Fast Computation of Capacitance Matrix and Potential Distribution for Multiconductor in Non-Homogenous Multilayered Dielectric Media

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Abstract: This paper presents the fast computational and modeling of multiconductor transmission lines interconnect in non-homogenous multilayered dielectric media using the finite element method (FEM). We illustrate the potential distribution of the multiconductor transmission lines for the models and their solution time. We mainly focus on designing of four-transmission lines embedded in two-layered dielectric media and six-transmission lines interconnect in three-layered dielectric media. We compared some of our results of computing the capacitance matrix with method of moment (MoM), method of lines (MoL), and semi-analytic Green's function (SAGF) method and found them to be closed.

Keywords: Capacitance per unit length, Multicondcutor transmission lines, Finite element Method, Multilayered dielectric media, Potential distribution

1. Introduction

Recently, the increases of advances of integrated circuits and multichip modules attracts researchers and designers to investigate the effectiveness of electromagnetic compatibility of the per-unit-length capacitances matrices for multilayer and multiconductor interconnects in very high-speed digital circuits. The management of the on-chip interconnects with respect to the internal parasitic parameter immunity is important for the IC designers. Therefore, the optimization of the electrical properties of IC using the estimation of capacitance matrix of the multilayer and multiconductor interconnects in very high speed ICs is essential to calculate.

Multiconductor transmission lines embedded in multilayered dielectric lossy media have been analyzed in several methods include the method of moments (MoM) [1-2], method of lines (MoL) [3-6], semi-analytic Green's function (SAGF) method [7], spectral domain analysis (SDA) [8-9], and boundary element method (BEM) [10].

We use COMSOL, a finite element multiphysics package, in designing the four-transmission lines embedded in two-layered dielectric media and six-transmission lines interconnect in three-layered dielectric media. The FEM is especially suitable and effective for the computation of electromagnetic fields in strongly inhomogeneous media. Also, it has high computation accuracy and fast computation speed. We show that FEM is as suitable and effective as other methods for modeling multiconductor transmission lines VLSI circuits.

We compared some of our results of computing the capacitance-per-unit length matrix with those in the literature. We specifically compared the modeling of designing of the structures with the MoM, MoL, and SAGF methods and found to be in agreement.

2. Results and Discussions

The models designed with finite elements are unbounded (or open), meaning that the electromagnetic fields should extend towards infinity. This is not possible because it would require a very large mesh. The easiest approach is just to extend the simulation domain "far enough" that the influence of the terminating boundary conditions at the far end becomes negligible. In any electromagnetic field analysis, the placement of far-field boundary is an important concern, especially when dealing with the finite element analysis of structures which are open. It is necessary to take into account the natural boundary of a line at infinity and the presence of remote objects and their potential influence on the field shape [11]. In all our simulations, the open multiconductor structure is surrounded by a W X H shield, where W is the width and H is the thickness.

The models are designed in 2D using electrostatic environment in order to compare our results with the other available methods. In the boundary condition of the model's design, we use ground boundary which is zero potential ($V=0$) for the shield. We use port condition for the conductors to force the potential or current to one or zero depending on the setting. Also, we use continuity boundary condition between the conductors and between the conductors and left and right grounds.

In this paper, we consider two different models. Case A investigates the designing of four-transmission lines embedded in two-layered dielectric media. For case B, we illustrate the modeling of six-transmission lines interconnect in three-layered dielectric media. The results from both models are compared with some other results in the literature such as MoM, MoL, and SAGF methods and found to be close.

The dimension of the coefficient capacitance matrix is proportional to the sum of widths of every dielectric layer and the parameters of all conductors. This results in long computing time and large memory especially when the structure to be analyzed has many layers and conductors [3].

We use one port at a time as the input to evaluate all the matrix entries. With the Forced Voltage method, the capacitance matrix entries are computed from the charges that result on each conductor when an electric potential is applied to one of them and all the others are set to ground. The matrix is defined as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \dots \\ Q_N \end{bmatrix} = \begin{bmatrix} C_{11}*V_1 + C_{12}*V_2 + \dots + C_{1N}*V_N \\ C_{21}*V_1 + C_{22}*V_2 + \dots + C_{2N}*V_N \\ \\ C_{N1}*V_N + C_{N2}*V_2 + \dots + C_{NN}*V_N \end{bmatrix}$$

For example, using port 2 as the input will provide the entries of the second column: C_{12}, C_{22}, ..., C_{N2}.

