Rationale
AO Principle
Motivation
PM energies
Virtualization
Mag-Coil
Mag-Mag
Crosstalk
Bias
Summary

Predicting the Parasitic Forces in the Magnetically Levitated Adaptive Optics Mirrors

C. Del Vecchio[1] R. Briguglio[1] M. Xompero[1] A. Riccardi[1]

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COMSOL CONFERENCE 2017 ROTTERDAM

Outline

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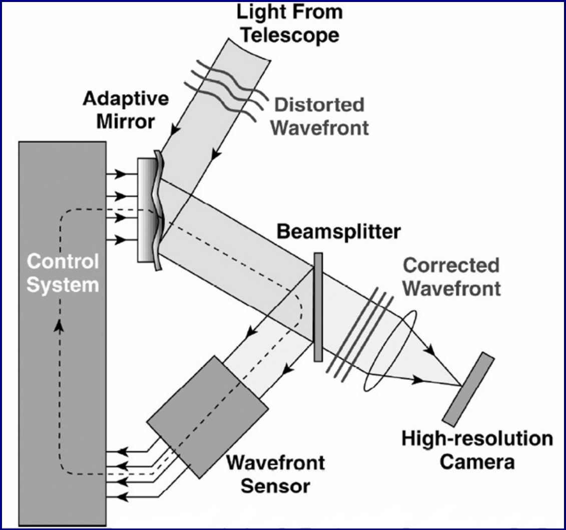
Compensating the Atmospheric Turbulence
The Control System Concept



Adaptive Optics on board the Telescope I

System Overview

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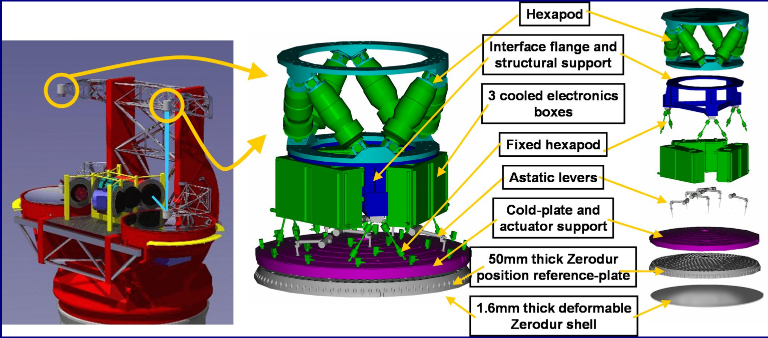
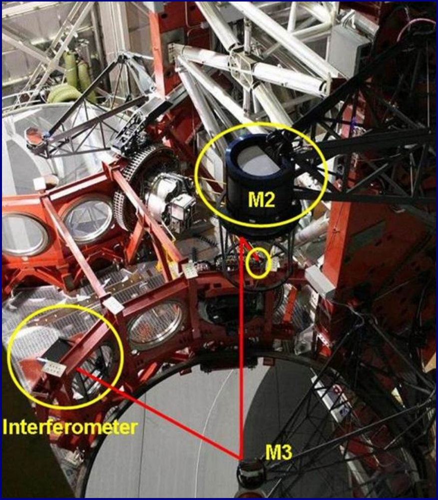


[Riccardi et al., 2004]

Adaptive Optics on board the Telescope II

DM Location



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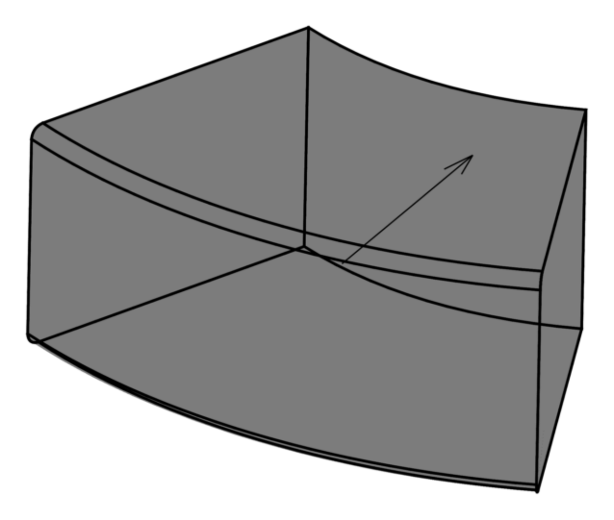
The Parasitic Forces
The Case Studies



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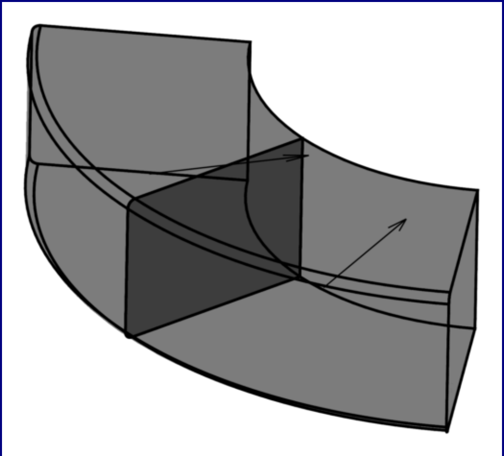

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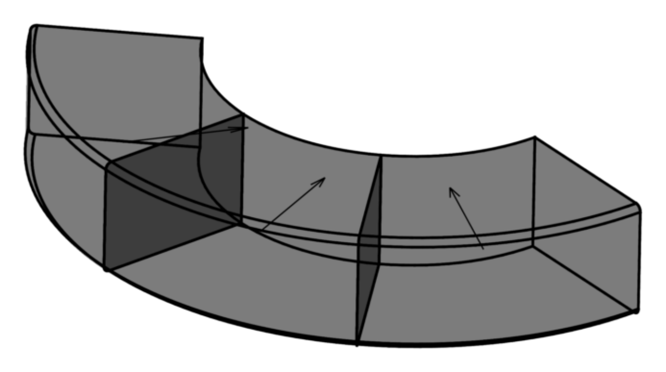

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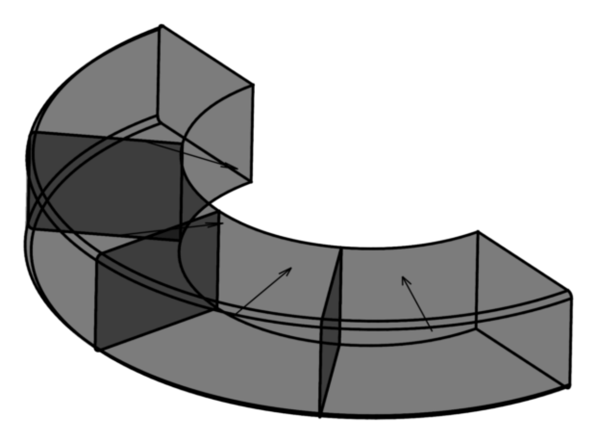

