if this position is irrelevant in a simplified description of the electromagnet as a circuit of magnetic resistances.

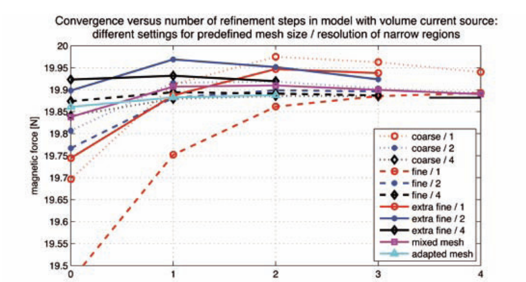

Figure 3

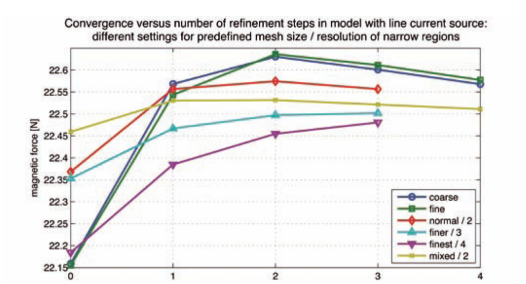

Figure 4

6. Prescribed armature movement via mesh deformation

This section contains the main part of this paper, the comparison of the COMSOL mesh deformation techniques. We study these techniques for a prescribed axial displacement of the magnetic armature. Thus we are considering a parameterized geometry. However, the corresponding COMSOL application mode is not available for axially symmetric two-dimensional geometries, so that we have to use the more general ALE application mode instead. In order to avoid the inversion of mesh elements, we introduce additional edges around the armature. Thus we create a rectangular net, which is shown in the following figure.

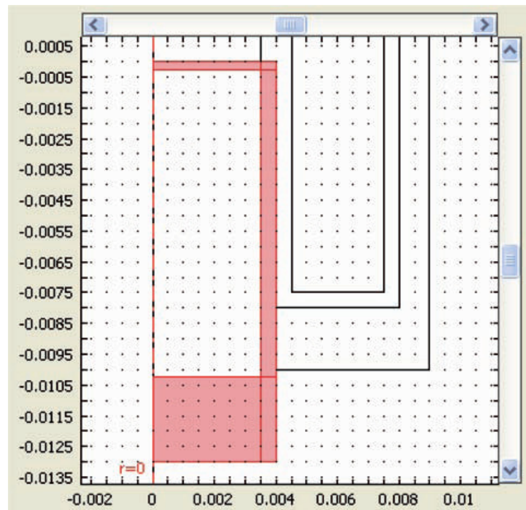

Figure 5: geometry modification for mesh deformation

The displacement in the shaded area around the armature has to be determined, the displacement of the armature is constant, and the displacement of the remaining geometry is zero. The displacement around the armature is prescribed on the six edges which are deformed. Let s be the edge parameter, which runs from 0 to 1, and let δ be the armature displacement. The displacement along the edge is prescribed as $\delta f(s)$ or as $\delta f(1-s)$, depending on the direction of the edge parameterization. Here f is a continuous function from the real line to the unit interval [0, 1] which is 0 on]- ∞, 0[, is 1 on]1, ∞[, and is a polynomial on [0, 1]. Specifically, we use the following definitions for the polynomial section of f, i.e. for $0 \le s \le 1$:

$$f_0(s) = s$$

$$f_1(s) = 3s^2 - 2s^3$$

$$f_2(s) = 10s^3 - 15s^4 + 6s^5$$

The graphs of these functions are shown in Figure 6. Thus the function f_0 is continuous, the function f_1 is continuously differentiable, and the function f_2 is twice continuously differentiable. One of the interesting questions is whether a higher degree of smoothness is important for the calculation.

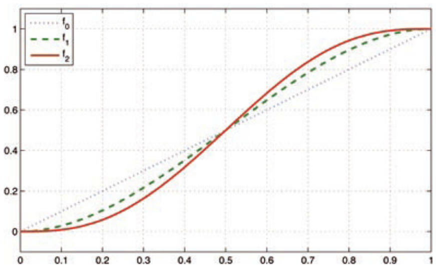

Figure 6: functions for mesh deformation

Simplicity is not the only advantage of f_0 over the smoother alternatives. Indeed, on an edge of length d which is compressed, we have to avoid self-overlap (i.e. mesh inversion). The condition for this is that the absolute value of the derivative of δf is less than d. Now the functions f_k attain their maximal derivatives at ½, and the values are

$$f_0'(\tfrac{1}{2}) = 1,\ f_1'(\tfrac{1}{2}) = \tfrac{3}{2}, \text{ and } f_2'(\tfrac{1}{2}) = \tfrac{15}{8}.$$

This means that the maximal displacement which can be described with f_0 is d, whereas the maximal displacement for f_1 is only ⅔ d, and the maximal displacement $8/15\, d$ for f_2 is even smaller.

