COMSOL CONFERENCE
2017 ROTTERDAM

UNIVERSITY of GREENWICH

Level set based Topology Optimisation of Convectively Cooled Heat Sinks

Mani S Santhanakrishnan, Timothy Tilford & Chris Bailey

Computational Mechanics and Reliability Group,
University of Greenwich, London, UK

October 2017, Rotterdam, the Netherlands

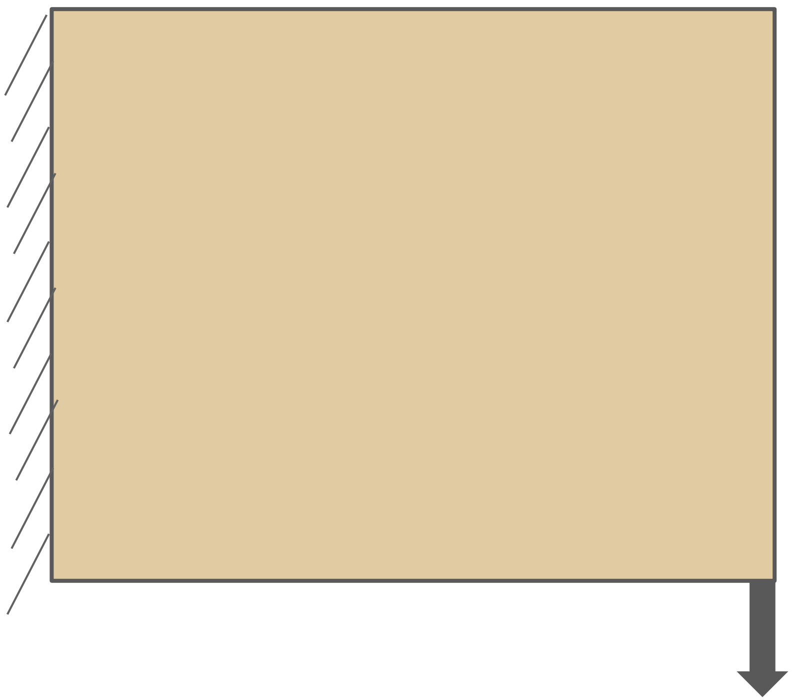
Contents

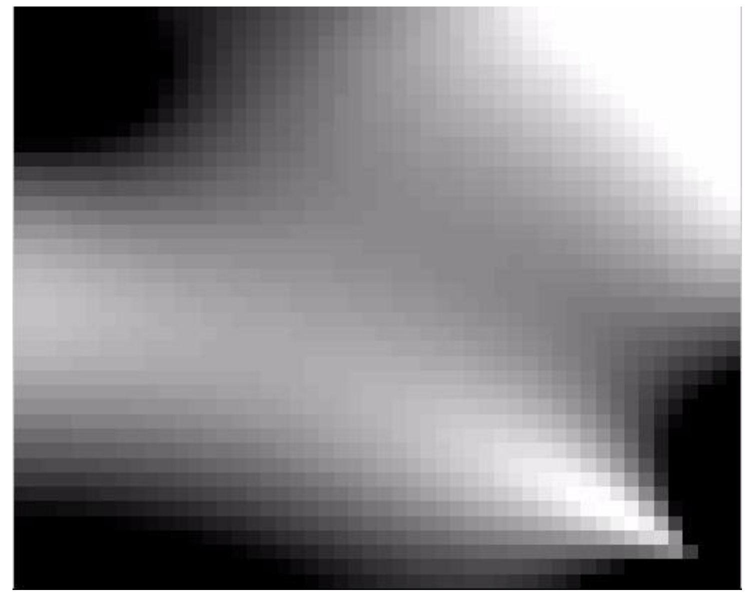
Introduction to Topology Optimisation

- Topology optimisation is a mathematical approach that **optimises material layout** for a given **set of constraints** meeting prescribed set of performance **objectives**.
- Concept is started for structural mechanics problems (by Bendsoe & Kikuchi) but now it finds application in Fluids, Acoustics, Electromagnetics, Optics etc.
- There are different methods for Topology optimisation they are,
  - Density Method
  - Level set methods
  - Topological derivative
  - Phase field method and
  - Evolutionary approaches.

Introduction to Topology Optimisation

Cantilever Subjected to Tip Load





Ref:O Sigmund, A 99 line Topology optimisation code written in Matlab, Struc & Multidisc Optim 2001

Objective: Maximum stiffness or minimum compliance [F^TU]

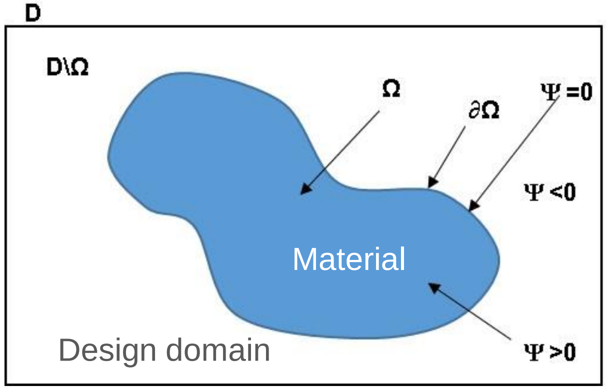
Constraints: 30% of material volume
KU= F (Governing equation)

■ Solid

□ No material/ Void

TO with the Level–set method

- Level set method is a concept developed for studying moving boundaries
- Major steps in Level-set TO
  - LSF parametrization (Polynomial shape function or Radial Basis function)
  - Mapping of geometry into mechanical model, Ersatz material, XFEM, Conforming mesh
  - Optimization strategy (Hamilton Jacobi solver or Mathematical programming)



$$\psi = \begin{cases} = 0 \ \forall x \in \partial\Omega \ (boundary) \\ > 0 \ \forall x \in \Omega^{+} \ (solid\ region) \\ < 0 \ \forall x \in \Omega^{-} \ (void\ region) \end{cases}$$

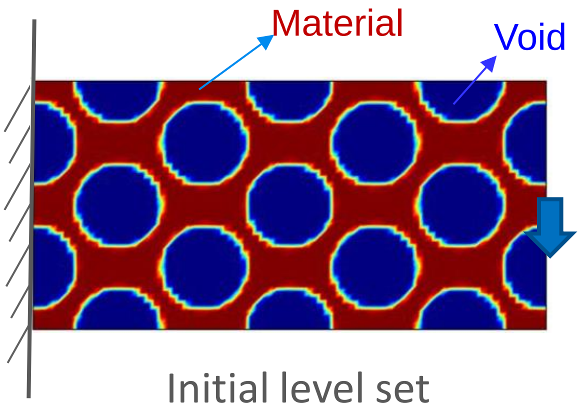
Hamilton-Jacobi equation $$\frac{\partial \psi}{\partial t} = -v|\nabla\psi| - \omega g$$