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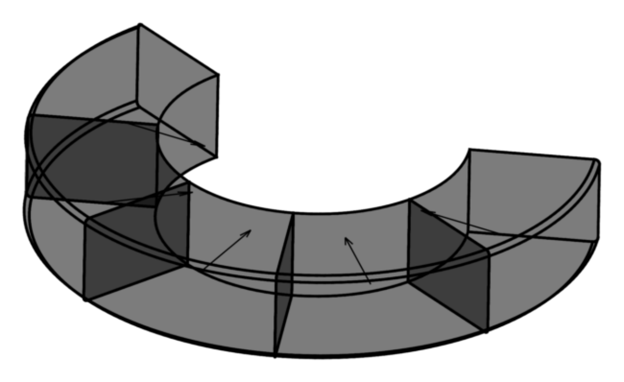

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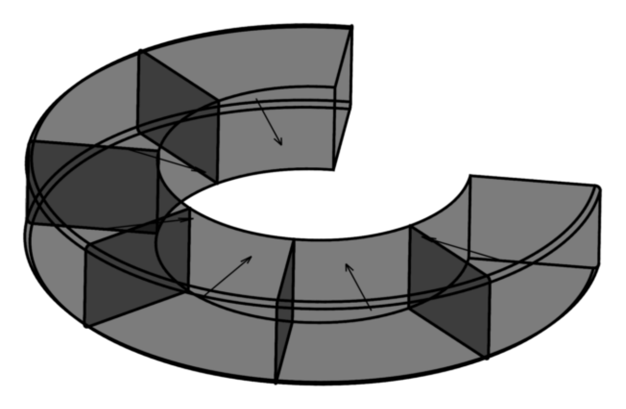

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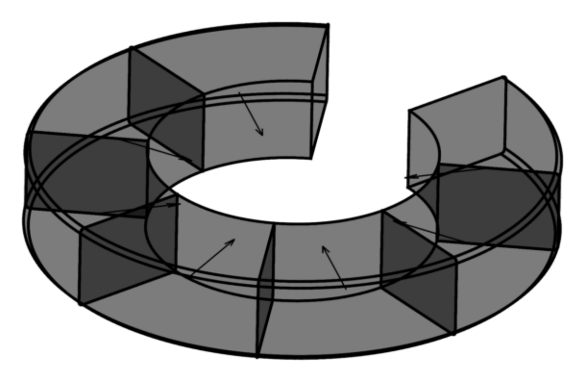

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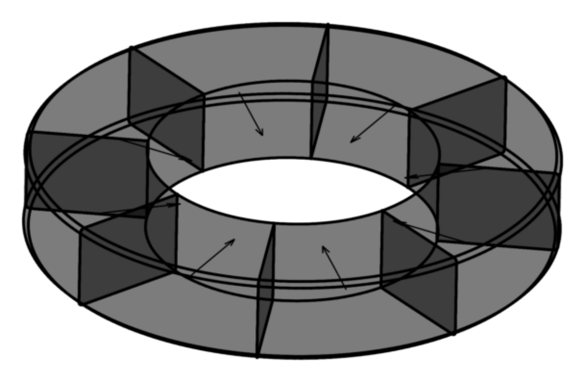

The Parasitic Forces
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AdOpt Parasitic Forces

Del Vecchio et al.

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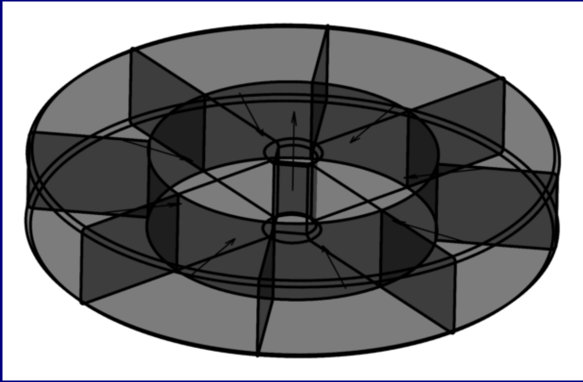

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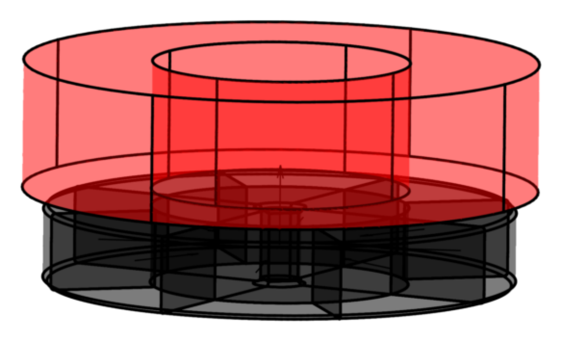

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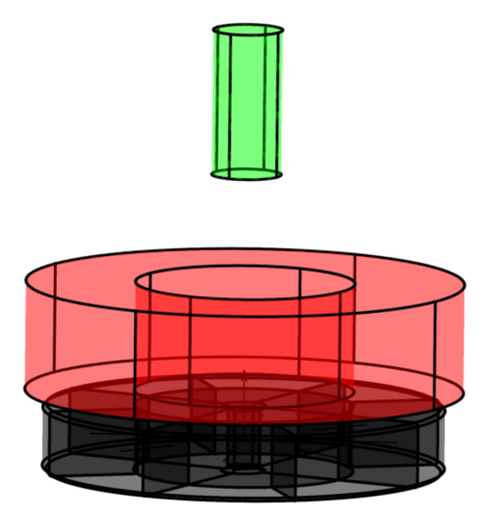

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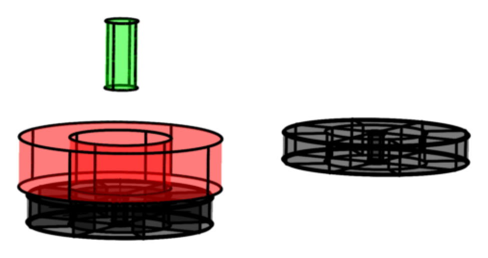

