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ANALYSIS OF FORCES ACTING ON SUPERPARAMAGNETIC BEADS IN FLUID MEDIUM IN GRADIENT MAGNETIC FIELDS

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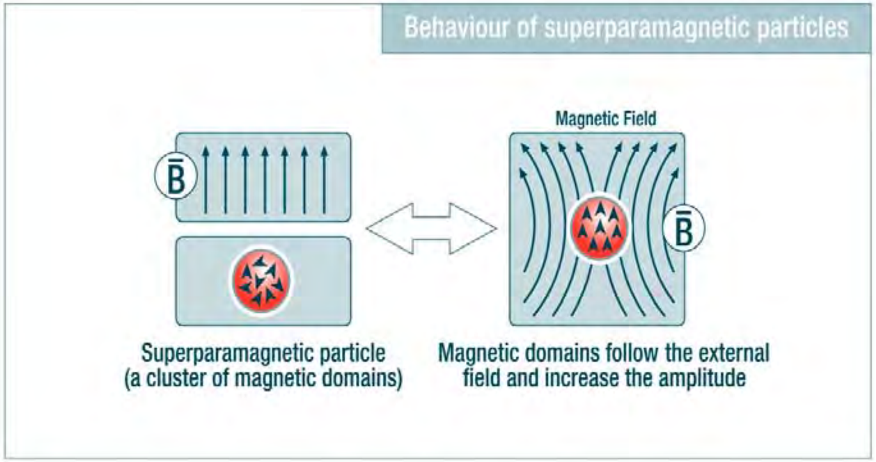
OUTLINE

- Introduction
- COMSOL Multiphysics
- Governing Equations
- Geometric Modeling
- Results
- Conclusions

INTRODUCTION

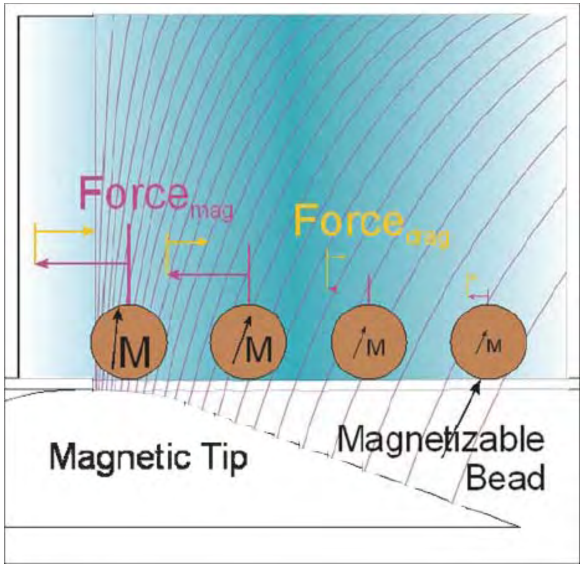
- Organization of superparamagnetic beads into crystalline structures.
- Wide use in both research and clinical medical diagnostic applications
- Two key properties
  1. Small in diameter
  2. Superparamagnetic



http://magnasense.com/index.php?page=superparamagnetic-nanospheres

INTRODUCTION - Mechanism

- As the magnetizable particle moves into the magnetic gradient, the bead magnetization increases, and the bead moves more rapidly, increasing the drag.



$$\text{Force}_{mag} = (\vec{m}.\nabla)\vec{B}$$

$$\text{Force}_{drag} = -bv$$

$$\text{Resultant} = \text{Force}_{mag} - \text{Force}_{drag}$$

INTRODUCTION – Magnetic Organization

Parameters effecting the magnetic organizations

1. Strength of the field.
2. Gradient of the magnetic field.
3. Magnetic and drag forces acting on the beads.

COMSOL Multiphysics

The model in this work consists of the following coupled applications:

- Magnetostatics
- Incompressible Navier Stokes
- Moving Mesh ALE

GOVERNING EQUATIONS

Magnetic Forces acting on the bead

Attraction Forces ← Magnetic Force → Interaction Forces

$$\frac{V\chi_{bead}}{\mu_0}(\vec{B}.\nabla)\vec{B}$$

$$\rho V(\overrightarrow{M}_0.\nabla)\vec{B}$$

$$\vec{F} = \rho V(\overrightarrow{M}_0.\nabla)\vec{B} + \frac{V\chi_{bead}}{\mu_0}(\vec{B}.\nabla)\vec{B}$$

$\overrightarrow{M}_0$ is the initial magnetization of the bead [Am^2/kg],

V is the volume of the particle [m^3],

χ_{bead} is the magnetic susceptibility of the bead,

μ_0 is the permeability of the vacuum [H/m],

$\vec{B}$ is the applied magnetic field [T].

