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Analytical Solution for the Steady Poroelastic State under Influence of Gravity

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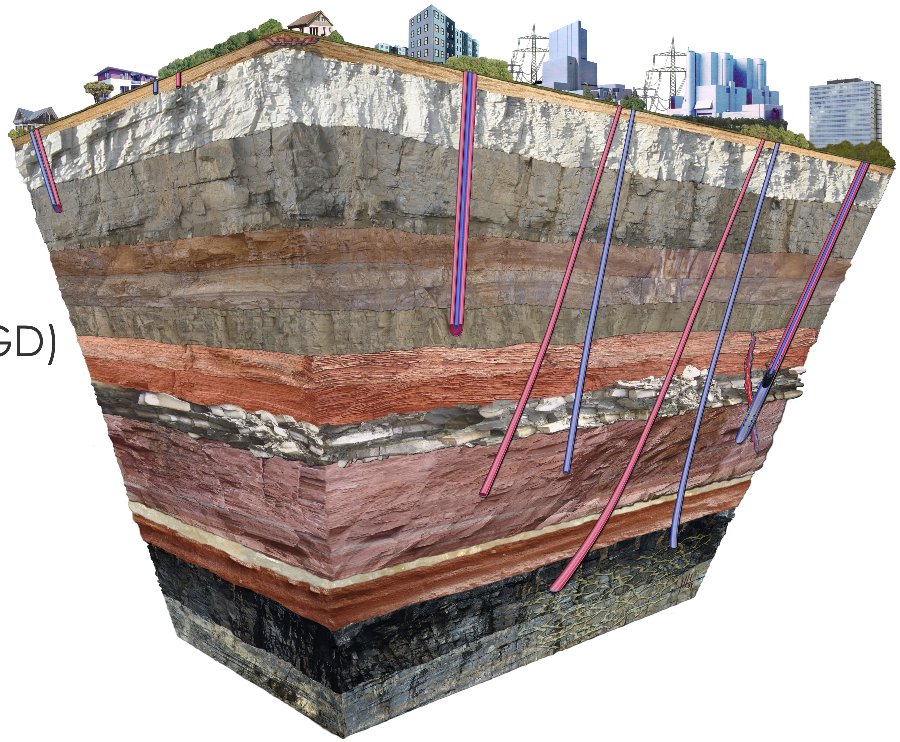
Introduction

Applications of poroelastic models

- Liquid waste injection
- Nuclear waste disposal
- CO2 sequestration
- Hydraulic stimulation
- Steam assisted gravity drainage (SAGD)
- Hydrate deposits
- Energy piles
- Soil liquefaction



Coupled hydraulic-mechanical regime (HM)

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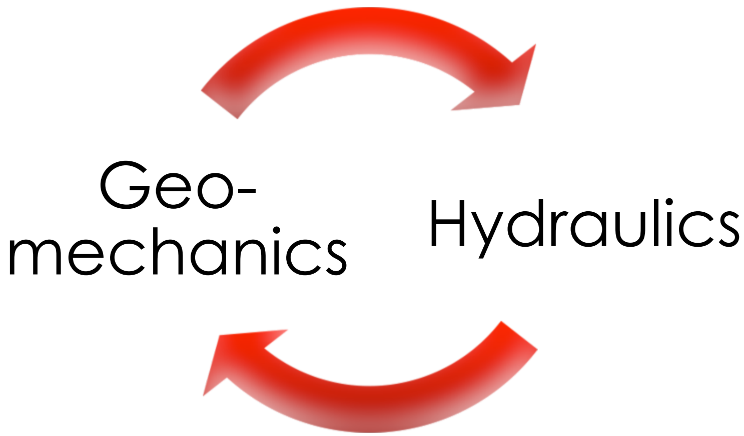
- Changed stress regime effects fluid flow
- Alteration of fluid flow changes stresses and strains