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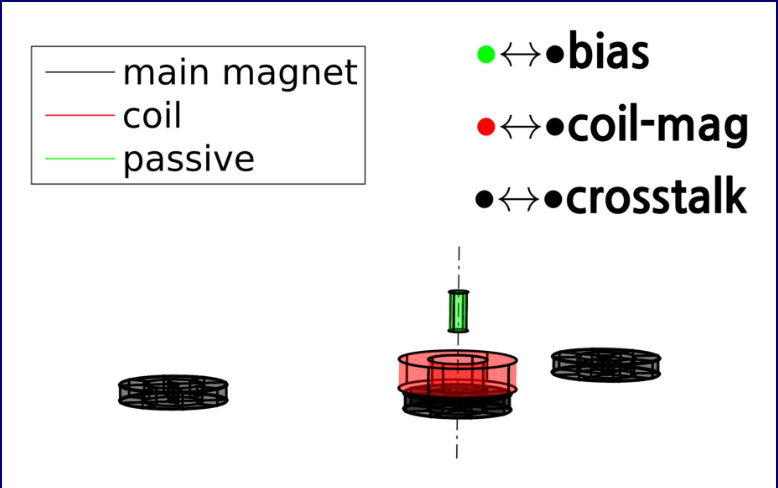
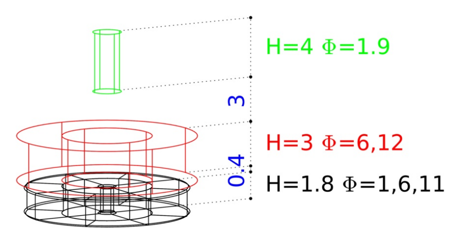


The Parasitic Forces

The Case Studies

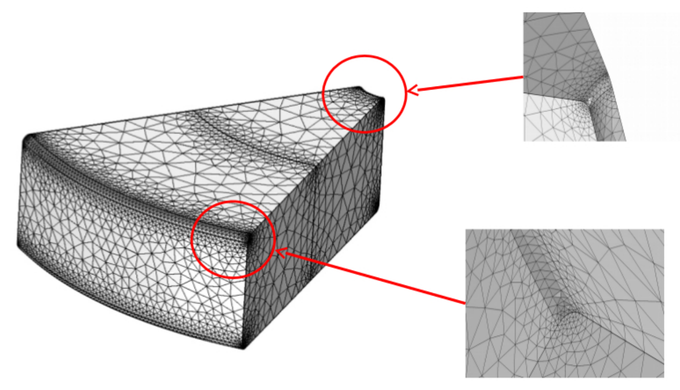
- NeFeBo 8 petals + 1 cyl core
- 270 turns
- SmCo cylinder



Magnet-Coil	Magnet-Magnet	
	Bias	Crosstalk
● ↔ ●	● ↔ ●	● ↔ ●
misalignments by manufacturing		*lifelong warping of DM*
Lorentz: $\int(\mathbf{J}\times\mathbf{B})dV$	VW or Maxwell: $\int(\mathbf{T}\times\mathbf{n})dS$	

Caveats

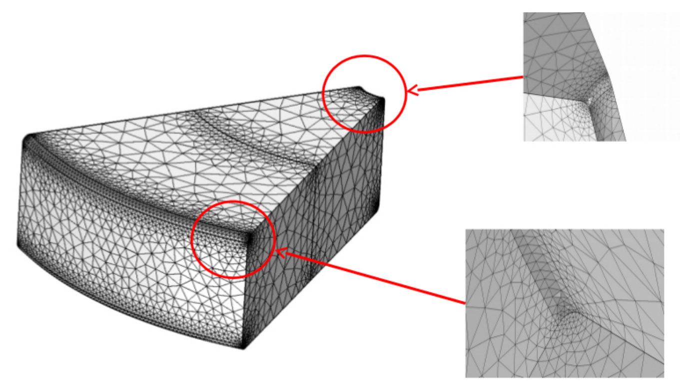
Some important requirements



- corner filleting
- verifying via 3rd principle of dynamics
- extending the BH curve of PM
- redefine the stored energy/coenergy density of PM

Caveats

Some important requirements



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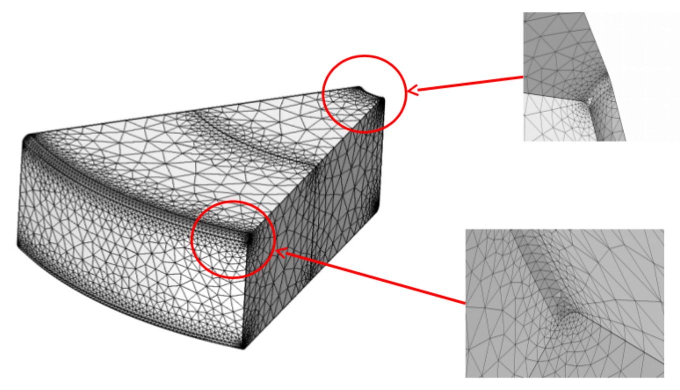

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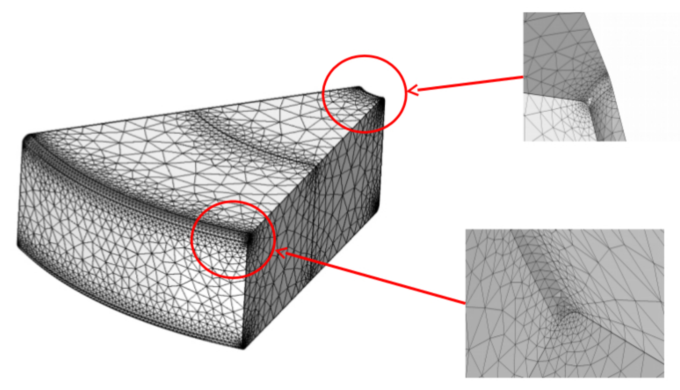

- corner filleting
- verifying via 3rd principle of dynamics
- extending the BH curve of PM
- redefine the stored energy/coenergy density of PM

Energy Densities in PM (1)

Energy and Coenergy: $E = \int_{B_0}^{B} HdB \quad C = \int_{H_0}^{H} BdH$

$$E = \int_{B_r}^{0} HdB \quad C = \int_{H_c}^{H} BdH$$

[Deliège et al., 2003]

$$E = \int_{0}^{B} HdB \times (B < 0) + \int_{B}^{0} HdB \times (B \leq B_r) \times (B \geq 0) + \left(\int_{B_r}^{0} HdB + \int_{B_r}^{B} HdB \right) \times (B > B_r) \quad (1)$$

$$C = \int_{H_c}^{H} BdH \times (H < H_c) + \int_{H_c}^{H} BdH \times (H \leq 0) \times (H \geq H_c) + \left(\int_{H_c}^{0} BdH + \int_{0}^{H} BdH \right) \times (H > 0)$$

because of the consecutiveness of the limits

$$C = \int_{H_c}^{H} BdH$$

Energy Densities in PM (2)

Energy: $E = \int_{B_0}^{B} HdB$

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$E = \int_{B_r}^{B} HdB$ [Strahan, 1998], [Lovatt and Walterson, 1999], and [Campbell, 2000]