GOVERNING EQUATIONS

Magnetic forces acting on the beads

Attraction Forces ← Magnetic Force → Interaction Forces

$$\vec{F} = \rho V(\overrightarrow{M}_0.\nabla)\vec{B} + \frac{V\chi_{bead}}{\mu_0}(\vec{B}.\nabla)\vec{B}$$

$$\overrightarrow{F_m} = \rho V\begin{bmatrix} M_{0x}\dfrac{\partial B_x}{\partial x} + M_{0y}\dfrac{\partial B_x}{\partial y} + M_{0z}\dfrac{\partial B_x}{\partial z} \\ M_{0x}\dfrac{\partial B_y}{\partial x} + M_{0y}\dfrac{\partial B_y}{\partial y} + M_{0z}\dfrac{\partial B_y}{\partial z} \\ M_{0x}\dfrac{\partial B_z}{\partial x} + M_{0y}\dfrac{\partial B_z}{\partial y} + M_{0z}\dfrac{\partial B_z}{\partial z} \end{bmatrix} + \frac{V\chi_{bead}}{\mu_0}\begin{bmatrix} B_x\dfrac{\partial B_x}{\partial x} + B_y\dfrac{\partial B_x}{\partial y} + B_z\dfrac{\partial B_x}{\partial z} \\ B_x\dfrac{\partial B_y}{\partial x} + B_y\dfrac{\partial B_y}{\partial y} + B_z\dfrac{\partial B_y}{\partial z} \\ B_x\dfrac{\partial B_z}{\partial x} + B_y\dfrac{\partial B_z}{\partial y} + B_z\dfrac{\partial B_z}{\partial z} \end{bmatrix}$$

GOVERNING EQUATIONS

Drag Force acting on the beads

The equation for drag force is given as $F_{drag} = -6\pi\mu r v$

where,

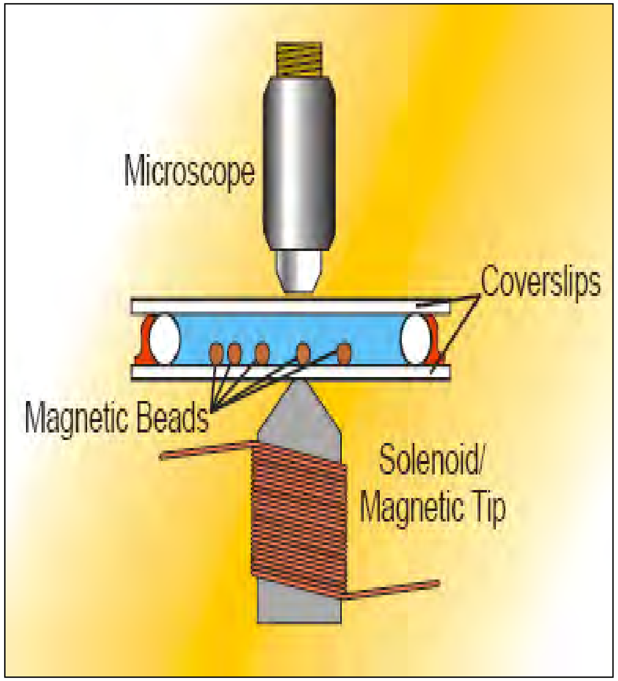
v is the velocity of the bead [m/s]

μ is the dynamic viscosity of the fluid medium (Pa.s)