Differential equations

- Hydraulics
  - Darcy's Law $$\mathbf{v} = -\frac{\mathbf{k}}{\mu}\nabla\left(p + \rho_f g z\right)$$
  - Mass Conservation $$\rho_f\left(S\frac{\partial\rho_f}{\partial t} + \alpha\frac{\partial\varepsilon_{vol}}{\partial t}\right) = -\nabla\cdot\left(\rho_f\mathbf{v}\right)$$
- Geomechanics
  - Force Equilibrium $$\nabla\cdot\boldsymbol{\sigma} = \nabla\cdot\boldsymbol{\sigma}_{eff} - \alpha\nabla p = -\rho\mathbf{g}$$
  - Poroelastics $$\frac{E\left(1-v\right)}{\left(1+v\right)\left(1-2v\right)}\cdot\frac{\partial^2 w}{\partial z^2} = \alpha\frac{\partial p}{\partial z} + \rho g$$

Steady State under Influence of Gravity

Hydraulics $$\frac{\mathbf{k}\rho_f}{\mu}\nabla\left(p+\rho_f g z\right)=0$$

Boussinesq Assumption $$\nabla\left(p+\rho_f g z\right)=0$$

Hydrostatics $$p=-\rho_f g z$$

Geomechanics $$\frac{E\left(1-\nu\right)}{\left(1+\nu\right)\left(1-2\nu\right)}\cdot\frac{\partial^2 w}{\partial z^2}=-\alpha\rho_f g+\rho g$$

The second derivative of vertical deformation w is nonzero!

Analytical Solution

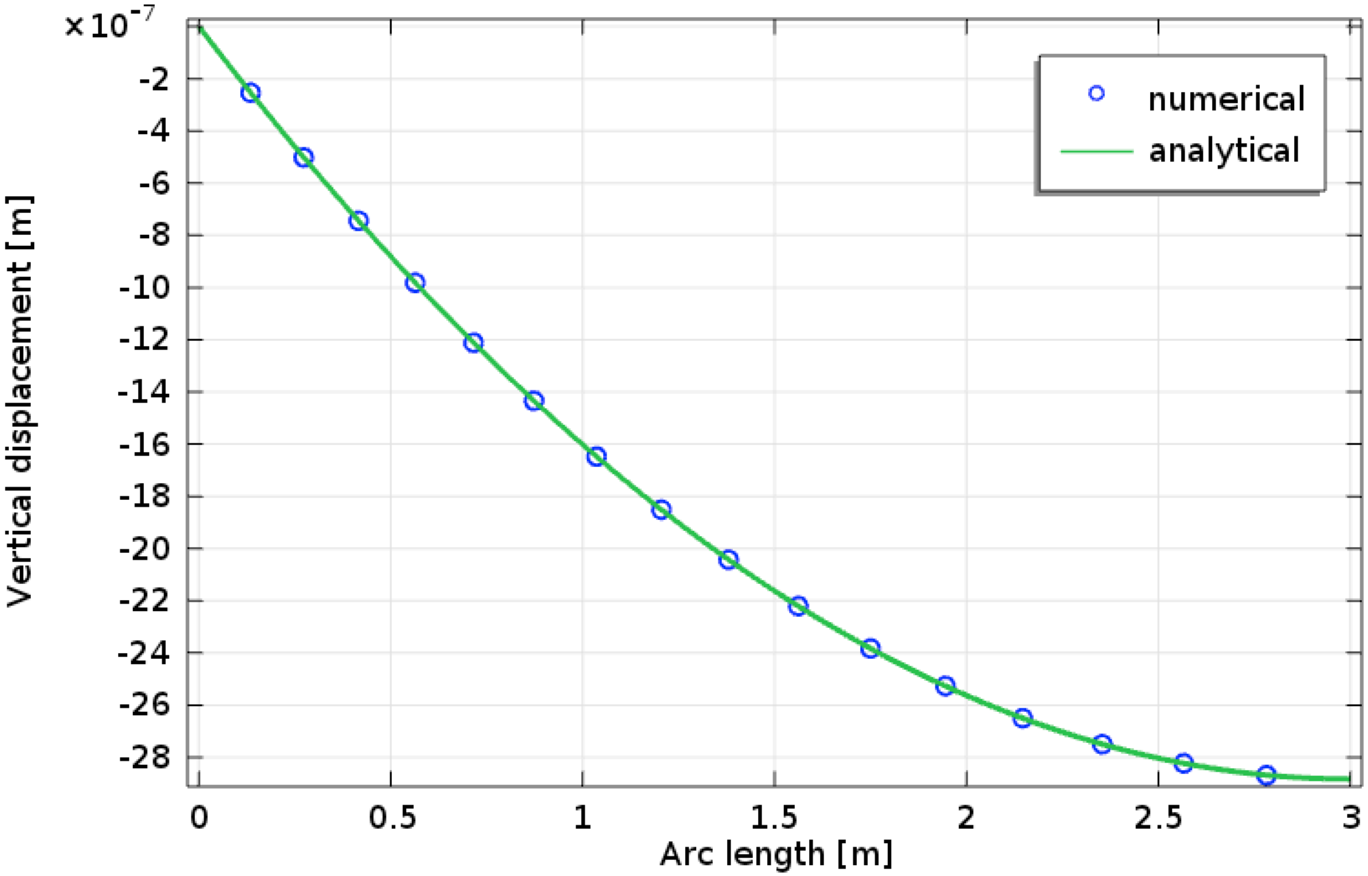
In case of constant parameters the second derivative is a constant.