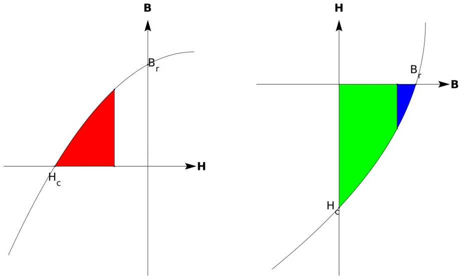
$$E = \left(\int_{B_r}^{0} HdB + \int_{0}^{B} HdB \right) \times (B < 0) + \int_{B_r}^{B} HdB \times (B \leq B_r) \times (B \geq 0) + \int_{B_r}^{B} HdB \times (B > B_r) \quad (2)$$

because of the consecutiveness of the limits

$$E = \int_{B_r}^{B} HdB$$

Energy Densities in PM (3)

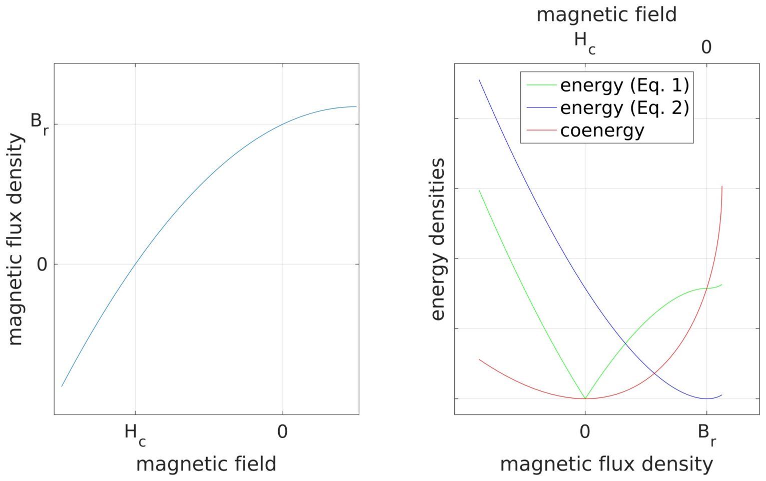
Energy and Coenergy: $E = \int_{B_0}^{B} HdB \quad C = \int_{H_0}^{H} BdH$



coenergy	$\int_{H_c}^{H} BdH$	$\int_{H_c}^{H} BdH$	$\int_{H_c}^{0} BdH + \int_{0}^{H} BdH$
energy (Eq. 1)	$\int_{0}^{B} HdB$	$\int_{B}^{0} HdB$	$\int_{B_r}^{0} HdB + \int_{B_r}^{B} HdB$
energy (Eq. 2)	$\int_{B_r}^{0} HdB + \int_{0}^{B} HdB$	$\int_{B_r}^{B} HdB$	$\int_{B_r}^{B} HdB$
domain	$H < H_c$ $B < 0$	$H_c \le H \le 0$ $0 \le B \le B_r$	$H > 0$ $B > B_r$

Energy Densities in PM (3)

Energy and Coenergy: $E = \int_{B_0}^{B} H dB \quad C = \int_{H_0}^{H} B dH$



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Energy Densities in PM (4)

Choosing Coenergy

- $\int HdB$ and $\int BdH$ not available
  - via Matlab `cumtrapz` command + Comsol BH table
  - via Comsol `integrate` command

Comsol `integrate` slightly more accurate than Matlab `cumtrapz`

Coenergy *typically* more accurate and faster than energy

Using Comsol-defined coenergy

- allows to manage
  1. non linear PM's
  2. BH curve outside the 2nd quadrant

Transformation Matrices (1)

Virtual Displacements and Magnetic Variables as as Functions of θ and β

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} \quad \text{principal axes directions} \qquad \boldsymbol{\beta} = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix} \quad \text{virtual rotations}$$

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta_y \cos\theta_z & \cos\theta_z \sin\theta_x \sin\theta_y - \cos\theta_x \sin\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \cos\theta_z \sin\theta_y \\ \cos\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \sin\theta_y \sin\theta_z & \cos\theta_x \sin\theta_y \sin\theta_z - \cos\theta_z \sin\theta_x \\ -\sin\theta_y & \cos\theta_y \sin\theta_x & \cos\theta_x \cos\theta_y \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} \cos\beta_y \cos\beta_z & \cos\beta_z \sin\beta_x \sin\beta_y - \cos\beta_x \sin\beta_z & \sin\beta_x \sin\beta_z + \cos\beta_x \cos\beta_z \sin\beta_y \\ \cos\beta_y \sin\beta_z & \cos\beta_x \cos\beta_z + \sin\beta_x \sin\beta_y \sin\beta_z & \cos\beta_x \sin\beta_y \sin\beta_z - \cos\beta_z \sin\beta_x \\ -\sin\beta_y & \cos\beta_y \sin\beta_x & \cos\beta_x \cos\beta_y \end{bmatrix}$$

$$\boldsymbol{\delta} = \boldsymbol{R}\left((\boldsymbol{D} - \boldsymbol{I})\boldsymbol{R}^{-1} \begin{bmatrix} X - x_c \\ Y - y_c \\ Z - z_c \end{bmatrix} \right)$$

Transformation Matrices (2)

Avoiding any Extra Coordinate System

if $\beta_x \ll 1$ and $\beta_y \ll 1$ and $\beta_z \ll 1$

$$\boldsymbol{D} = \begin{bmatrix} 1 & \beta_x\beta_y - \beta_z & \beta_y + \beta_x\beta_z \\ \beta_z & \beta_x\beta_y\beta_z + 1 & \beta_y\beta_z - \beta_x \\ -\beta_y & \beta_x & 1 \end{bmatrix}$$

if, for instance, $\theta_x = 30^\circ$ and $\theta_y = \theta_z = 0$, splitting $\boldsymbol{\delta}$ becomes:

$\beta_x \neq 0,\ \beta_y = 0,\ \beta_z = 0$ $\quad$ $\beta_y \neq 0,\ \beta_x = 0,\ \beta_z = 0$ $\quad$ $\beta_z \neq 0,\ \beta_x = 0,\ \beta_y = 0$