Velocity of propagation

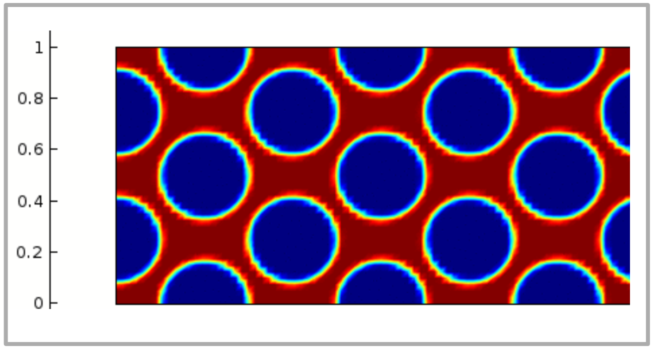
Nucleation of new holes

TO with the Level–set method

- Advantages
  - Accurate prediction of interphases
  - No pressure diffusion in fluid flow problems (in XFEM & Conformal Mapping)
- Compared to Density method convergence of Level-set method is slow

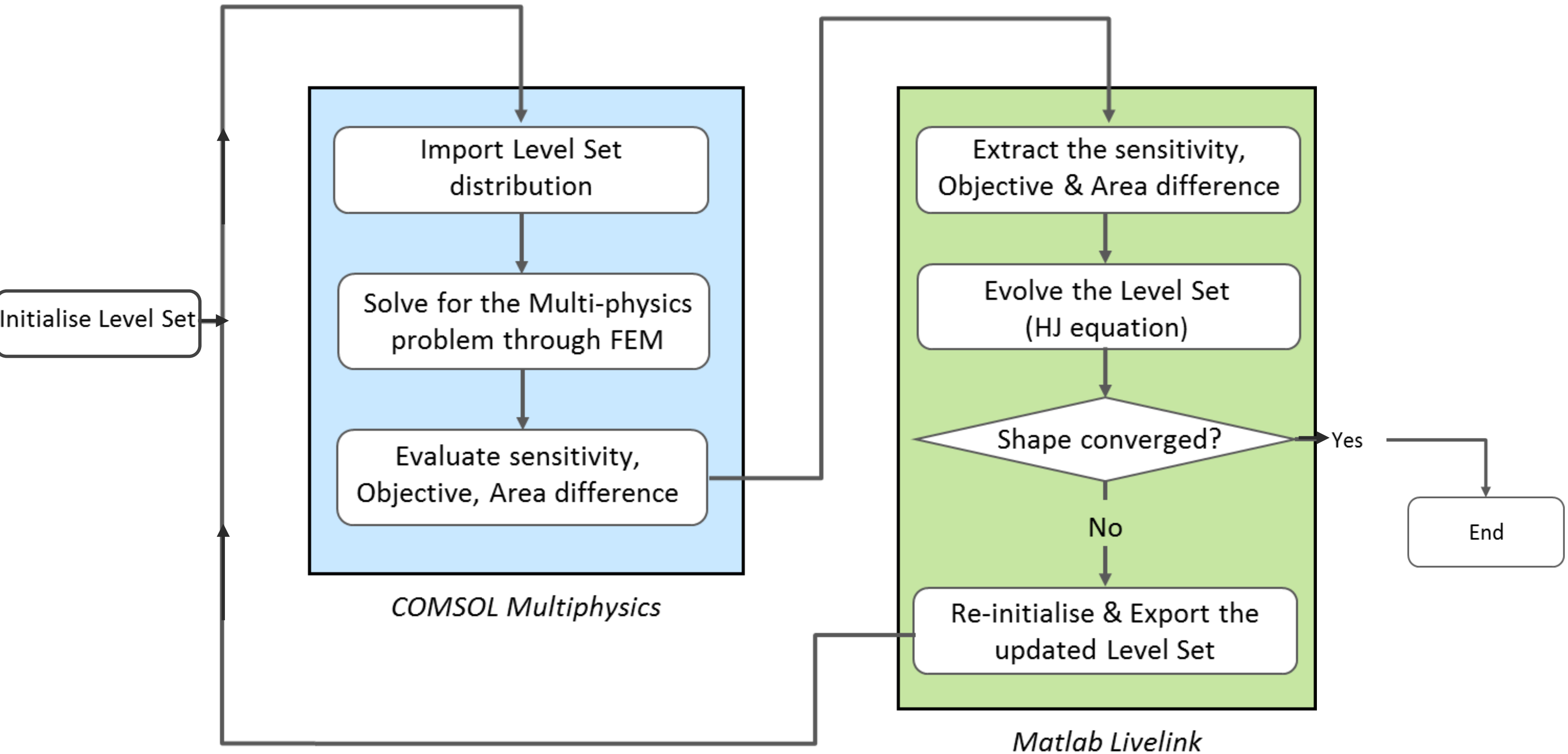


Initial level set distribution on cantilever beam

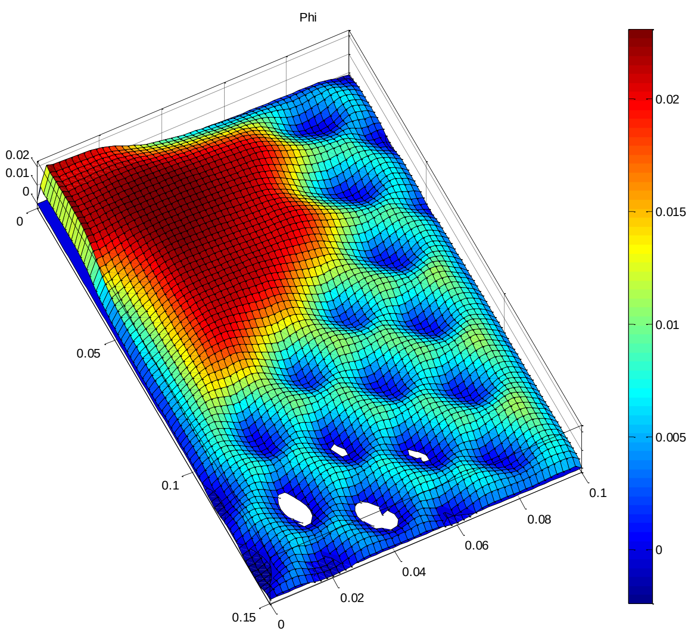


Optimised Shape

Level Set TO - Numerical Implementation



Re-initialisation of Level sets



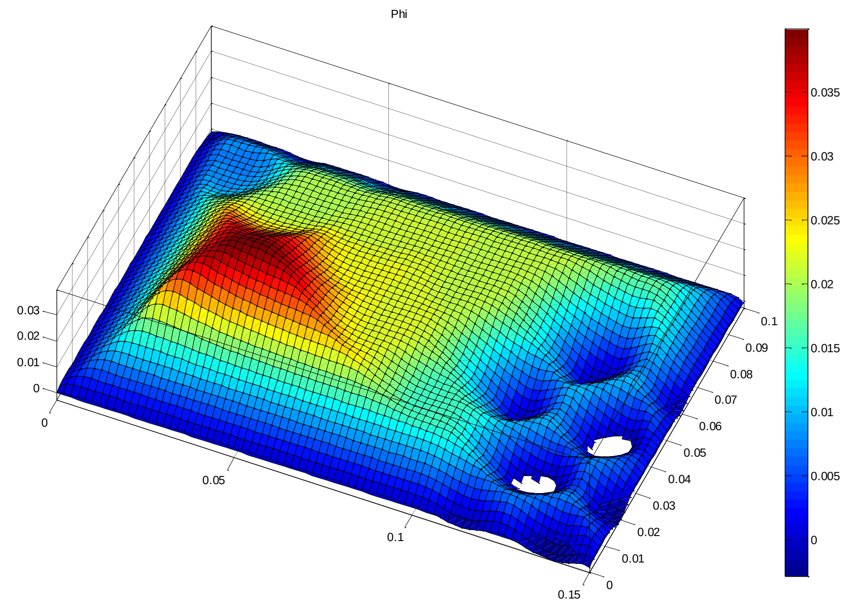
Level Set after few TO iterations, before Re-initialisation