➔ The vertical deformation is a quadratic function of depth:

$$w(z) = \frac{(1+\nu)(1-2\nu)}{2E(1-\nu)}\left((\theta-\alpha)\rho_f + \rho_s\right)gz\left(z-2H\right)$$

with Poisson ratio ν , Young modulus E, porosity θ , Biot parameter α , fluid density ρ_f and solid density ρ_s

Check



COMSOL Multiphysics

Zheng/Burridge 2003

Parameter	Value [unit]	Parameter	Value [unit]
Length L	2 [m]	Biot parameter a	0.79
Height H	3 [m]	Permeability k	$2 \cdot 10^{-13}$ [m^2]
Young modulus E	14.4 [GPa]	Porosity θ	0.2
Poisson ratio ν	0.2	Compressibility	$8.13 \cdot 10^{-11}$ [Pa^{-1}]
Fluid density	940 [kg/m^3]	Viscosity μ	0.122 [Pa•s]
Solid density	1600 [kg/m^3]		

The Analytical Solution can be used

- for computing steady states
- as initial state in unsteady numerical models

  Especially in transient simulations it is not necessary anymore to determine the initial steady state by a steady or unsteady pre-run of the numerical code
- as boundary conditions at vertical edges in numerical simulations
- comparison of steady states
- comparison of steady and unsteady states
- benchmark for code developers

Utilization in Numerical Modelling

Problem setting:

What is the original thickness of a system without any outer force that shrinks to a given thickness under influence of gravity?

Solutions:

In order to obtain the solution by a numerical model, several (maybe lots) of trial and error runs would be necessary to obtain an approximate solution.

Using the derived analytical solution, the original thickness can be calculated exactly and used in the numerical model

The Inverse Problem

Lets define H_0 as the height of the deformed steady state under the influence of gravity. In contrast H represents the thickness of the layer in a hypothetical environment without gravity. The two heights are connected by the formula:

$$H = H_0 + w_{\max}$$

Combining the above equations holds:

$$w_{\max} = \frac{1 - AH_0}{A} \pm \sqrt{\frac{\left(1 - AH_0\right)^2}{A^2} - H_0^2}$$

with

$$A = \frac{\left(1+\nu\right)\left(1-2\nu\right)}{E\left(1-\nu\right)}\left(-\alpha\rho_f + \rho\right)g$$

Generalization for Multi-layer systems

- The described solution can be extended to systems of horizontal layers with interface conditions:

$$w_n(0) = 0$$

$$\frac{\partial w_i}{\partial z}(H) = 0 \qquad \text{for } i = 1 \ldots n-1$$

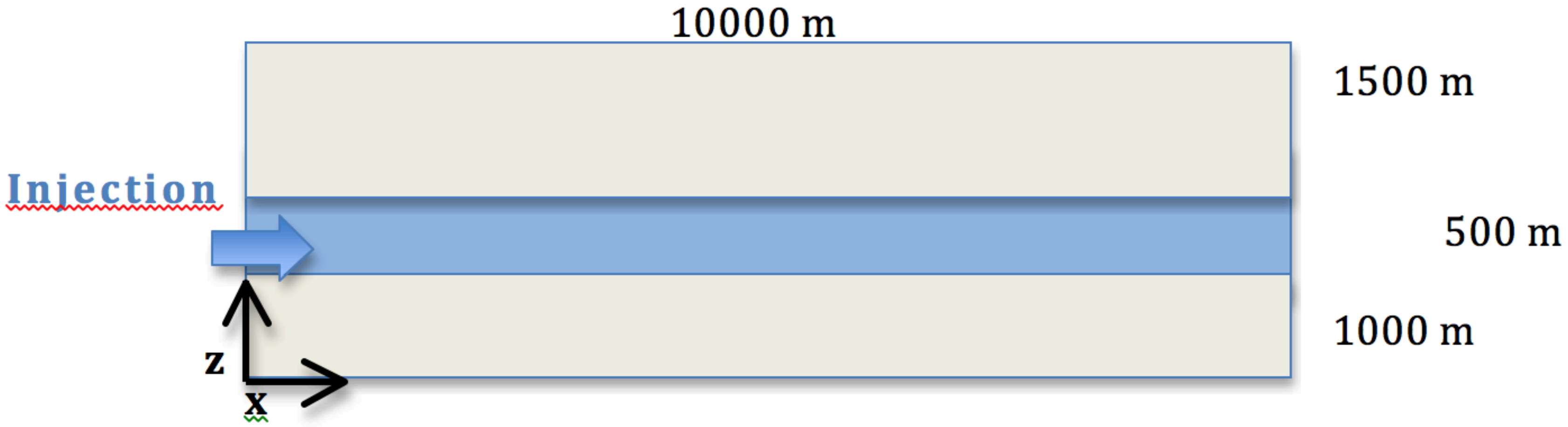
$$w_i(z_{i+1}) = w_{i+1}(z_{i+1}) \qquad \text{for } i = 1 \ldots n-1$$

- The inverse task can also be solved for a system of layers#

A system of quadratic systems has to be solved. This can be performed easily by a numerical iteration