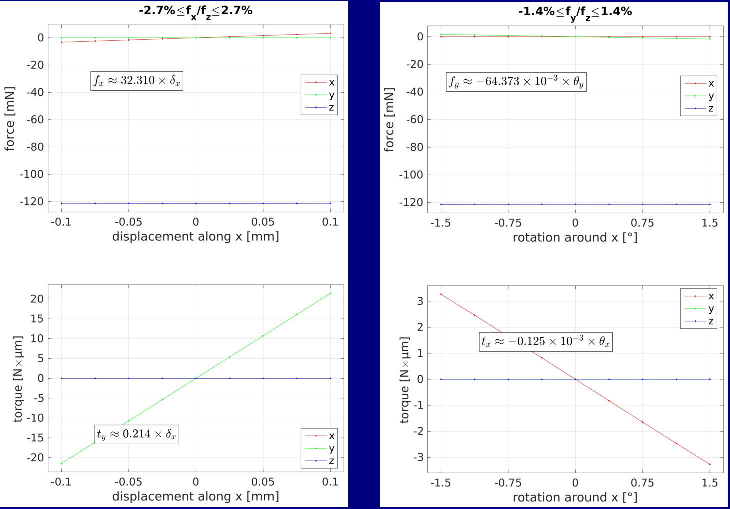
$$\begin{bmatrix} 0 \\ -\beta_x\,(Z - z_c) \\ \beta_x\,(Y - y_c) \end{bmatrix} \quad \begin{bmatrix} \frac{\sqrt{3}\,\beta_y\,(Z-z_c)}{2} - \frac{\beta_y\,(Y-y_c)}{2} \\ \frac{\beta_y\,(X-x_c)}{2} \\ -\frac{\sqrt{3}\,\beta_y\,(X-x_c)}{2} \end{bmatrix} \quad \begin{bmatrix} -\frac{\beta_z\,(Z-z_c)}{2} - \frac{\sqrt{3}\,\beta_z\,(Y-y_c)}{2} \\ \frac{\sqrt{3}\,\beta_z\,(X-x_c)}{2} \\ \frac{\beta_z\,(X-x_c)}{2} \end{bmatrix}$$

$$\boldsymbol{F} = (\boldsymbol{RD})\boldsymbol{p} \qquad \boldsymbol{p} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \boldsymbol{F}\boldsymbol{B}_{loc} \quad \boldsymbol{H} = \boldsymbol{F}\boldsymbol{H}_{loc} \quad \boldsymbol{M} = \boldsymbol{F}\boldsymbol{M}_{loc} \quad \boldsymbol{\mu} = \boldsymbol{F}^{-1}\boldsymbol{\mu}_{r\,loc}\boldsymbol{F}$$

Magnet-Coil Linear and Angular Misalignments

$\boldsymbol{f} = f(\delta, \theta) \quad \boldsymbol{t} = f(\delta, \theta)$



Magnet-Coil Linear and Angular Misalignments

$\boldsymbol{f} = f(\delta, \theta) \quad \boldsymbol{t} = f(\delta, \theta)$

Rotations and displacements of main magnet
$\boldsymbol{f}$ and $\boldsymbol{t}$ via Lorentz

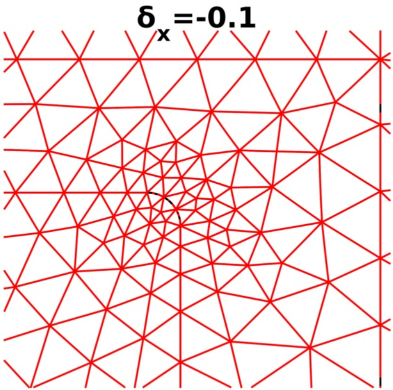
- *parasitic* forces and torques $\approx$ linear
  - $f_x \approx k\delta_x$ and $t_y \approx k\delta_x$
  - $f_y \approx k\delta_y$ and $t_x \approx k\delta_y$
- *active* $f_z \approx$ constant

Even for alignments tolerances as large as $\pm.1$ mm *or* $\pm 1.5^\circ$

$$\frac{f_x}{f_z} < 3\% \text{ and } \frac{f_y}{f_z} < 3\%$$

Exploiting the Deformable Mesh I

Sweeping the Displacements in a Single Run



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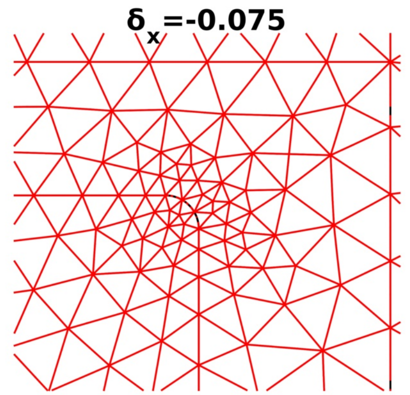

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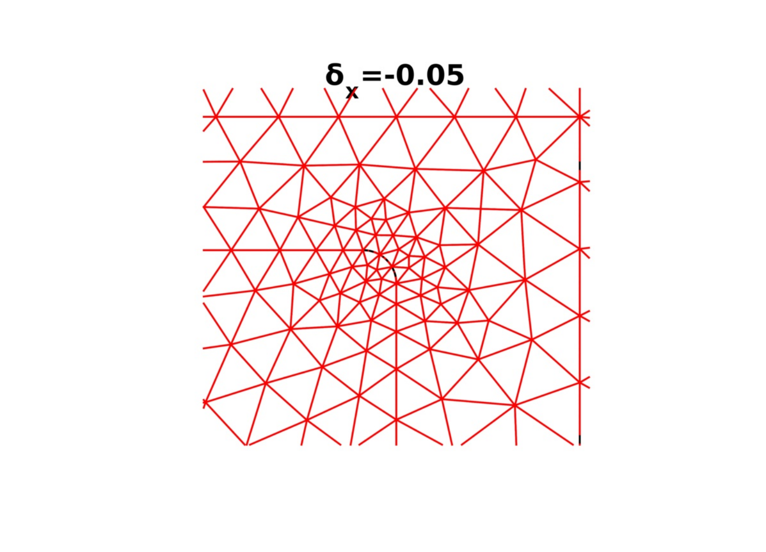

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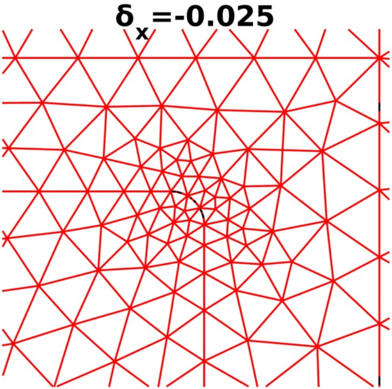

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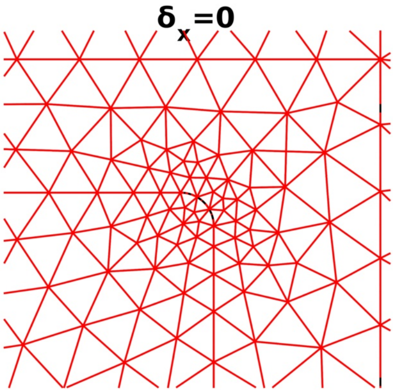

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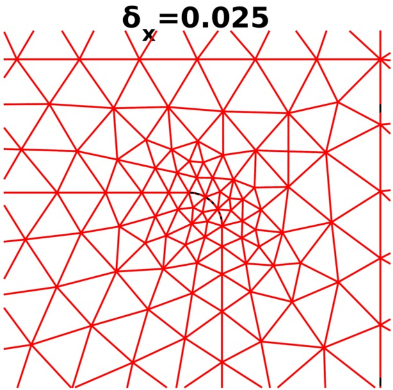