Eikonal Equation

$$\frac{\partial \psi}{\partial t} + w.\nabla\psi = S(\psi o)$$

$$w = S(\psi o)\frac{\nabla\psi}{|\nabla\psi|}$$

where S is smoothed sign function



Level Set after Re-initialisation

Heat sink Design: Problem Formulation

Objective: Thermal Compliance: $\min \int_{\Omega} kgam * [(\nabla T)^2] d\Omega$

Viscous Dissipation: $\min \mu \int_{\Omega} (\frac{\partial u_i}{\partial x_j})^2 \, d\Omega$

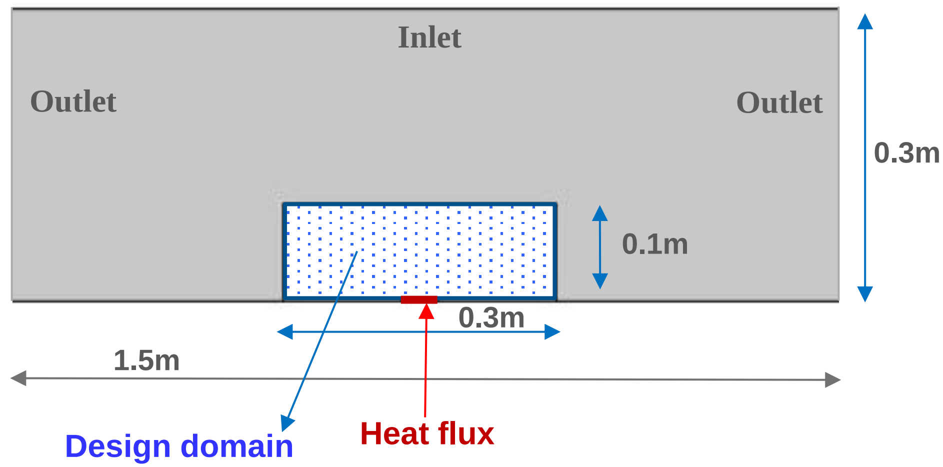
Governing Eqns:

$\rho Cp(u. \quad T) = \nabla.(k\nabla T) + Q$

$(\nabla . u) = 0$

$(u. \quad u) = -\nabla p + \quad .\{\mu\{\nabla u + (\nabla u)^T\}\} - \alpha u$

$H(\Psi)u = 0$



Variable	Expression
Kgam	$(K_s - K_f)*H + K_f$
Cpgam	$(Cp_s - Cp_f)*H + Cp_f$
ρgam	$(\rho_s - \rho_f)*H + \rho_f$
α	$(\alpha max - \alpha min)*H + \alpha min$

Material: k_f/k_s=0.001 High conductivity solid
k_f/k_s=0.1 Low conductivity solid

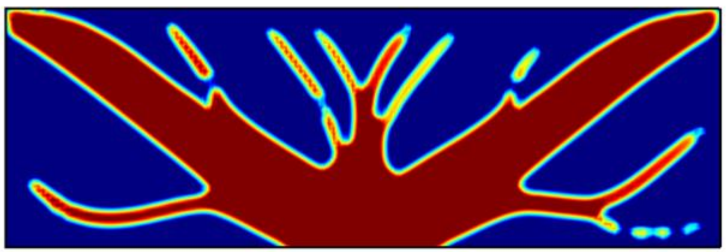
Heat flux =700W/m2

Reynolds number=600

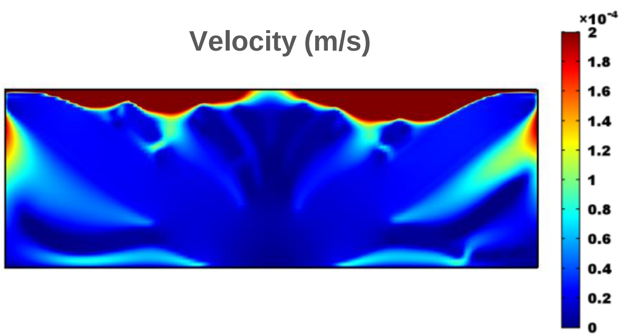
Constraints: 40% volume constraint for solid material

High conductivity solid- Results

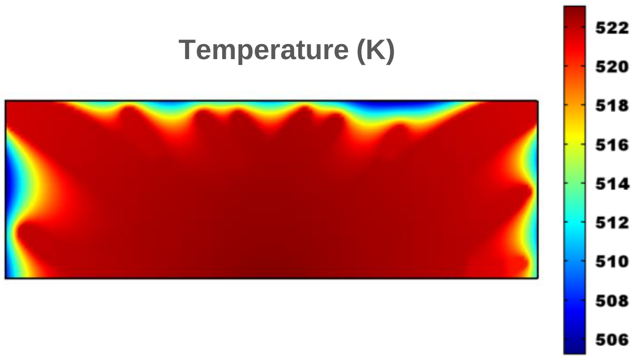
Design variable

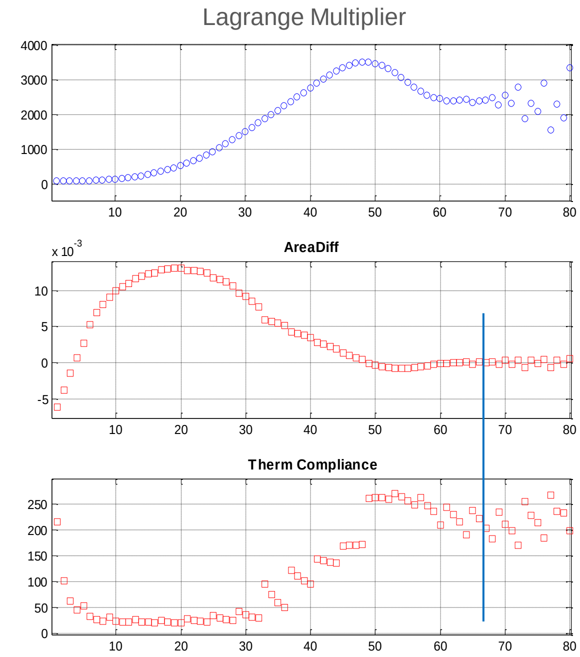


Velocity (m/s)



Temperature (K)



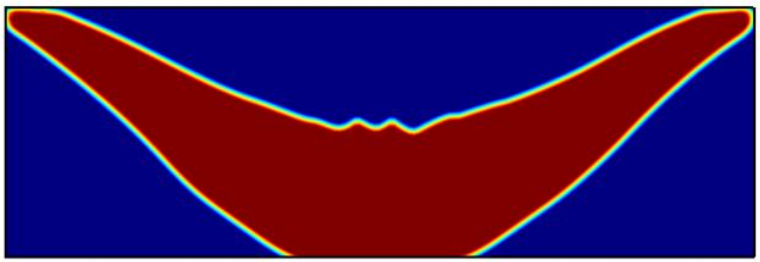


Iteration = 67
Area Difference= 2.4384e-05
Thermal Compliance= 202.51
Max Temperature=523.10K