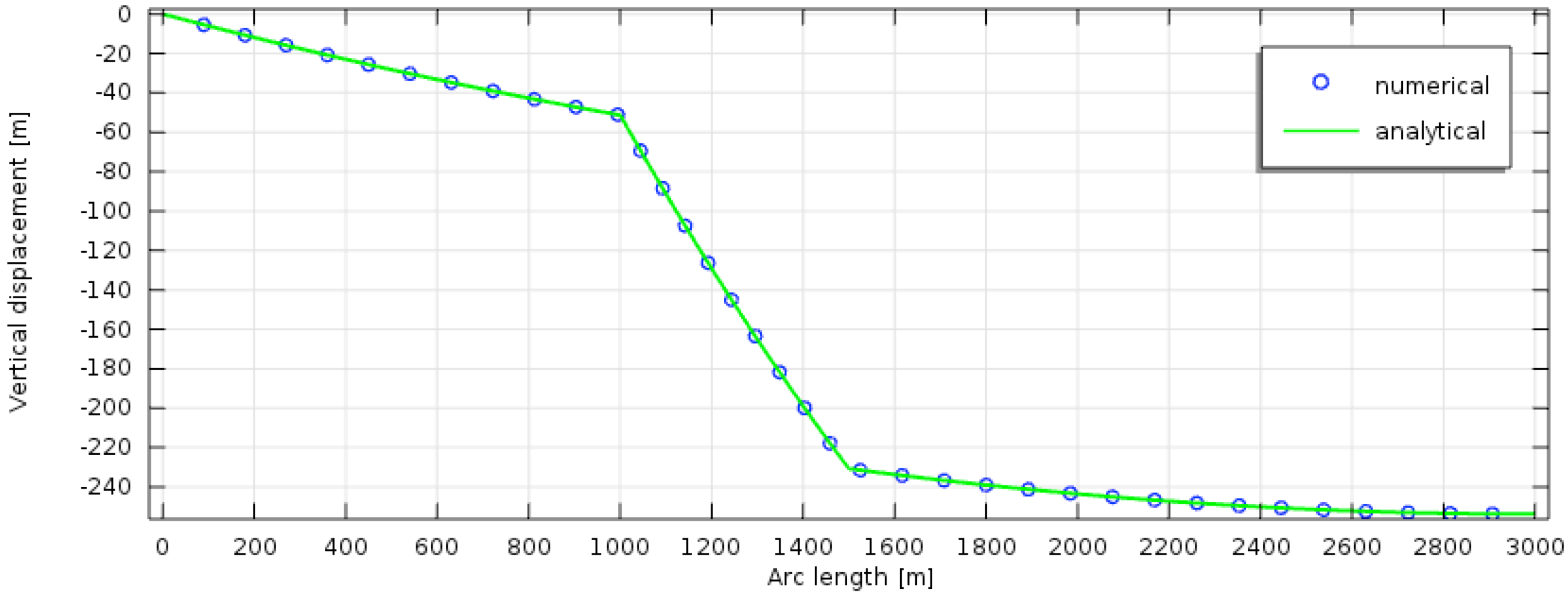
Application

- Example, following Nopper et al. (2012)



Parameter	Value [unit]	Parameter	Value [unit]
Length *L*	10000 [m]	Biot parameter *a*	1
Young modulus *E*	80, 800 [MPa]	Permeability *k*	$2.9 \bullet 10^{-11}$, $1.18 \bullet 10^{-14}$ [m^2]
Poisson ratio ν	0.25	Porosity θ	0.25
Well radius	0.1 [m]	Compressibility	$4.4 \bullet 10^{-10}$ [Pa^{-1}]
Fluid density	990 [kg/m^3]	Viscosity	0.001 [Pa•s]
Solid density	2750 [kg/m^3]	Pressure rise s_0	1000 [psi]

Steady Initial State



COMSOL Multiphysics

Reference State

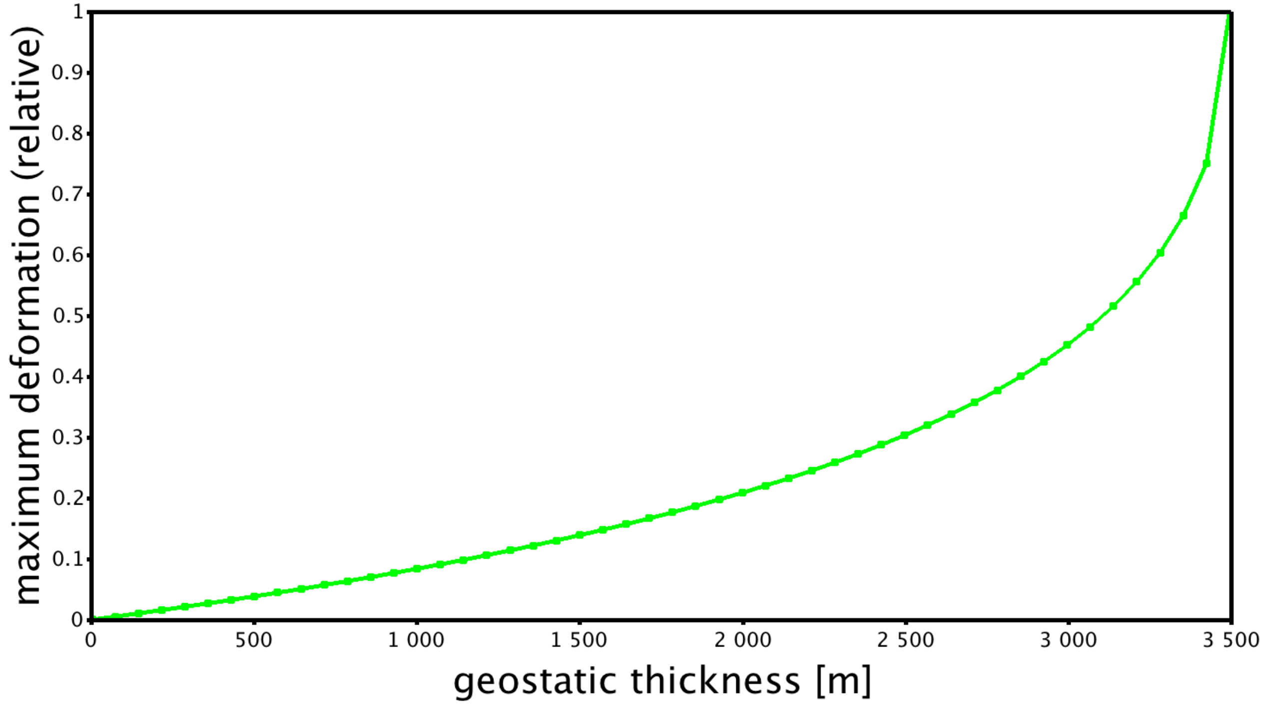
A common parameter for a geological set-up is used to explore the applicability of the presented approach.