The results of the various calculations are summarized in Table 1 at the end of this section. As in the convergence study, we used a line current source of 500 ampere turns and a gap of 0.25 mm. This gap is obtained by deforming the initial (drawn) gap of 0.4 mm. The solution times were obtained with COMSOL 3.2 on an AMD Opteron 280 processor with 2.4 GHz. Every version of the calculation was run both on a triangular mesh and on a mixed mesh. The results indicate that the accuracy of the result is already very good when the displacement is described by f_0, and is increased slightly when we use f_1 or even f_2 instead. This holds true for both the Laplace and the Winslow smoothing technique provided by COMSOL. (In Laplace smoothing, the space coordinates satisfy a Laplace equation as functions of the reference coordinates. In Winslow smoothing, the opposite relation holds.) The results for Laplace smoothing and for Winslow smoothing are exactly the same. However, Winslow smoothing sometimes leads to warnings about inverted mesh elements (red table entries) or requires to mark the problem as "strongly non-linear" in the solver parameters setting (blue table entry).

The geometry was tiled by rectangles in order to avoid mesh inversion. The resulting geometry is particularly simple, which allows us to prescribe the displacement in the deformed mesh areas explicitly. Thus we save calculation time. Moreover, a prescribed deformation may also be smoother than a deformation which is calculated numerically. Let us describe the deformation on a rectangle $[R_1, R_2] \times [Z_1, Z_2]$ in the R-Z-plane of reference coordinates. Say that the lower and the left-hand edges are to be fixed, and the upper right-hand corner is to be displaced by δ in the z-direction. Then the r-displacement vanishes on the rectangle, and the z-displacement at (R, Z) can be prescribed by the formula

$$\delta\, f\left(\frac{R-R_1}{R_2-R_1}\right) f\left(\frac{Z-Z_1}{Z_2-Z_1}\right).$$

On a rectangle where the displacement is 0 on one edge and δ on the opposite edge, one may drop the second or the third factor from this formula. In the linear case, i.e. if $f = f_0$, this formula describes in fact the solution to the Laplace equation with linear boundary conditions. Thus Laplace smoothing and prescribed displacement coincide in this case.

The spread between the results for the different deformation techniques is in the order of magnitude of the numerical error if we assume that the latter is indicated by the spread of results for different meshes. Although this is already very satisfactory, it can still be improved by drawing a narrow non-magnetic area around the armature and moving this area rigidly with the armature. It turns out that the force calculated in such a model is perfectly independent of the choice of a mesh deformation technique.

Therefore, the recommendation which results from this investigation is to use a rigid hull around the moving armature and a prescribed liner / bilinear mesh deformation.

without armature air hull

	model P_05	C0	C1	C2
triangular mesh	Laplace	22.440 N 8 s	22.472 N 8 s	22.480 N 8 s
	Winslow	22.440 N 10 s	22.474 N 9 s	22.483 N 11 s
	prescribed	22.440 N 8 s	22.595 N 9 s	22.584 N 9 s

	model P_07	C0	C1	C2
mixed mesh	Laplace	22.405 N 9 s	22.529 N 11 s	22.546 N 11 s
	Winslow	22.404 N 12 s	22.529 N 11 s	22.545 N 12 s
	prescribed	22.405 N 10 s	22.676 N 11 s	22.684 N 13 s

with armature air hull

	model P_06	C0	C1	C2
triangular mesh	Laplace	22.538 N 17 s	22.538 N 16 s	22.538 N 19 s
	Winslow	22.538 N 23 s	22.538 N 27 s	22.538 N 50 s
	prescribed	22.538 N 10 s	22.537 N 11 s	22.537 N 11 s

	model P_08	C0	C1	C2
mixed mesh	Laplace	22.559 N 12 s	22.559 N 12 s	22.558 N 12 s
	Winslow	22.559 N 12 s	22.559 N 16 s	22.558 N 28 s
	prescribed	22.559 N 9 s	22.558 N 10 s	22.557 N 11 s