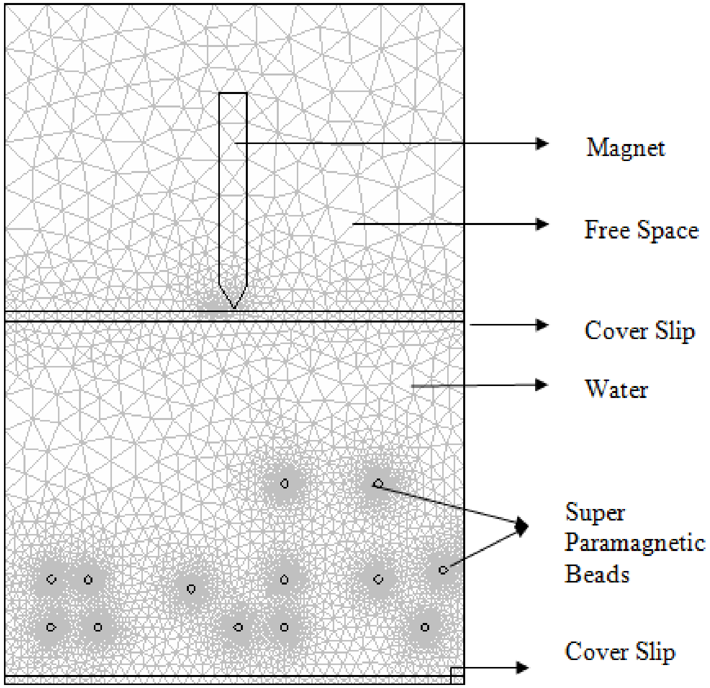
Finally, the velocity v of a particle described by the differential equation