Parameter	**Value [unit]**	**Parameter**	**Value [unit]**
Young modulus E	100 [MPa]	Biot parameter α	1
Poisson ratio ν	0.25	Porosity θ	0.25
Fluid density ρ_f	1000 [kg/m^3]	Gravity acceleration g	9.81 [m/s^2]
Bulk density ρ_b	2500 [kg/m^3]	Thickness H	3000 [m]

Dependence of Maximum Deformation on Thickness

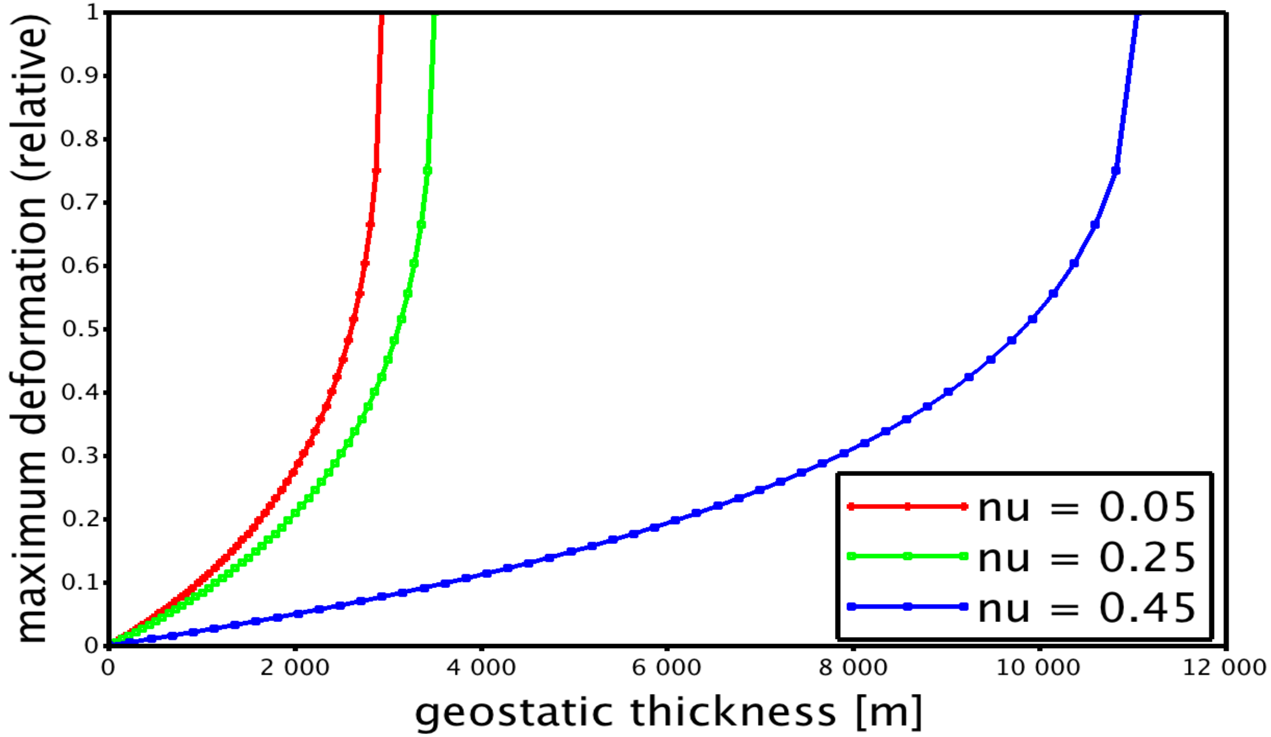
Maximum deformation (normalized) in dependence of thickness H_0 for the reference parameter set

For values up to 3500 m the proposed approach delivers real(istic) values. For higher values the results are complex values.