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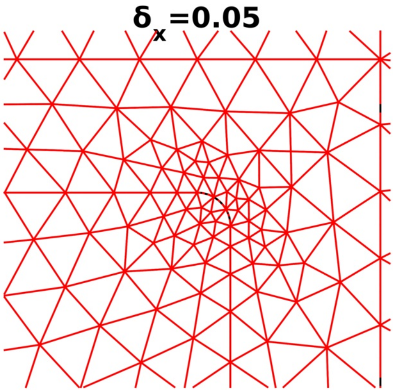

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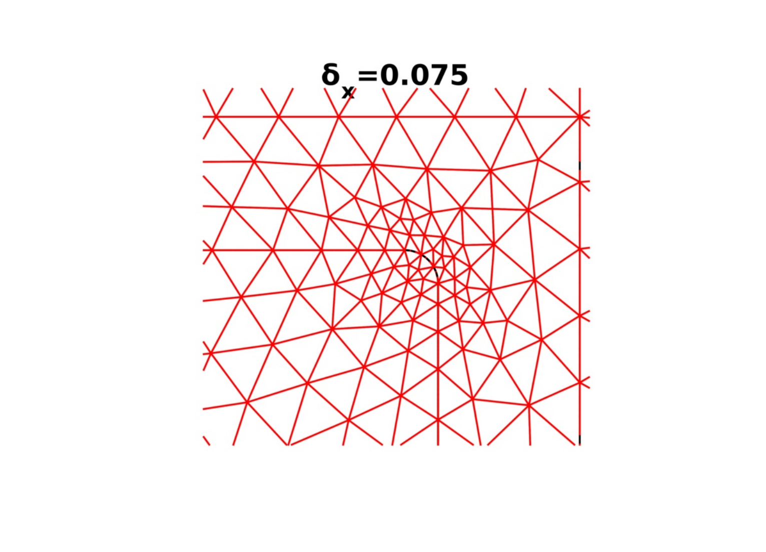

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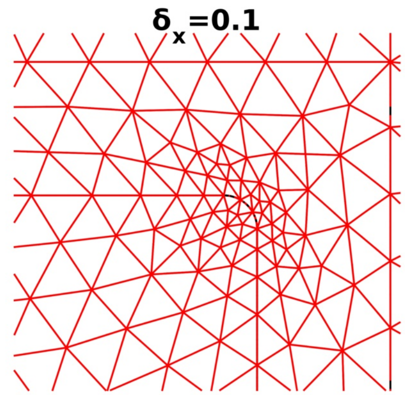

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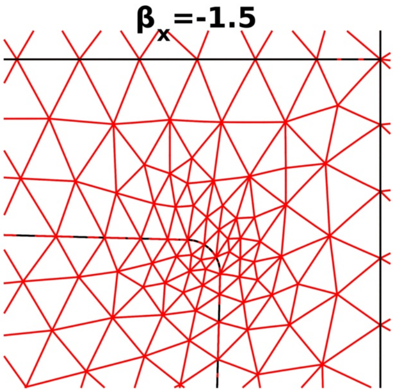
Sweeping the Rotations in a Single Run



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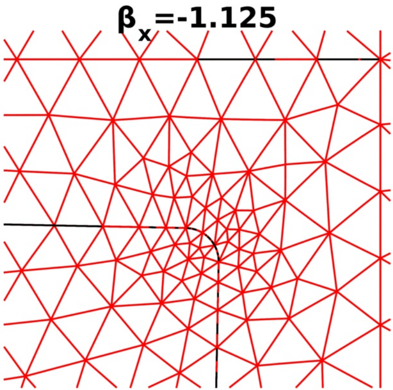

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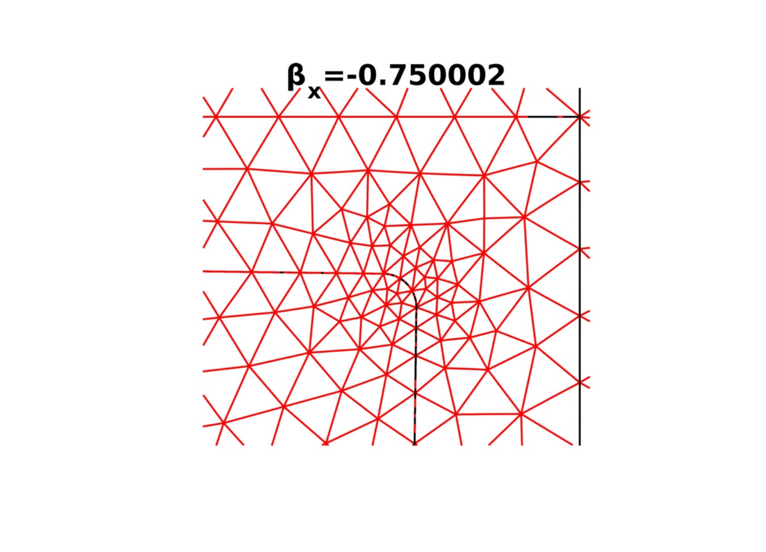

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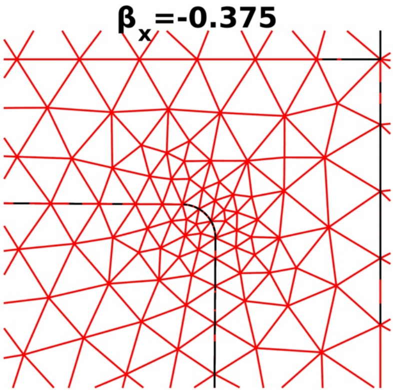
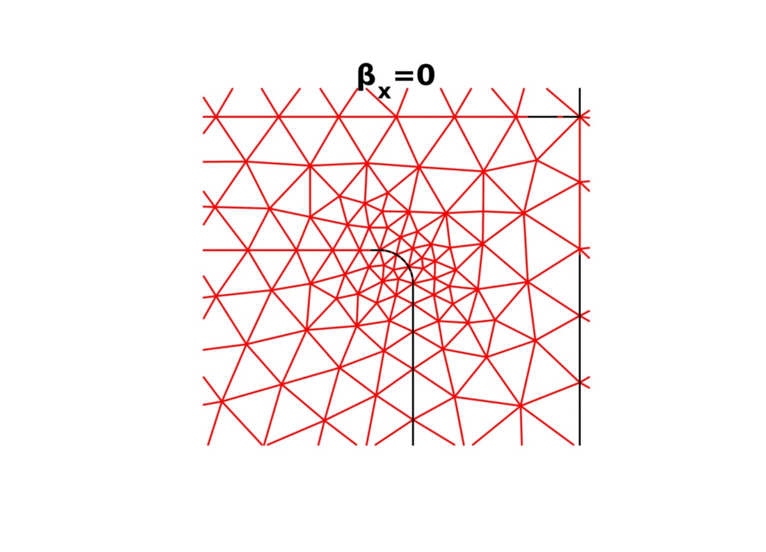