2.1 Four-transmission lines embedded in two-layered dielectric media

Figure 1 shows the cross section for four-transmission lines embedded in two-layered dielectric media with the following parameters:

ε_{r1} = dielectric constant of the dielectric material 1 = 9.5

ε_{r2} = dielectric constant of the dielectric material 2 = 4.65

ε_{r3} = dielectric constant of the free space = 1.0

W = width of the shield = 10mm
w = width of a single conductor line = 1mm
h = height of each of the dielectric materials = 1mm
H = height of the shield = 5mm
s = distance between the two coupled conductors = 1mm
t = thickness of the strips = 0.01mm

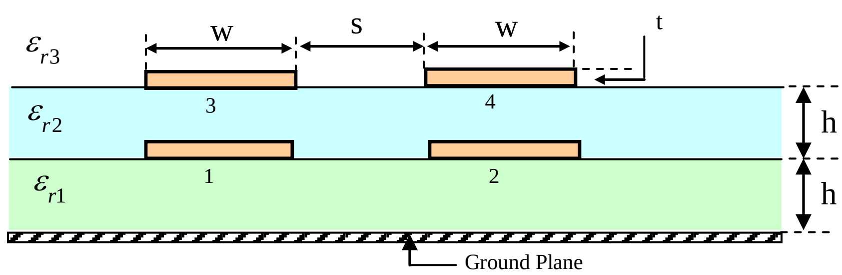

Figure 1. Cross section of four-transmission lines embedded in two-layered dielectric media.

The geometry is enclosed by a 10 X 5mm shield. From the model, we generate the finite elements mesh with 9,782 elements and number of degrees of freedom solved for 108, 999 with 23.422 seconds in solution time as in Figure 2. Figure 3 shows the contour plot of the potential distribution with port 1 as input. The potential distribution along the line that goes from (x,y) = (0,0) to (x,y) = (10mm, 5mm) with port 1 as input is show in Fig. 4. It shows that the approximate primary peak at 0.305 volts and secondary peak at 0.07 volts.

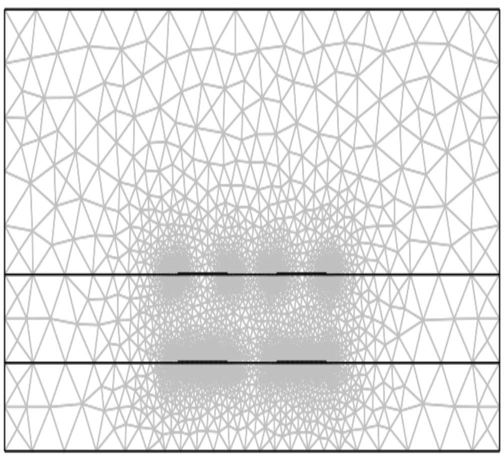

Figure 2. Mesh of four-transmission lines embedded in two-layered dielectric media.

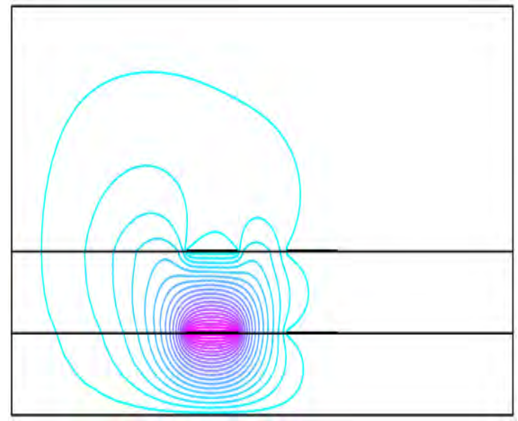

Figure 3. Contour plot of the potential distribution of four-transmission lines embedded in two-layered dielectric media with port 1 as input.

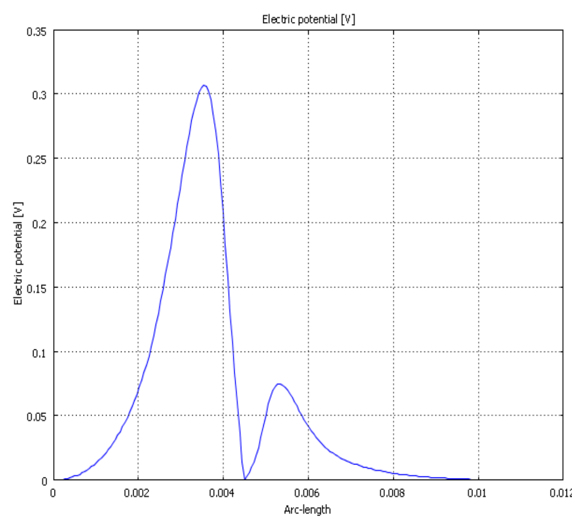

Figure 4. Potential distribution of four-transmission lines embedded in two-layered dielectric media using port 1 as input along a line from (x,y) = (0,0) to (x,y) =(10 mm, 5 mm).