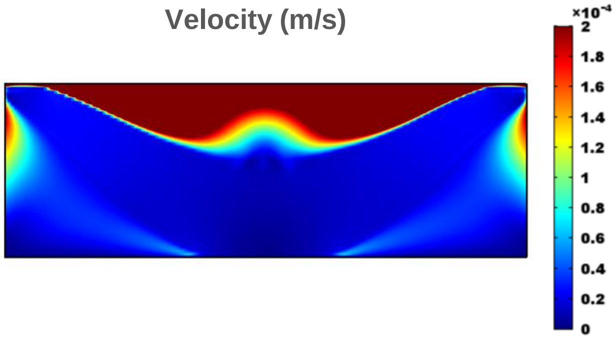
- Heat sink has tree like/Dendritic shape
- Temperature is uniformly distributed throughout the design domain

Low conductivity solid- Results

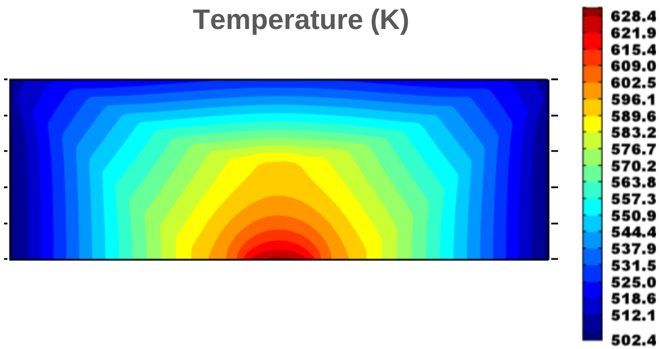
Design variable

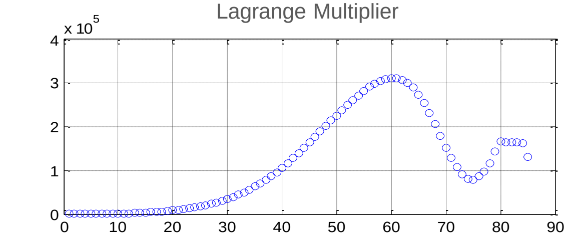


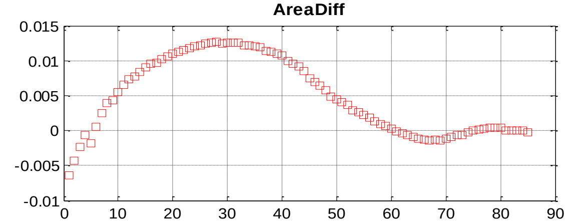
Velocity (m/s)

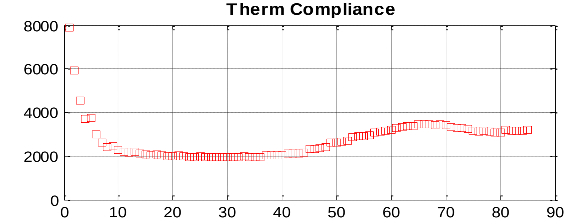


Temperature (K)







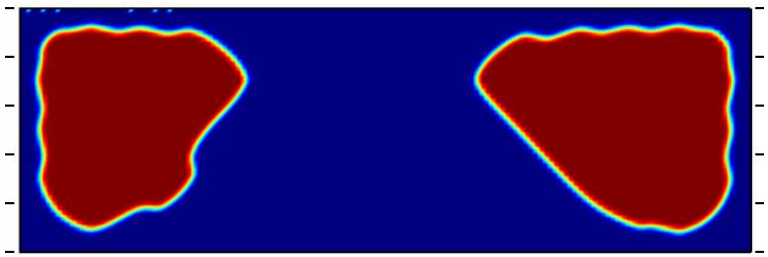


Iteration = 83
Area Difference= 6.5897e-07
Thermal Compliance= 3154.40
MaxTemperature=631.59 K

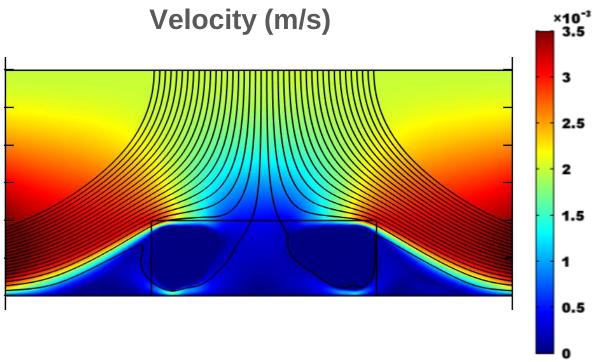
- Secondary branches have disappeared for low conductivity solid

Minimum Viscous Dissipation - Results

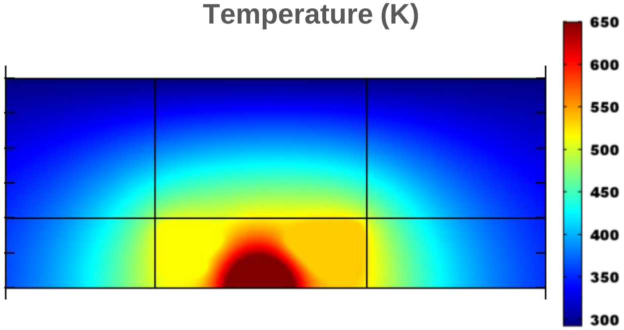
Design variable

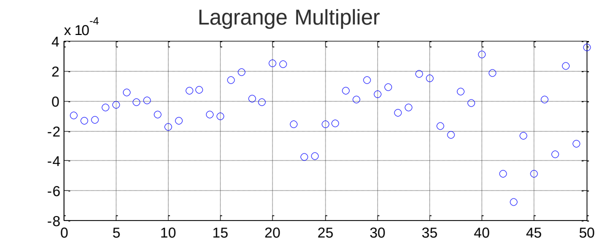


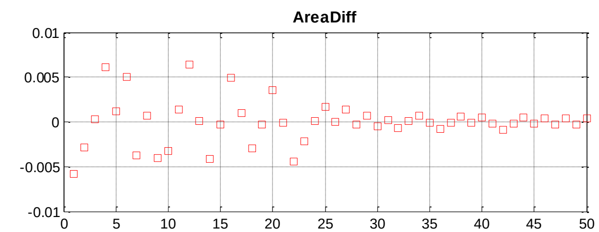
Velocity (m/s)

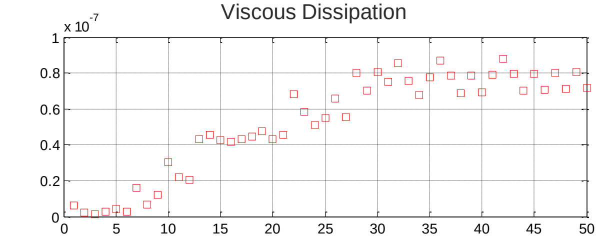


Temperature (K)