$$\frac{dv}{dt} = \frac{F_{mag} - F_{drag}}{m_{par}}$$

GEOMETRIC MODELING

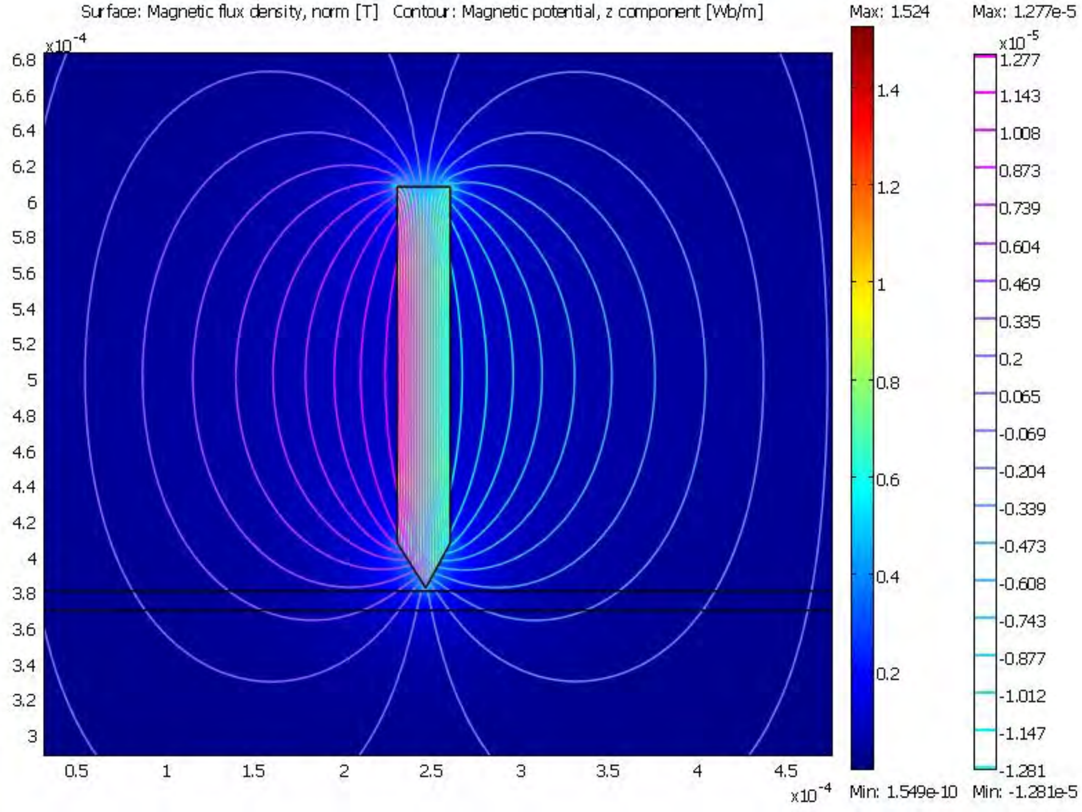


Experimental setup



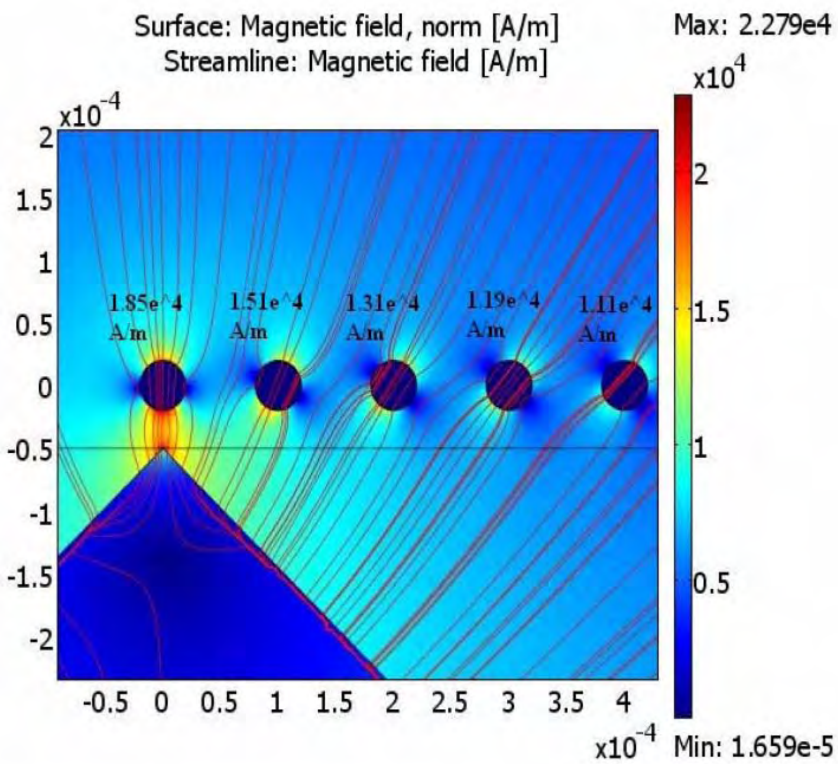
Geometric Modeling and Meshing

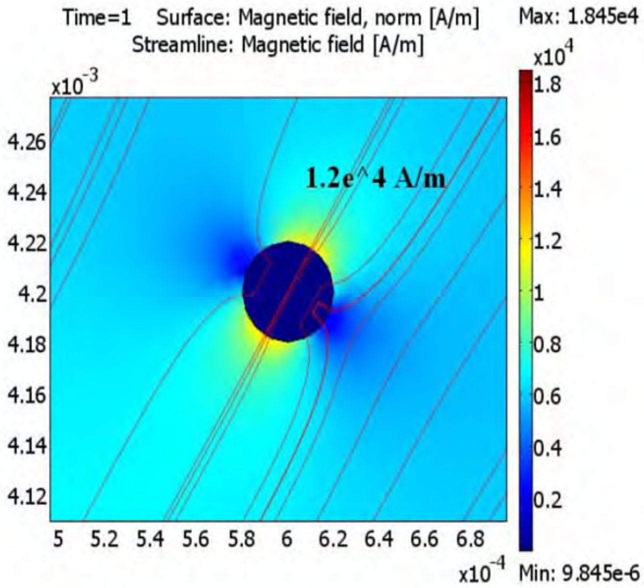
RESULTS - Magnetostatics



Surface plot showing the magnetic flux density and contour plot showing the magnetic potential, Z component