Exploiting the Deformable Mesh II

Sweeping the Rotations in a Single Run



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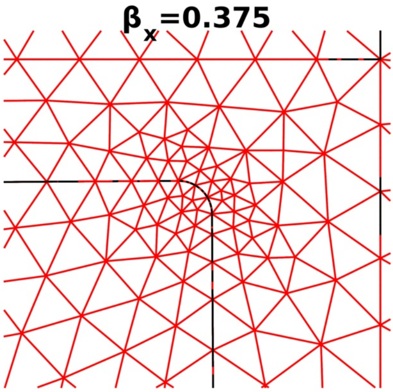

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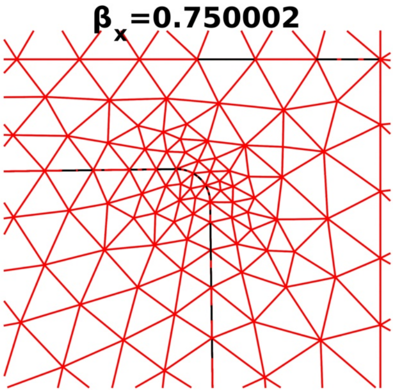

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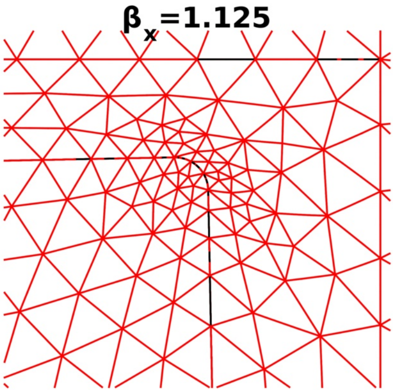

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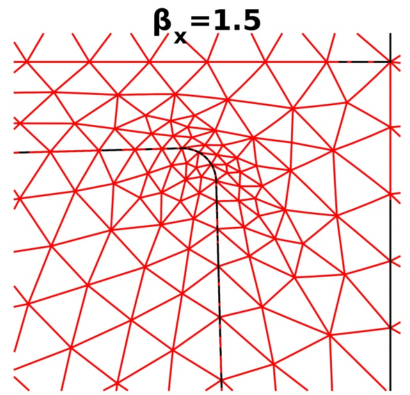

AdOpt Parasitic Forces

Del Vecchio et al.

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AO Principle
Motivation
PM energies
Virtualization
Mag-Coil
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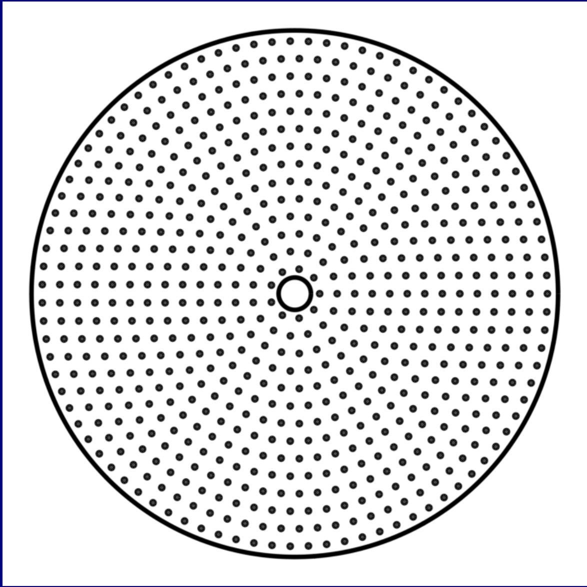
Magnet-Magnet Interaction may Wrap the DM
Assumptions and Restrictions



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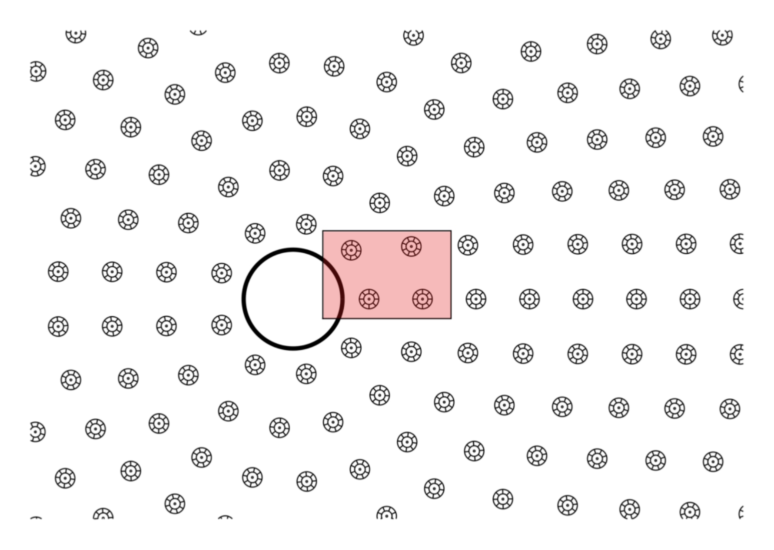

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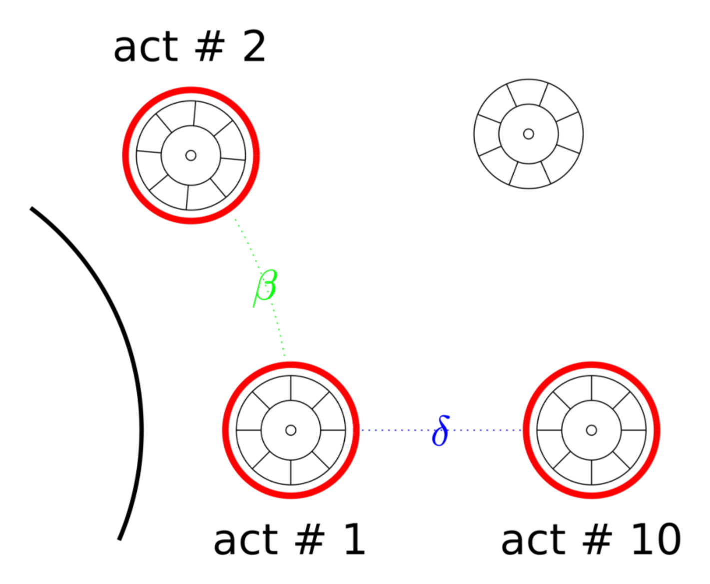

