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COMSOL in a new Tensorial Formulation of Non-Isothermal Poroelasticity

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OUTLINE

- **INTRODUCTION - Motivation**
  - Geothermal rocks. Poroelastic effects.
- **BIOT'S THEORY OF POROELASTICITY**
  - Poroelastic Hookean Rocks. Main parameters.
  - Quaternions and poroelastic 4D formulation.
  - Gibbs function: Thermoporoelasticity in 4D.
- **TENSOR POROELASTIC MODEL AND FEM**
  - Some results (applications).
- **CONCLUSIONS**

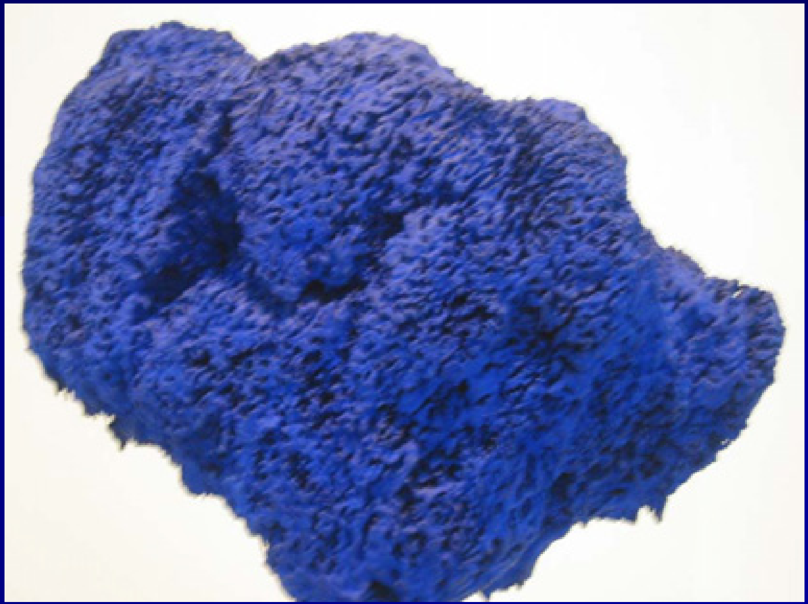
The words of a pioneer:

- *… soils* and rocks *are made by nature and not by man, and the products of nature are always complex… As soon as we pass from steel and concrete to earth, the omnipotence of theory ceases to exist. Soil mechanics arrived at the borderline between science and art. I use the term "art" to indicate mental processes leading to satisfactory results without the assistance of step-for-step logical reasoning…*"

Karl Terzaghi

(1883-1963)