Dependence on Poisson Ratio

Maximum deformation (normalized) in dependence of thickness H_0 for the reference parameter set



Dependence on Young Modulus

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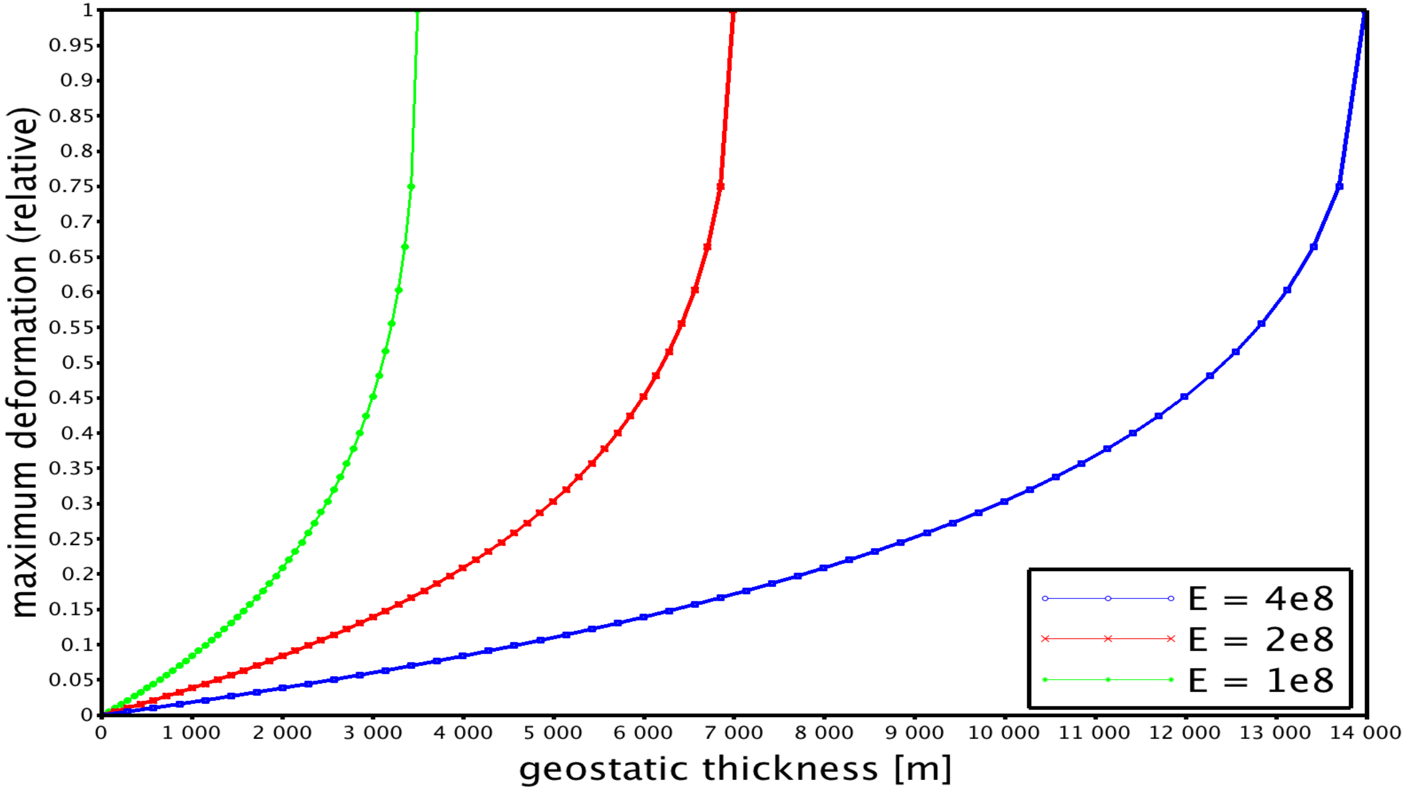
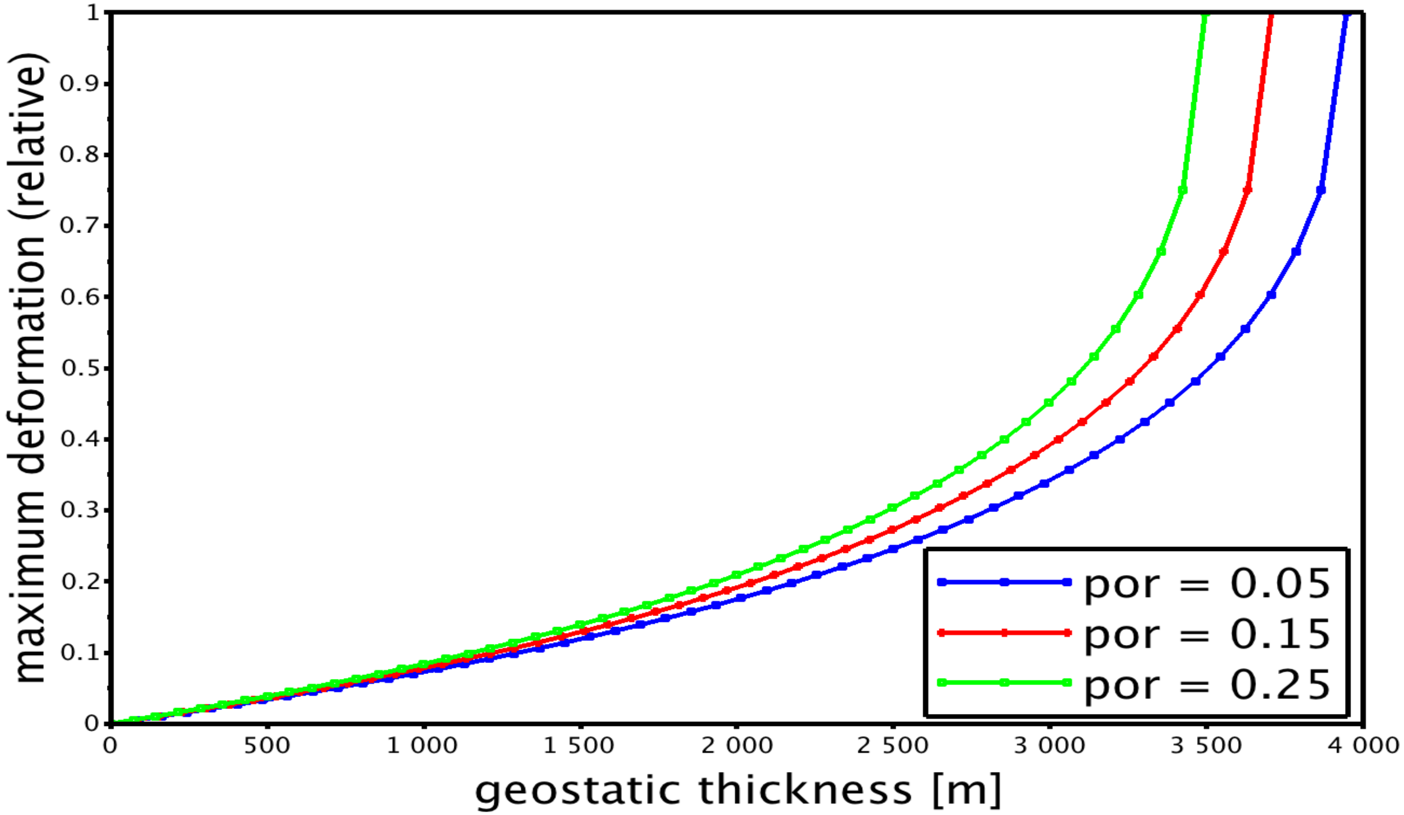
Maximum deformation (normalized) in dependence of thickness H_0 and Young modulus E



Dependence on Porosity

Maximum deformation (normalized) in dependence of thickness H_0 and Young modulus E



Summary & Conclusion

- If gravity is taken into account, vertical deformation is a quadratic function of depth
- In order to take that into account the modeller has to use a (hypothetical) thickness of a layer without gravity influence
- That hypothetical thickness can be computed by the presented analytical approach
- The approach can be extended to mult-layer systems
- The proposed approach delivers real(istic) values for the thickness of the system without gravity influence

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