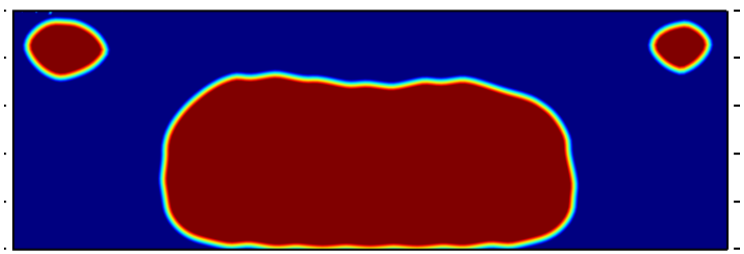


Iteration:43
Area Difference= 2.3e-4
Viscous Dissipation= 7.9642e-8

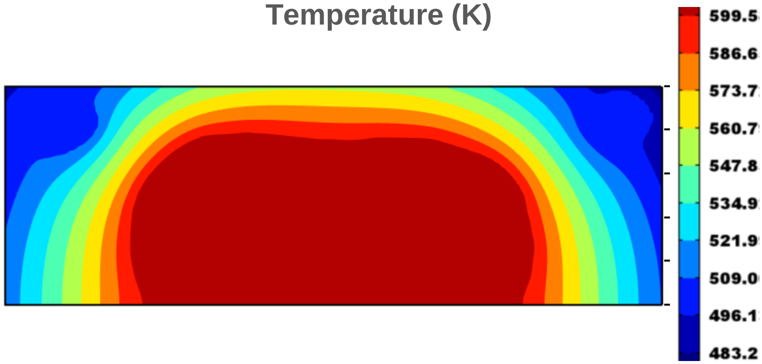
- Viscous Dissipation objective leads to a shape guiding the flow with least resistance

Combined Thermal Compliance and Viscous Dissipation

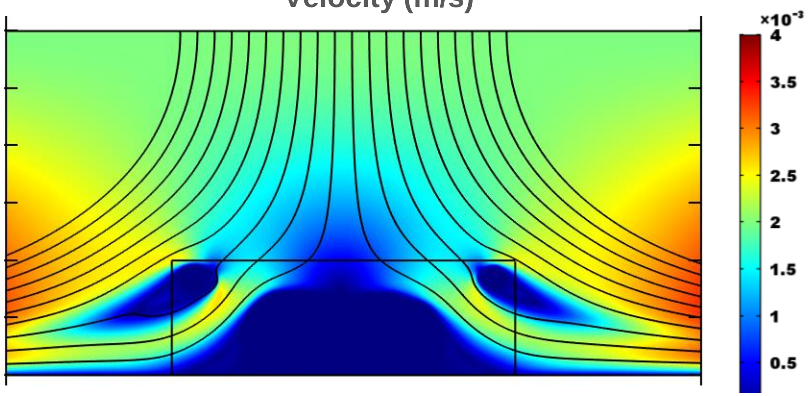
Design variable



Temperature (K)



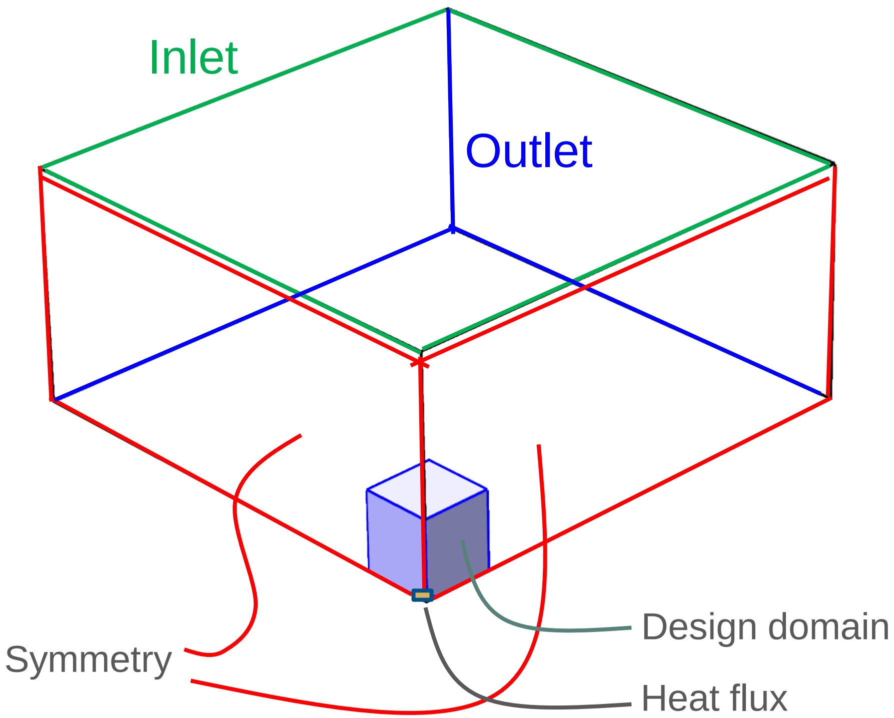
Velocity (m/s)



(F1,F2)	Thermal Compliance (WK/m)	Viscous Dissipation (N/s)
(0, 1)	14197.6	7.9642e-8
(1e-9,1)	2357.1	8.8307e-8

- Combined objective, tries to minimize both Thermal Compliance & Viscous Dissipation.

Three dimensional Heat sink design



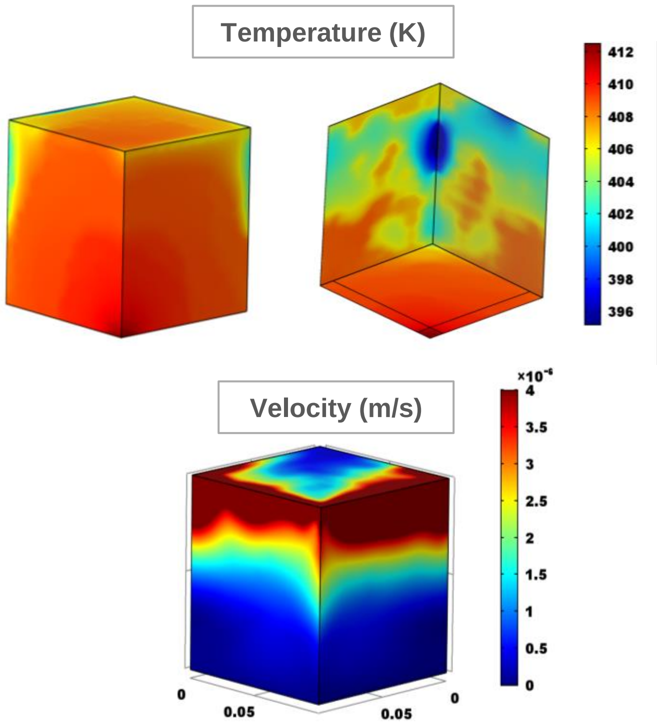
Computational Domain

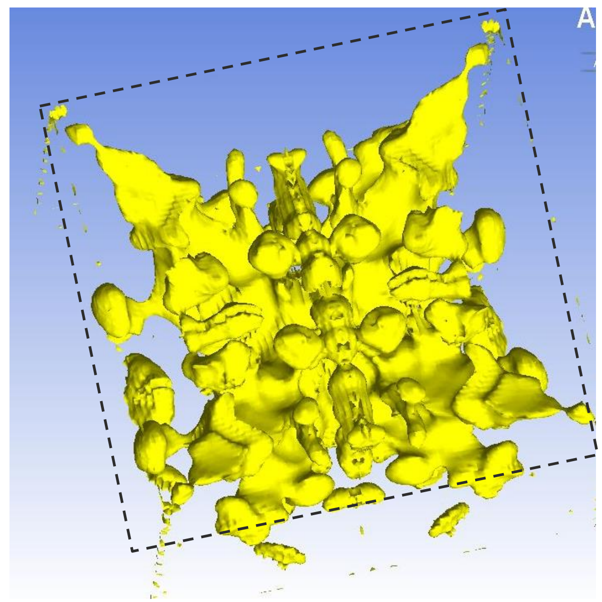
- Design domain of size 0.1x0.1x0.1m is discretised with 43x43x43 hexahedral cells
- Material: K_f/K_s=0.001 (High conductivity solid)
- Heat flux=1000W/m^2
- Re=8 (vel=4e-5m/s)