RESULTS - Magnetostatics

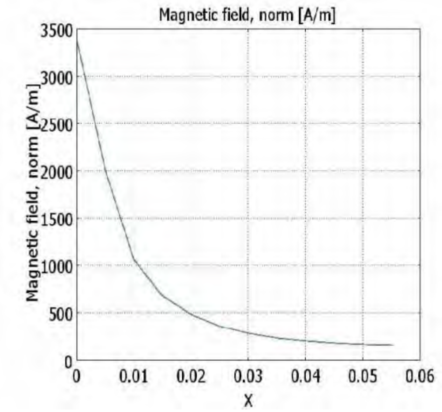


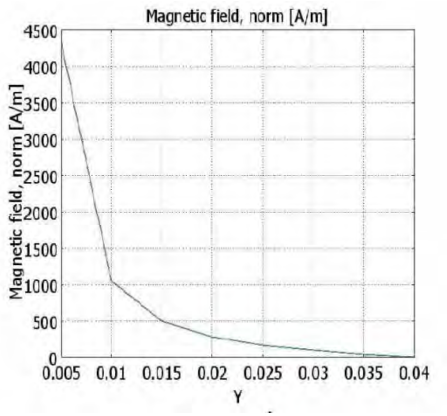


Variation of Magnetic field on the beads with respect to distance

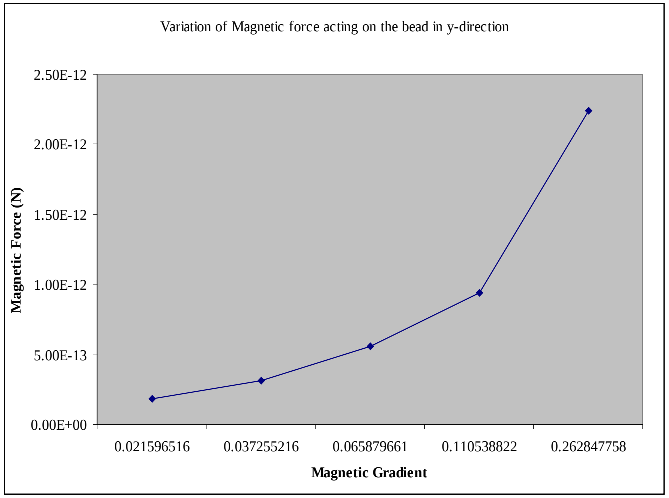
Variation of Magnetic Flux Density across the bead