Table 1: comparison of mesh deformation techniques

7. Current build-up at fixed armature under voltage step

The aim is to solve a time-dependent model with a moving armature. As a last preparatory step, we have to determine suitable tolerance requirements for the time-dependent solver. This is again done in a model with fixed armature. We apply a constant voltage. Since the initial value of the field is zero, a constant voltage represents a voltage step at time 0. Eddy currents are induced in the armature, and the current-build up is delayed by the voltage induced in the coil. We calculate the induced voltage by averaging the electric field over the coil area,

$$U_{\text{ind}} = \frac{w}{S} \iint_S 2\pi \, r \, E_\varphi \, dr \, dz,$$

where w is the winding number, and S is the section area of the coil in a plane which contains the symmetry axis. The solution time depends significantly on the settings for the absolute and relative tolerance of the time-dependent solver. The following table shows the time needed to calculate the solution over an interval of 10 ms for several combinations of tolerance parameters. A good setting is 10^{-5} for both the relative and the absolute tolerance. The solution time increases if the tolerance is either larger or smaller.

log(rtol) \ log(atol)	**-3**	**-4**	**-5**	**-6**	**-7**
-3	96 s	199 s	195 s	276 s	226 s
-4	168 s	219 s	85 s	111 s	244 s
-5	179 s	103 s	80 s	117 s	121 s
-6	179 s	201 s	86 s	124 s	164 s
-7		219 s	97 s	121 s	146 s

Table 2: time-dependent solver tolerance settings

8. Armature movement under voltage step

In this final section, we combine all the modelling techniques discussed so far. This yields a model in which the armature moves under the influence of the magnetic force, and the build-up of the magnetic field and of the coil current is delayed by eddy currents and by the voltage induced in the coil both by the magnetic field change and by the armature movement. To set up such a model in COMSOL, we can take together the elements of the preceding models and add an ordinary differential equation for the armature movement in the global equations dialogue. At time 0, the armature rests at a gap of 0.5 mm. At a gap of 0.1 mm, the armature hits a stop. This stop is modelled by a spring of stiffness $c = 5\times10^6$ N/m and a velocity-proportional damping, in which the damping constant has the a-periodic limit value $k = 2\,(c\,m)^{½}$ (here m is the armature mass). The stop stiffness is a rather arbitrary value, for which the stop elasticity is just visible in the plot of the armature position versus time. This plot is shown in Figure 7 and Figure 8 below, together with the development of the current, the magnetic force, and the powers of the eddy currents in the various parts.

The calculation time for the combined model is much longer than for the constituent models. We use a mesh which is rather fine in the gaps, at the corners, and close to the edges where the eddy currents are built up. This mesh results in 48871 degrees of freedom. For an interval of 15 ms, calculation time is 8755 s[1].

Figure 7 and Figure 8 also show the behaviour of a model in which the conductivity of the steel parts is set to 0, which means that the eddy currents are suppressed. This entails a faster increase in the magnetic field. Hence the current development is initially slowed down, and the current reaches its stationary value earlier. The armature movement is also slightly accelerated. However, the difference between the situation with and without eddy currents is small enough to be neglected for many purposes. (Note that when eddy currents are neglected, we may consider magnetic force and magnetic flux as stationary functions of current and armature position. Using these two functions, the development of current and position can be described by a system of ordinary differential equations. This is worked out in Section 9.) Damping by eddy currents seems to be advantageous for the time-dependent solver algorithm. When eddy currents are switched off, the solution time increases to 20064 s.

Finally, Figure 7 contains the armature movement in a model in which the coil is replaced with a circular line current source. (Here eddy currents are again considered.) The influence on the result is even higher than in the static force calculation. However, solution time is reduced to 1628 s, so that the disturbance of the result may be acceptable under certain circumstances.