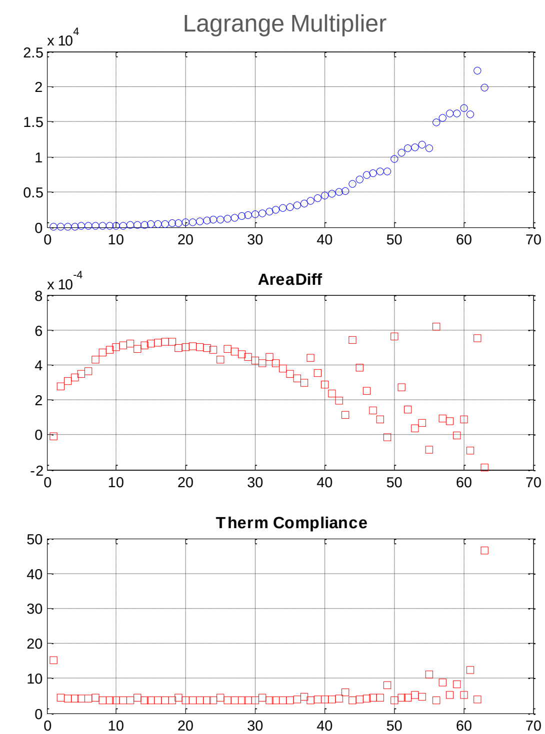
Objective: Minimizing the thermal compliance

Constraints: 25% volume constraint for solid material

Three Dimensional High conductivity solid - Results







Therm Compliance=8.2257
Max Temp= 412.519 K

- Tree like structure with primary branches starting from heat source reaching to corners of the domain.
- Use of symmetry condition & Global optimality of the shape needs to be verified

Conclusions

- Implemented Level set based Topology optimisation methodology with Re-initialisation in Comsol 5.2 using MATLAB Livelink® feature
- Demonstrated the application of this methodology for Heat sink designs for different objectives
- Heat sink for thermal compliance objective leads to Dendritic shape whereas for Viscous dissipation objective leads to solid shape guiding the flow with least resistance
- Three Dimensional Heat sink also designed for minimum Thermal Compliance Objective.
- Further research is needed to ensure the global optimality of the obtained shapes

Thank You

- **Email: m.santhanakrishnan@greenwich.ac.uk**

- **Web: cmrg.cms.gre.ac.uk**

Funded through University of Greenwich Vice Chancellor's Scholarship scheme

Additional slides

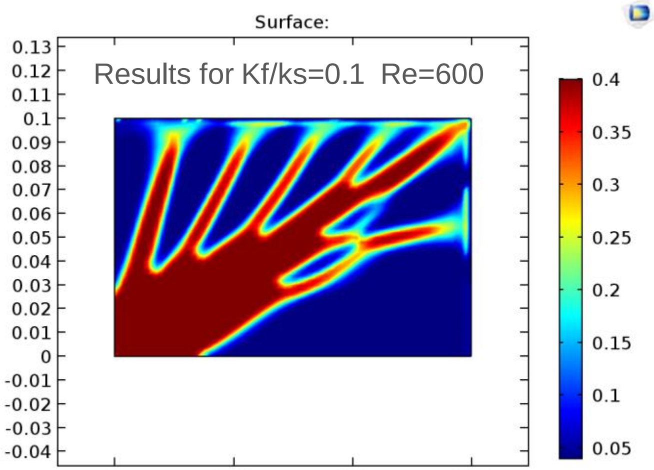
Results Comparison

Re	Pr of fluid	Kf/Ks	Heatflux	Coupled Level Set	SIMP	Re-initialized LS
60	105	0.001	700	29.5 ★ T=509K	43.7 (g=0.2) 510K	202.506 ★ 523.1K
60	105	0.1	700	2687 T=606K	2569 (g=0.7) Temp:618K	3154.4 631.59K

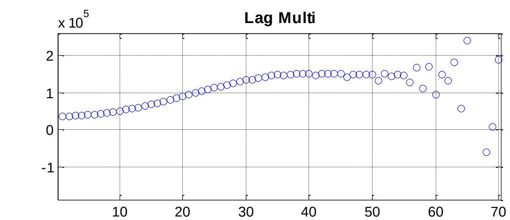
	SIMP TO	Coupled LS	Re-initialised LS
Thermal compliance (kgm^2K/s^3)	6.518	2.05 ★	8.2257 ★
Maximum Temperature (K)	383.9	378.58	412.52

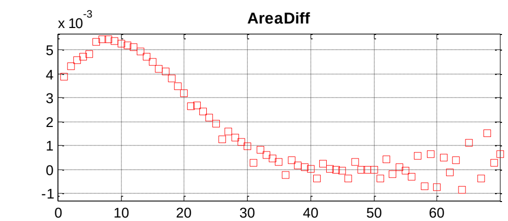
- Coupled LS results show lower objective value than re-initialised LS due to presence of grey cells.
- CFD study on optimal shapes are required to validate the results

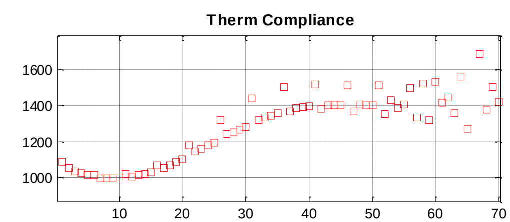
Results with Symmetry Boundary condition

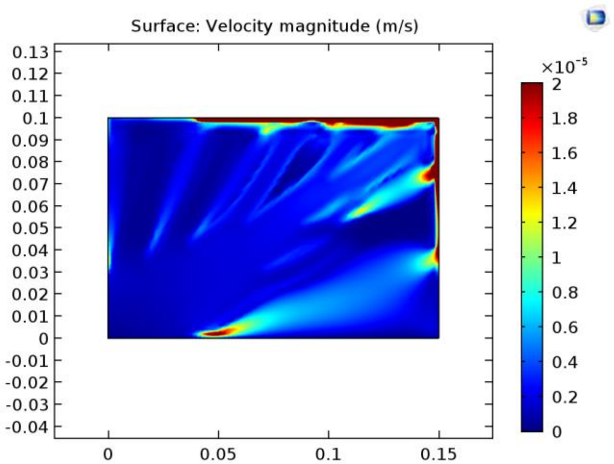


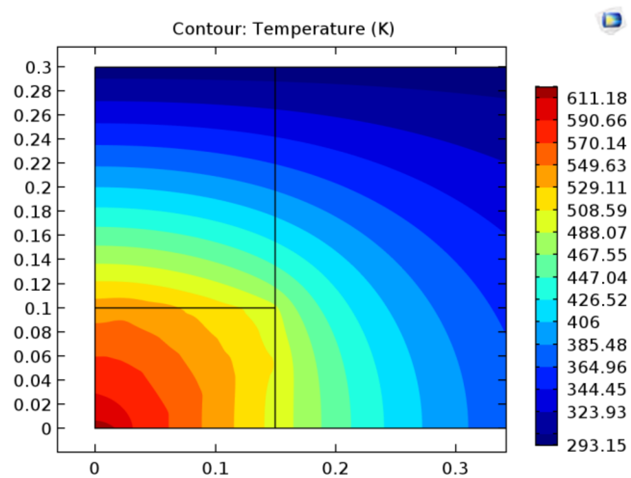
Lm_area=0.00598 (-0.3%)
Thermcomp=1400.10
Max T=621.44k
Limiter=950,000