RESULTS – Magnetostatics - Graphs



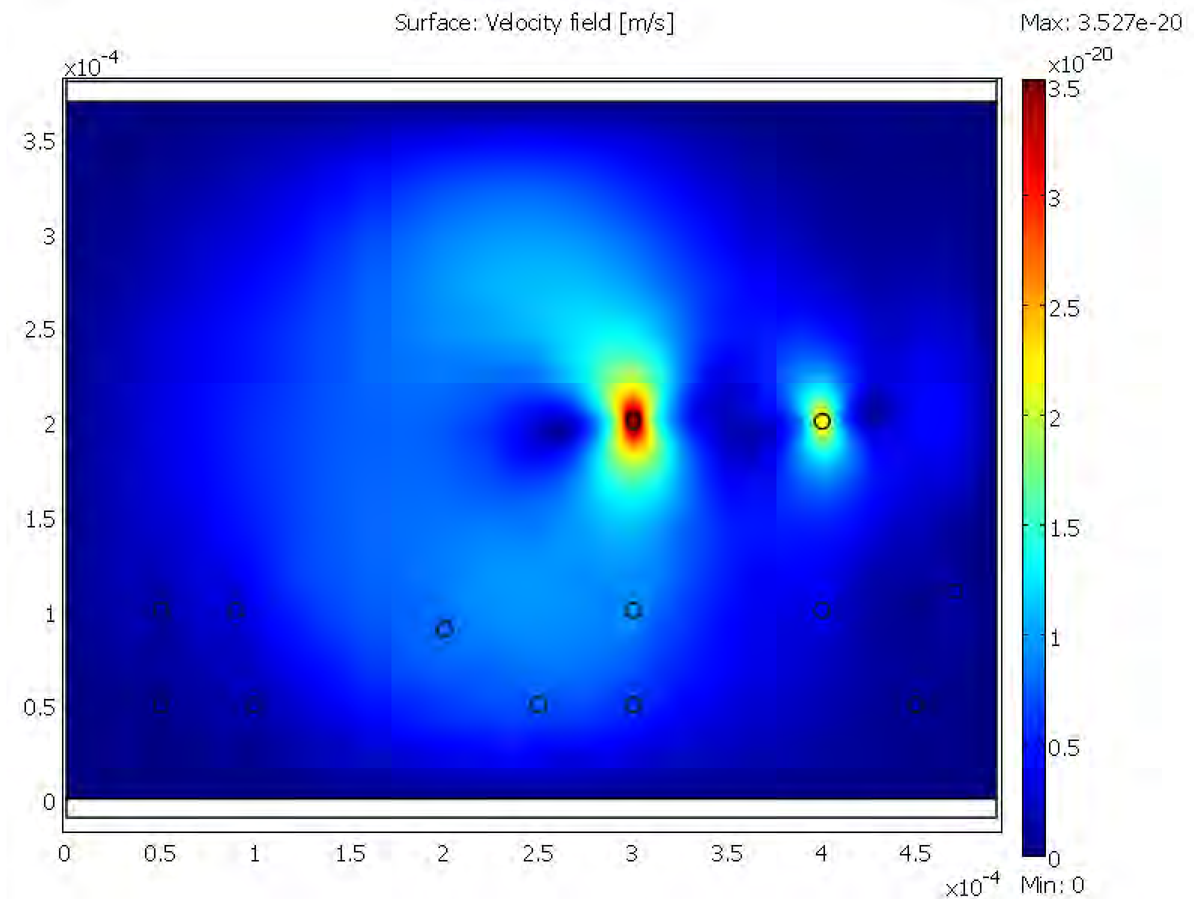


Variation of the magnetic field across the x and y directions



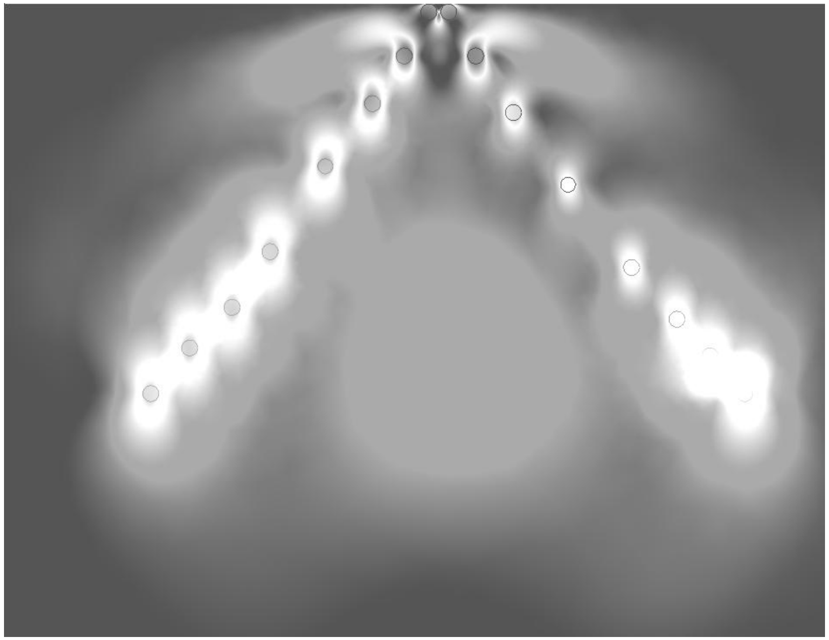
Variation of Magnetic Force along the Magnetic Gradient acting on the bead

RESULTS – Incompressible Navier Stokes

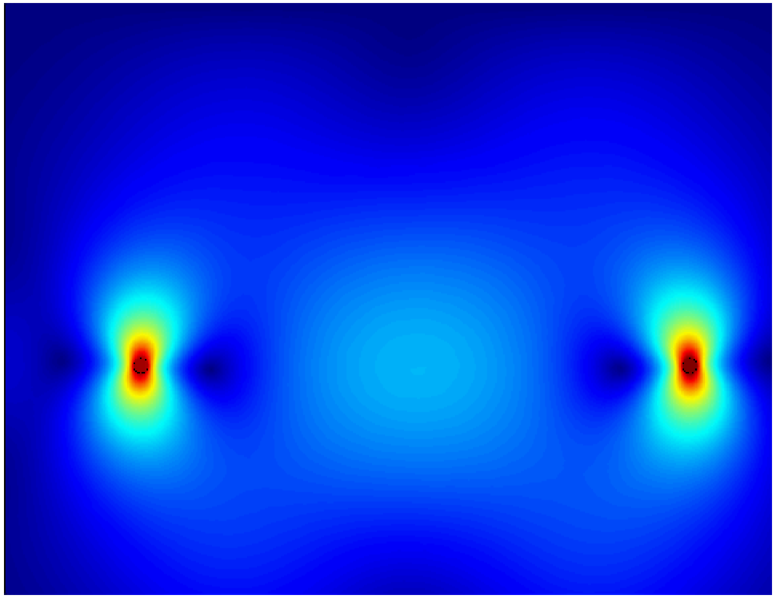


Surface plot showing the velocity field created around the beads

RESULTS – Moving Mesh ALE



Displacement of two beads to a higher gradient point



Movie

CONCLUSIONS

- Beads organization in 2D.
- Magnetic Field and response of beads.
- Fluid behavior.
- Magnetic attraction and interaction forces, Drag forces in x and y directions

Future Work

- Greater number of beads
- 3D modeling

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