Table 1 shows the finite element results for the capacitance-per-unit length of four-transmission lines embedded in two-layered dielectric media. It compares the results based on our work with those from MoM, MoL, and SAGF methods.

Table 1: Values of the Capacitance Matrix (in pF/m) for Four-Transmission Lines Embedded in Two-Layered Dielectric Media as Shown in Fig. 1

Capacitance per unit length	MoM[2]	MoL[3]	SAGF [7]	This work
C_{11}	216.70	222.68	216.91	233.43
C_{12}	-15.08	-15.06	-15.08	-18.73
C_{13}	-44.82	-45.62	-44.83	-48.09
C_{14}	-5.77	-5.96	-5.77	-6.44
C_{22}	216.70	222.68	216.91	233.45
C_{23}	-5.77	-5.96	-5.77	-6.45
C_{24}	-44.82	-45.62	-44.83	-48.09
C_{33}	81.08	83.44	81.25	88.19
C_{34}	-8.26	-8.21	-8.26	-9.09
C_{44}	81.08	83.44	81.25	88.19

2.2 Six-transmission lines interconnect in three-layered dielectric media

Figure 5 shows the cross section for the six-transmission lines interconnect in three-layered dielectric media with the following parameters:

ε_{r1} = dielectric constant of the dielectric material 1 = 2.0

ε_{r2} = dielectric constant of the dielectric material 2 = 3.0

ε_{r3} = dielectric constant of the dielectric material 3 = 4.0

ε_{r4} = dielectric constant of the free space = 1.0

W = width of the shield = 8mm
w = width of a single conductor line = 1mm
h = height of each of the dielectric materials = 1mm
H = height of the shield = 5mm
s = distance between the two coupled conductors = 1mm
t = thickness of the strips = 0.01mm

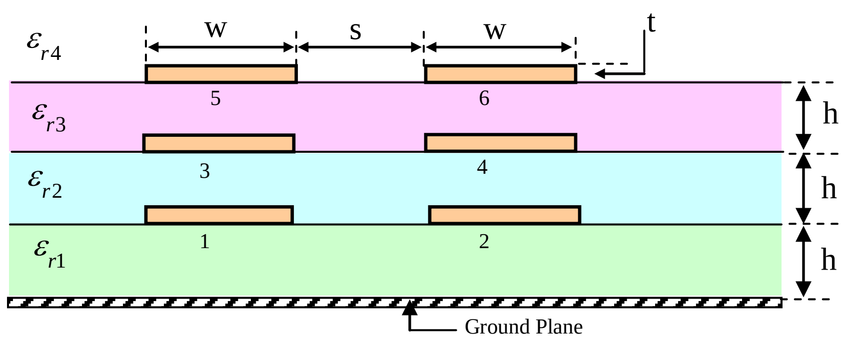

Figure 5. Cross section of six-transmission lines interconnects in three-layered dielectric media.

The geometry is enclosed by a 8 X 5mm shield. From the model, we generate the finite elements mesh with 11,274 elements and number of degrees of freedom solved for 125,851 with 25.469 seconds in solution time as in Figure 6. Figure 7 shows the contour plot of the potential distribution with port 1 as input. The potential distribution along the line that goes from (x,y) = (0,0) to (x,y) = (10mm, 5mm) with port 1 as input showing that approximate primary peak at 0.357 volts and secondary peak at 0.047 volts is portrayed in Figure 8.

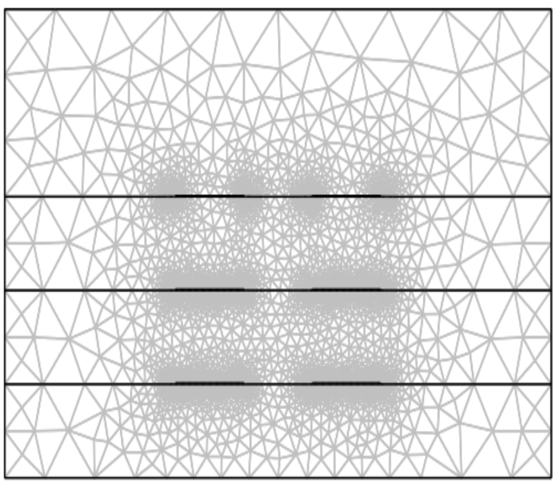

Figure 6. Mesh of the six-transmission lines interconnects in three-layered dielectric media.