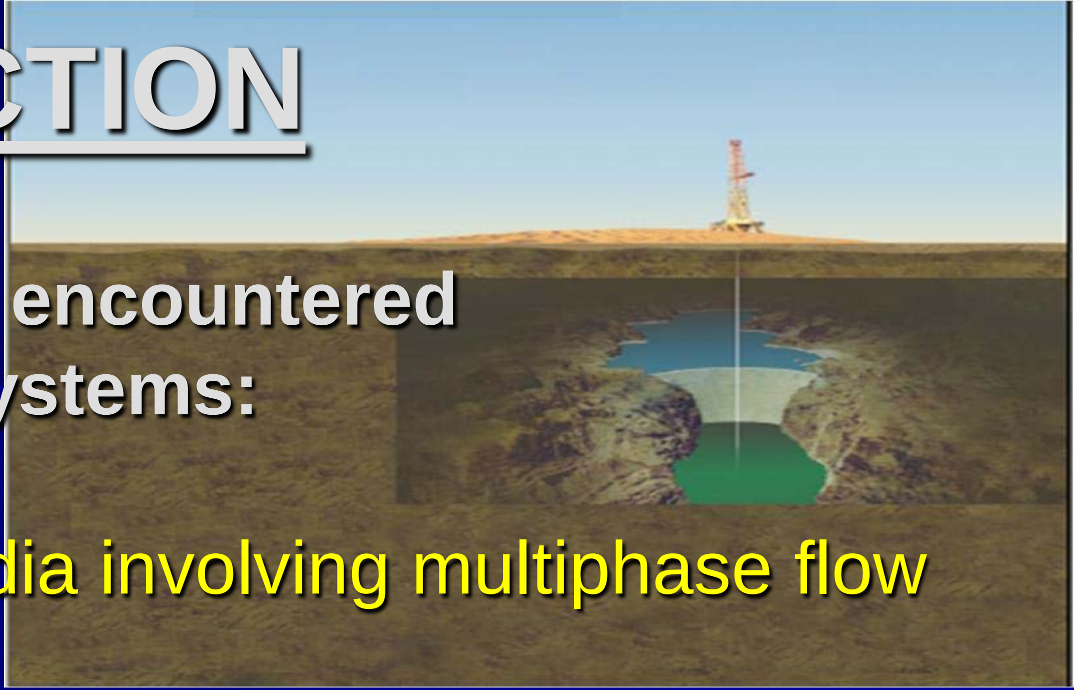
Examples of porous media



INTRODUCTION

- **Porous media phenomena are encountered in many man-made systems:**
  - A lot of industrial processes involve porous media flows.
  - processes in fuel cells, paper pulp drying, food production and safety, filtration,
  - concrete, ceramics, moisture absorbents, textiles, paint drying, polymer composites,
  - detergent tablets, various wood applications.

INTRODUCTION



- **Porous media are encountered in many natural systems:**

- Natural porous media involving multiphase flow and transport :
  - soils, aquifers, geothermal, oil and gas reservoirs.
  - biological tissues and plants. Wood.
  - Recently: growing interest in biomechanics of porous tissues, engineering tissues, and in-tissue drug delivery.

- **This is the Motivation of the subject.**

Geothermal rock properties:

- 95% of geothermal reservoirs are in igneous matrices.
- Crystal structure & total amount of interconnected pores = geomechanical properties.
- Rock deformation: elastic, plastic or anelastic.
- Cohesive structure of rocks is weakened by the fluid.
- This property tends to reduce rock elasticity and stiffness.

Some Poroelastic effects:

Reduction of reservoir storage capacity

Field observation of well casing failure

Land Subsidence and other effects:

Land fissure near Picacho, Arizona.

Photo courtesy of Mary Wheeler.

- rock deformations, creep and subsidence.
- stress changes lead to changes in seismic velocities affecting time-lapse seismic response.
- Many processes are thermodynamically irreversible, producing:
- permanent plastic deformations and
- permanent reduction of reservoir storage capacity.

Land Subsidence and other effects:

- rock compaction, faulting, deformations, fracturing,
- subsidence.

Land fissure in Chalco, Mexico, 06/09

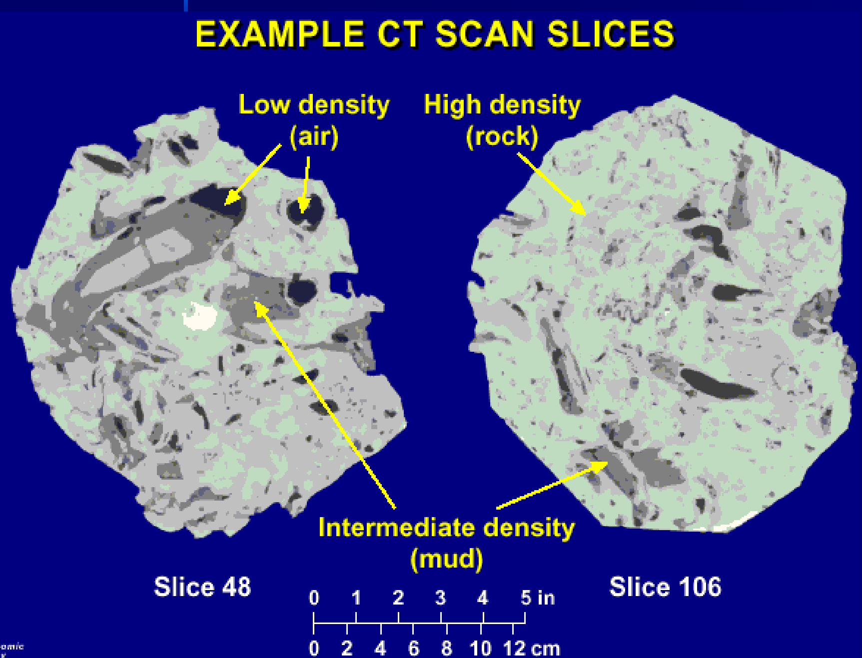
Photo courtesy of La Jornada.