Magnet-Magnet Interaction may Wrap the DM

Assumptions and Restrictions

- Magnets all spaced by $\approx 30\,\mathrm{mm} \rightsquigarrow$ 2 interactions
  - act. $\#1$ vs. act. $\#2$, on the first ring of actuators @ $\mathrm{r} = 43.044\,\mathrm{mm}$, separated by $\beta = 40°$
  - act. $\#1$ vs. act. $\#10$, on the *x* axis, separated by $\delta = 30.31\,\mathrm{mm}$
- Virtual works computation poorly verifies the third principle of dynamics $\rightsquigarrow$ 2 runs + fitting
  - increasing from $\beta = 18°$ by steps of $1°$
  - increasing from $\delta = 14\,\mathrm{mm}$ by steps of $1\,\mathrm{mm}$

Magnet-Magnet Interaction may Wrap the DM

Computation

- the analysis is halted if
  - $\arccos\left(\frac{F_1}{|F_1|}\cdot\frac{F_2}{|F_2|}\right) < 170^\circ$ or $\arccos\left(\frac{T_1}{|T_1|}\cdot\frac{T_2}{|T_2|}\right) < 170^\circ$
  - $\|\frac{|F_1|}{|F_2|} - 1\| > 2\%$ or $\|\frac{|T_1|}{|T_2|} - 1\| > 2\%$
- fit f and t with $f(x) = C_1 e^{k_2 x} + C_2 e^{k_2 x}$
  - fitting errors
    - $\leq .25\%$ for $14\,\mathrm{mm} \leq \delta \leq 18\,\mathrm{mm}$
    - $\leq .6\%$ for $18^\circ \leq \delta \leq 25^\circ$

$f(x) = C_1 e^{k_2 x} + C_2 e^{k_2 x}$					
$f(x)$	x	C_1	k_1	C_2	k_2
F		16550.3	-37.4582	5.09809	-12.8304
T_1	β	18.9089	-35.088	0.0097198	-11.1418
T_2		12.7918	-33.9237	0.00780015	-10.8013
F		21777.2	-900.256	6.56426	-313.77
T_1	δ	10.64	-775.193	0.00771193	-250.873
T_2		21.5468	-836.01	0.0112864	-269.964

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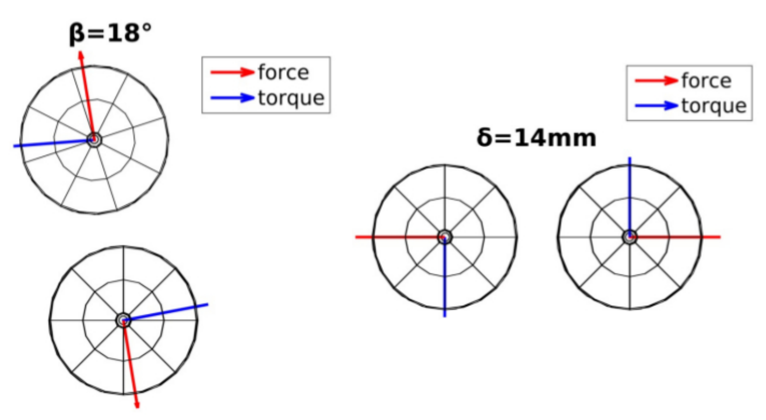
		$\beta = 40^\circ$	$\delta = 28\,\mathrm{mm}$
F	mN	0.657	0.959
T_1	$\mathrm{N} \times \mu\mathrm{m}$	4.070	6.617
T_2	$\mathrm{N} \times \mu\mathrm{m}$	4.143	5.656

- negligible disturbances:
  - the typical turbulence-correction force is $\approx .4\,\mathrm{N}$ rms
  - the maximum dynamic force is $\approx 1.3\,\mathrm{N}$
- as the sums of these forces and torques are null, they don't affect the DM global statics
- fit+vw allows to determine a lower limit for the actuator separation

Magnet-Magnet Interaction may Wrap the DM

Results



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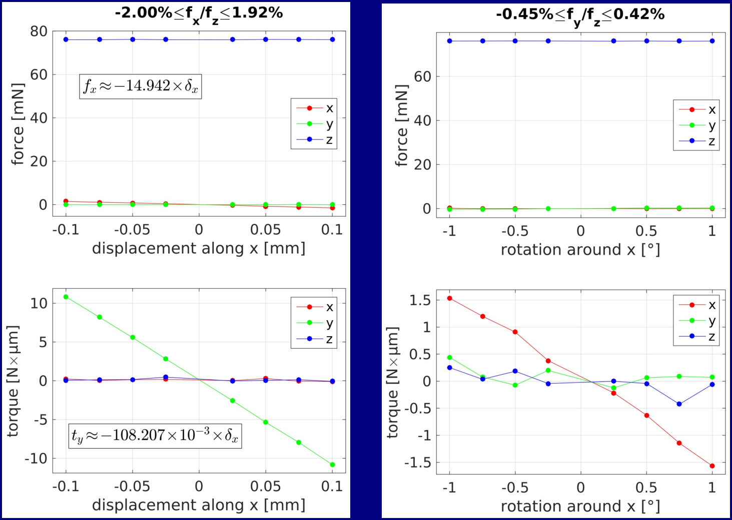
Main Magnet-Bias Magnet Displacements and Angular Misalignments

$\boldsymbol{f} = f(\delta, \theta) \quad \boldsymbol{t} = f(\delta, \theta)$



Main Magnet-Bias Magnet Displacements and Angular Misalignments

$\boldsymbol{f} = f(\boldsymbol{\delta}, \boldsymbol{\theta}) \quad \boldsymbol{t} = f(\boldsymbol{\delta}, \boldsymbol{\theta})$

Rotations and displacements of main magnet
f and ***t*** via vw

- the force parallel to the displacement is $\leq 1.5\%$ for $|\delta| \leq 1\,\mathrm{mm}$ and $|\theta| \leq 30\,^\circ$ of f_z
- the torques are $\leq 100\,\mathrm{N} \times \mathrm{\mu m}$

The design tolerances are much lower than the displacement and rotation ranges considered in the computations

Lessons Learned & Future Work

Exploiting the virtual work

Although more complex and cpu consuming, the vw turns out to be the only available method to compute PM-to-PM interactions, provided that stored magnetic energy density are properly re-defined

Lessons Learned & Future Work

The good result

Very weak parasitic forces and torques in the current DM's

- main magnet - coil
- main magnet - bias magnet
- main magnet - main magnet

Lessons Learned & Future Work

The powerful of simulations

Any type of magnetic force of an Adaptive Optics Deformable Mirror can be truthfully evaluated by numerical methods, including possible undesired, although weak, interactions.

A method to determine the minimum actuator spacing is at hand

For Further Reading I

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- Lovatt, H. and Walterson, P. (1999).
  Energy stored in permanent magnets.
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For Further Reading II

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