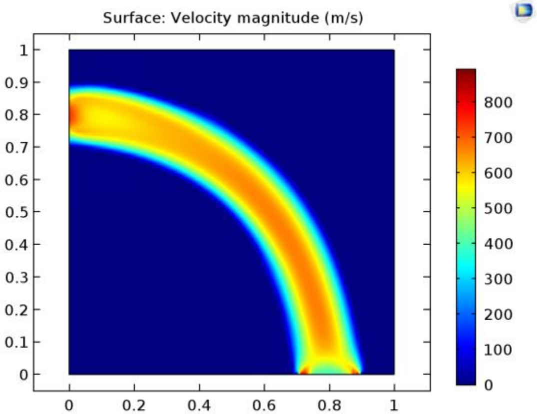


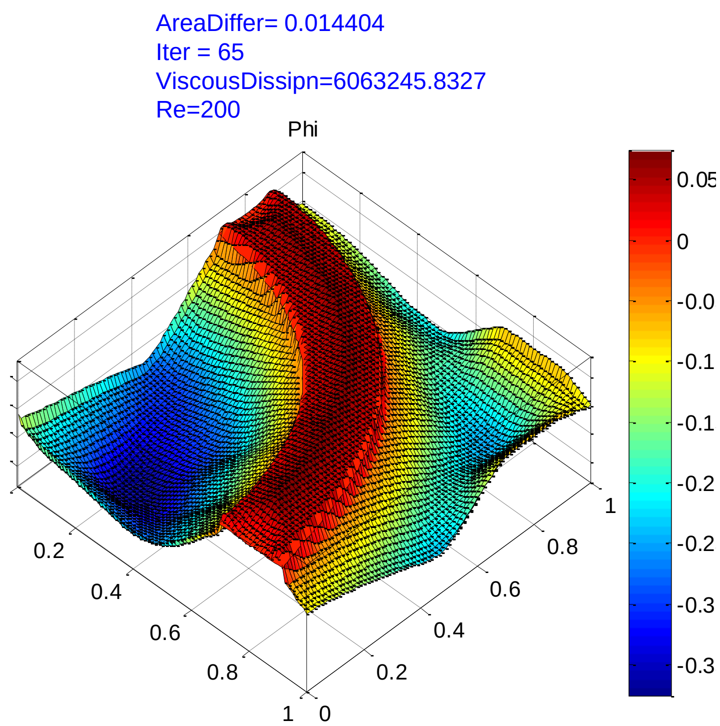


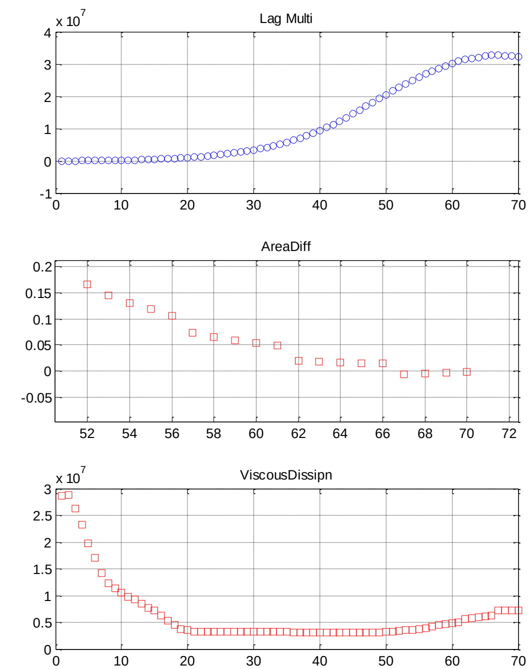




Low Reynolds number Duct flow

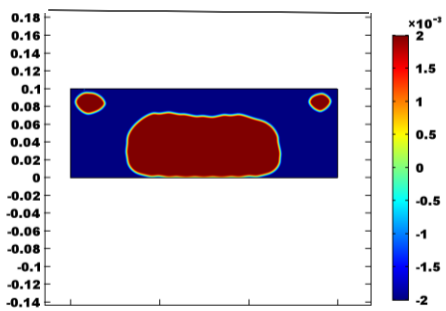


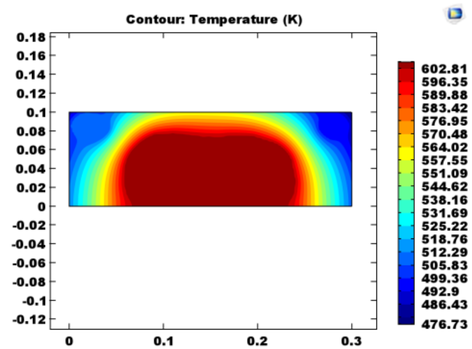


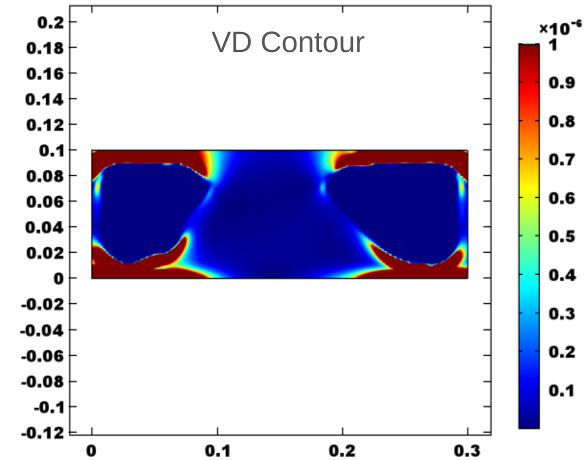


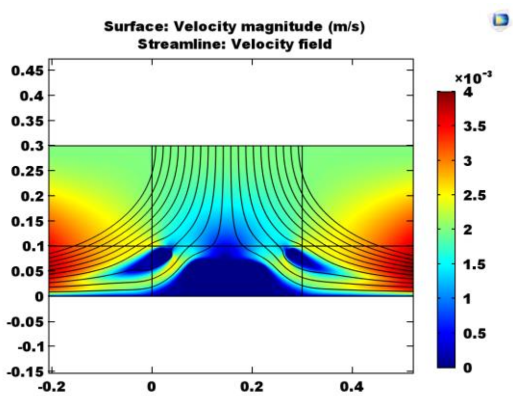
- Initial Values: if(H<1,0[m/s], v1) & Additional ‘ leaked Wall’ condn with No surfaces selected. (**Leaked wall condn not necessary; *Lsnoleakwall.mph**)