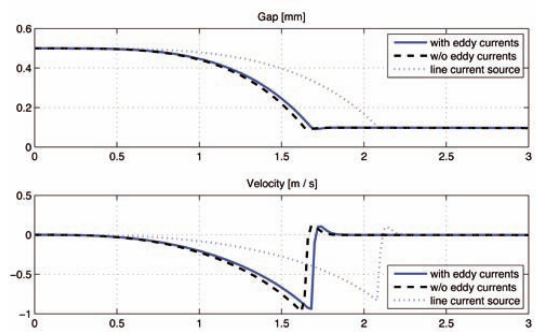

Figure 7: armature movement

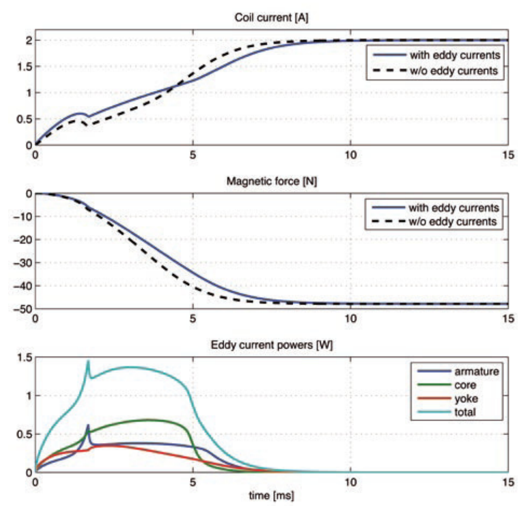

Figure 8: currents and force

9. Extraction of a small dynamic system

The method which has been described up to this point yields a very faithful solution of the Maxwell equation model of an electromagnet. However, the computational workload is considerable. We would like to solve the model for many different external voltage patterns. More generally, we would like to incorporate the model into a larger dynamic system which describes, on the one hand, the parts driven by the electromagnetic actuator and, on the other hand, the electronic driver and a control algorithm. Therefore, we would like to simplify the full

[1] When we introduce an artificial electric conductivity of 10^4 S/m in the air and winding regions, calculation time is reduced by 23 % to 6761 s.

model to a small set of ordinary differential equations. The key step is a preliminary neglect of eddy currents. This will lead to a small and fast model based on look-up tables, which we only need to create once for every design. The look-up tables are based on a stationary COMSOL calculation. In particular, the stationary behaviour of Maxwell's equations is reproduced exactly. Eddy currents are then taken into account by an effective model with parameters which are adapted to a transient COMSOL calculation.

We begin with Maxwell's equation

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \mathrm{rot}\left(\frac{1}{\mu}\,\mathrm{rot}\,\mathbf{A}\right) - \sigma\,\mathbf{v} \times \mathrm{rot}\,\mathbf{A} = \mathbf{J}_{\mathrm{ext}}$$

as given above, but we will now drop the first summand on the left-hand side. This amounts to neglecting the eddy currents induced by the change of the magnetic field. Note that we may keep the third summand, i.e. the eddy currents induced by the armature motion. The resulting equation does not depend on time any more. In the rotationally symmetric cases in which we are now interested, we can express velocity and current density by

$$\mathbf{v} = v\,\mathbf{e}_z \quad \text{and} \quad \mathbf{J}_{\mathrm{ext}} = \frac{\Theta}{S}\,\mathbf{e}_\varphi,$$

where v is the scalar velocity, Θ is the number of ampere turns, and S is the section area of the solenoid in a plane containing the axis. The resulting equation

$$\mathrm{rot}\left(\frac{1}{\mu}\,\mathrm{rot}\,\mathbf{A}\right) - \sigma\,v\,\mathbf{e}_z \times \mathrm{rot}\,\mathbf{A} = \frac{\Theta}{S}\mathbf{e}_\varphi$$

yields $\mathbf{A} = A_\varphi(\Theta, x, v)\,\mathbf{e}_\varphi$, the magnetic vector potential as a function of ampere turns, armature position, and armature velocity.

The number of ampere turns is determined by the external and induced voltages according to

$$\Theta = \frac{w}{R}(U_{\mathrm{ext}} + U_{\mathrm{ind}}),$$

where w is the number of turns and R is the electric resistance of the solenoid. The induced voltage is the average voltage over one turn times the winding number w:

$$U_{\mathrm{ind}} = \frac{w}{S}\iint_S 2\pi\, r\, E_\varphi\, dr\, dz.$$

Using $E_\varphi = -\partial A_\varphi/\partial t$ and exchanging time derivative and integration, we find the relation $U_{\mathrm{ind}} = -\,w\, d\Phi/dt$, where