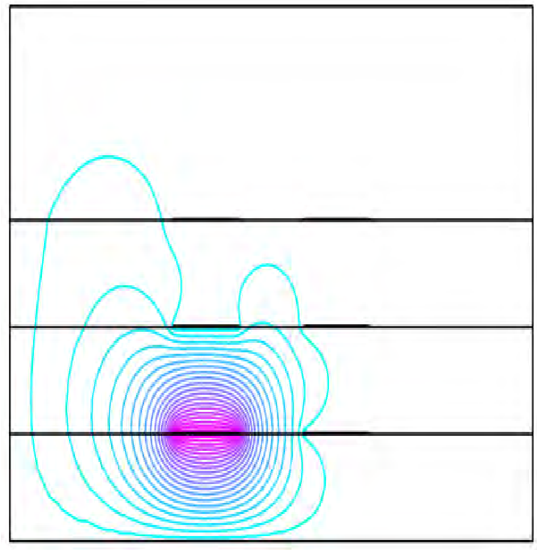

Figure 7. Contour plot of the potential distribution of the six-transmission lines interconnects in three-layered dielectric media with port 1 as input.

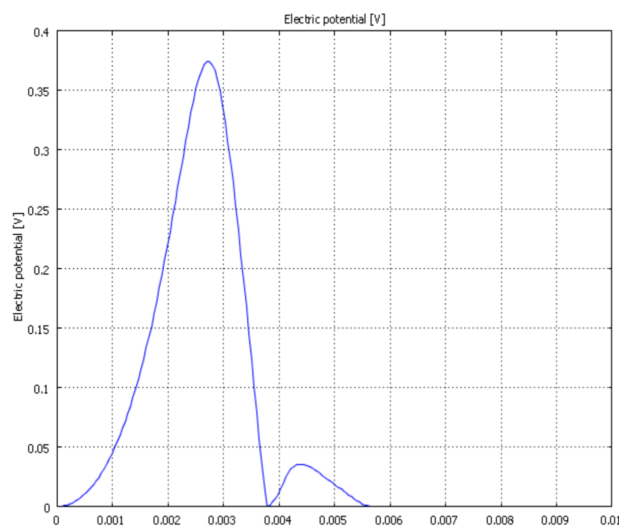

Figure 8. Potential distribution of the six-transmission lines interconnects in three-layered dielectric media using port 1 as input along the line from (x,y) = (0,0) to (x,y) = (8mm, 5mm).

Table 2 shows the finite element results for the capacitance-per-unit length of six-transmission lines interconnect in three-layered dielectric media. It compares the results from our work with those from other methods.

Table 2: Values of the Self and Mutual Capacitances Coefficient (in pF/m) for Six-Transmission Lines Interconnect in Three-Layered Dielectric Media as Shown in Fig. 8

Capacitance per unit length	(MoM) [2]	(MoL) [3]	SAGF [7]	This work
$C_{11} = C_{22}$	76.39	77.44	76.47	83.18
$C_{12} = C_{21}$	-5.26	-5.10	-5.25	-6.69
$C_{13} = C_{31} = C_{24} = C_{42}$	-29.54	-29.54	-29.54	-31.49
$C_{14} = C_{41} = C_{23} = C_{32}$	-3.59	-3.98	-3.58	-4.16
$C_{15} = C_{51} = C_{26} = C_{62}$	-3.43	-3.62	-3.43	-2.70
$C_{16} = C_{61} = C_{25} = C_{52}$	-0.86	-0.65	-0.88	-0.809
$C_{33} = C_{44}$	100.80	103.37	100.72	111.49
$C_{34} = C_{43}$	-8.88	-9.14	-8.88	-10.85
$C_{35} = C_{53} = C_{46} = C_{64}$	-40.84	-40.84	-40.84	-42.38
$C_{36} = C_{63} = C_{45} = C_{54}$	-5.36	-5.72	-5.37	-5.72
$C_{55} = C_{66}$	68.27	69.73	68.35	76.94
$C_{56} = C_{65}$	-8.23	-8.01	-8.23	-8.18

Tables 1 and 2 provide the results of FEM computations for the characteristics of two-layered multiconductor transmission lines and three-layered multiconductor transmission lines. The results of capacitance matrices, which are useful for the analysis of crosstalk between high-speed signal traces on the printed circuit board, are compared with other published data for the validity of the proposed method.

3. Conclusions

In this paper, we have presented the modeling in 2D of four-transmission lines embedded in two-layered dielectric media and six-transmission lines interconnect in three-layered dielectric media. We have shown that FEM is suitable and effective as method of moment (MoM), method of lines (MoL), and semi-analytic Green's function (SAGF) method for modeling multiconductor transmission lines in VLSI circuits. Some of the results obtained using FEM with COMSOL multiphysics for the capacitance-per-unit length agree well with those found in the literature. We illustrated the potential distribution of the multiconductor transmission lines for the models and their solution time. The results obtained in this research are encouraging and motivating for further study.

4. References

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