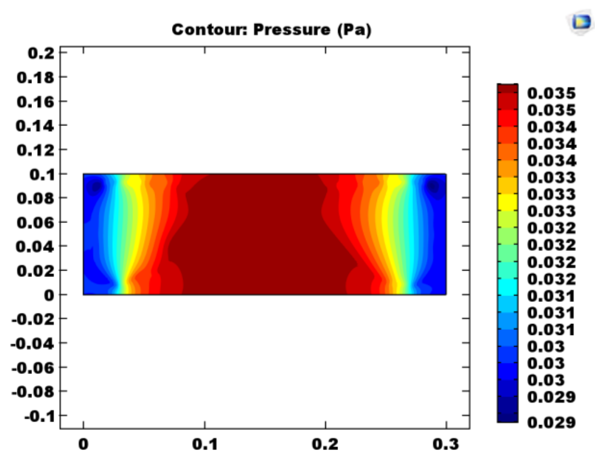
- Setting this initial value imposes noslip on solid walls
- Also corrected the force (alpha) term

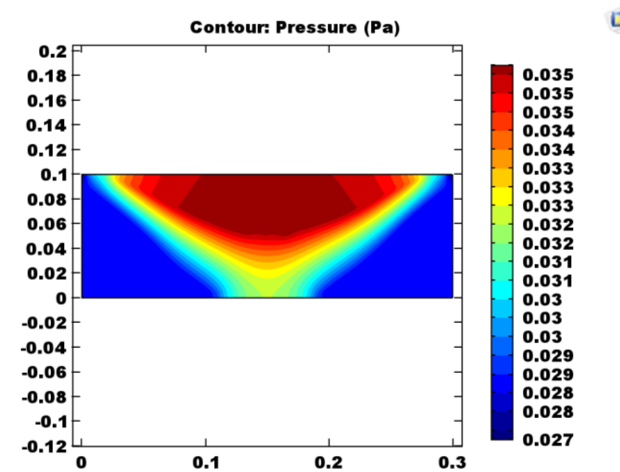


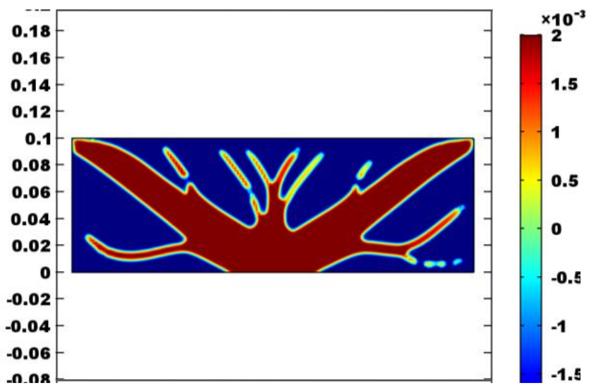

Contour: Temperature (K)

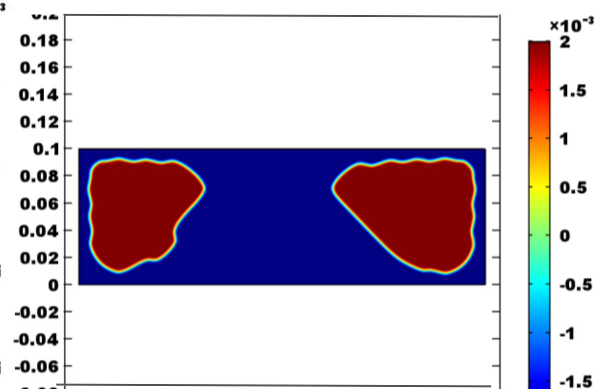

VD Contour

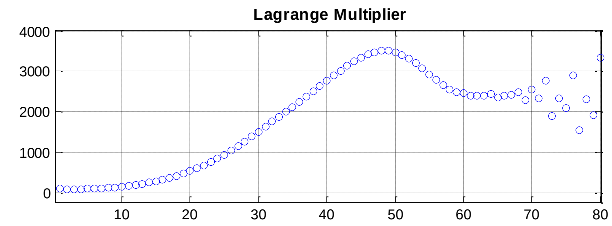

Surface: Velocity magnitude (m/s)
Streamline: Velocity field

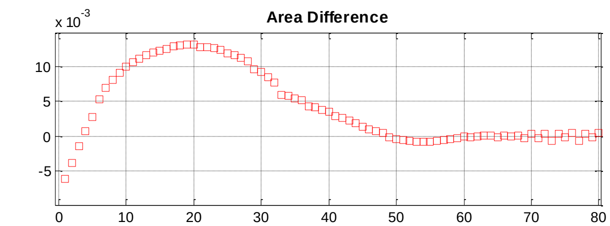

Contour: Pressure (Pa)

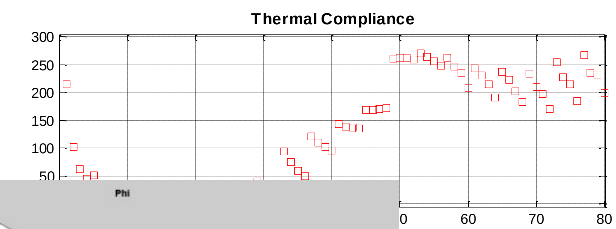

Contour: Pressure (Pa)

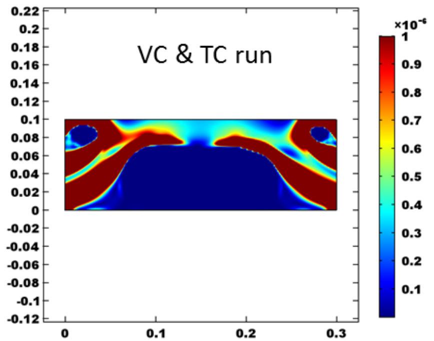


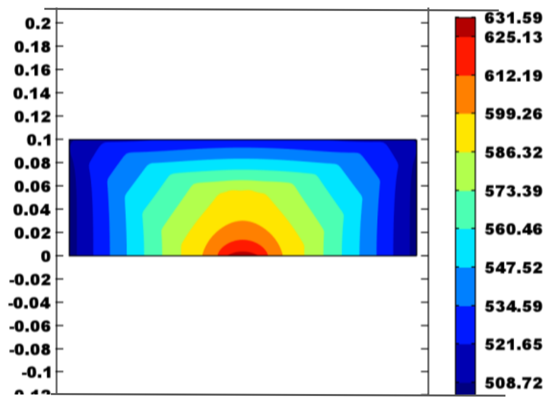


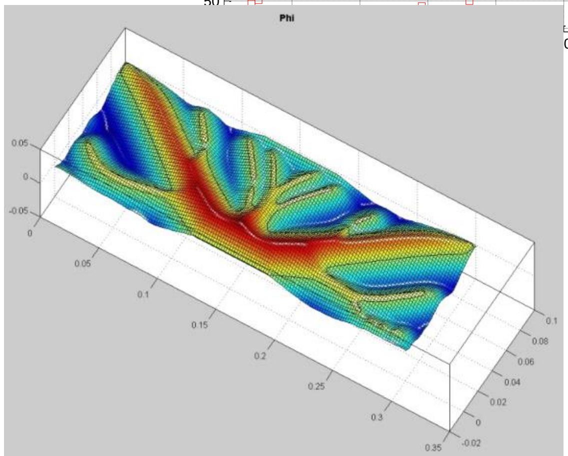

Lagrange Multiplier


Area Difference
x 10^-3


Thermal Compliance


VC & TC run




Phi

Level Set TO - Numerical Implementation