$$\Phi = \frac{1}{S}\iint_S 2\pi\, r\, A_\varphi\, dr\, dz$$

is the magnetic flux through the solenoid, averaged over the windings. Since the magnetic potential A_φ is determined by Θ, x, and v, so is Φ. For fixed x and v, the flux Φ is a strictly increasing function of Θ. Therefore, we can form the inverse function and consider $\Theta = \Theta_{\mathrm{stat}}(\Phi, x, v)$. Thus we obtain just one ordinary differential equation

$$w\,\Phi'(t) \;=\; U_{\mathrm{ext}}(t) \;-\; \frac{R}{w}\,\Theta_{\mathrm{stat}}(\Phi(t), x(t), v(t))$$

for the magnetic flux $\Phi(t)$ as a function of time. Together with the equation of motion for the armature, this equation is a small dynamic model of the electromagnet.

The calculation of the inverse function can be done directly in COMSOL. One adds a global equation for the scalar variable Θ. This equation implements the integral constraint that the average solenoid flux should equal some prescribed value.

10. Eddy current correction

We finish the reduced dynamic model by introducing an eddy current correction. To this purpose, we keep the main dynamic equation

$$w\,\Phi'(t) \;=\; U_{\mathrm{ext}}(t) \;-\; \frac{R}{w}\,\Theta(t)$$

and extend the relation $\Theta(t) = \Theta_{\mathrm{stat}}(\Phi(t))$ (which we now write without the dependence on position and velocity) by two linear correction terms:

$$\Theta(t) = \Theta_{\mathrm{stat}}(\Phi(t) + \mathbf{a}\cdot\psi(t)) + \mathbf{b}\cdot\psi(t)$$

Here $\mathbf{a}$ and $\mathbf{b}$ are constant vectors, and ψ is a vector-valued function of time. We determine ψ by a linear differential equation

$$\psi'(t) = \mathbf{C}\,\psi(t) + \mathbf{c}\,\Phi'(t)$$

with matrix and vector constants $\mathbf{C}$ and $\mathbf{c}$. Note that the stationary system behaviour is unchanged because the differential equation is excited by the time derivative Φ'. For generic $\mathbf{C}$, we can apply a linear transformation to ψ in order to diagonalize **C.** We can also rescale the components of ψ so that the vectors **a** and **b** consist of zeros and ones in the unit system which we have chosen. Thus the above linear correction is equivalent to the system

$$w\Phi'(t) = U_{\text{ext}}(t) - \frac{R}{w}\Theta(t)$$

$$\Theta(t) = \Theta_{\text{stat}}\left(\Phi(t) + \sum_{j=1}^{m}\varphi_j(t)\right) + \sum_{k=1}^{n}\vartheta_k(t)$$

$$\tau_j^{(1)}\varphi_j'(t) = \tau_j^{(2)}\Phi'(t) - \varphi_j(t)$$

$$\tau_k^{(3)}\vartheta_k'(t) = G_k\Phi'(t) - \vartheta_k(t)$$

with suitable time constants $\tau_\alpha^{(\beta)}$ and electrical conductance constants G_k. (Note, by the way, that a good assumption for the temperature dependence of these constants is proportionality to $(1 + \alpha(T-T_{\text{ref}}))^{-1}$, where $\alpha = 0.004\ \text{K}^{-1}$ is the temperature coefficient for the electric resistance of metals. This amounts to a uniform linear compression of the time scale as electric resistance increases.) The constant model parameters $\tau_\alpha^{(\beta)}$ and G_k are determined by fitting the model to a transient COMSOL calculation. More precisely, we determine the system response (both current and force) in COMSOL for various external voltage patterns, armature positions, winding numbers, and temperatures. We then choose the model parameters such that the response of the reduced system fits the COMSOL result as closely as possible in a least squares sense.

How large should the numbers m and n of correction quantities be chosen? This question has to be studied in concrete industrial examples. In my experience, $m = 0$ and $n = 1$ are sufficient for the response to a single voltage step. For more complicated voltage patterns, I currently use $m = 1$ and $n = 1$.

11. Conclusions

Three calculation scenarios for electromagnets have been presented, namely stationary layout, dynamic finite element calculation including armature motion by mesh deformation, and a model reduction technique. The comparison to experimental evidence, which could not be included here, is very satisfactory. These methods are suitable parts of the development process of electromagnetic actuators.

12. References

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2. Eberhard Kallenbach et al., *Elektromagnete*. Teubner, Wiesbaden (2nd ed. 2003)