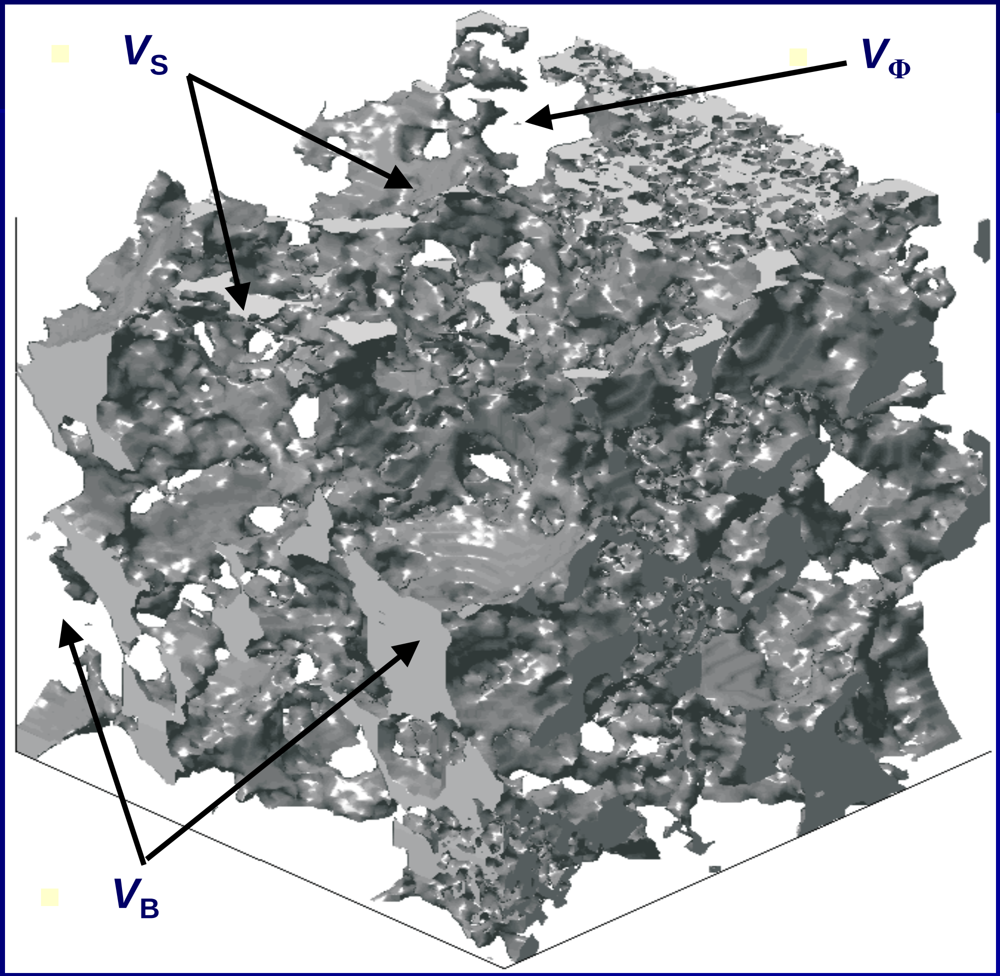
Experimental and theoretical results of Linear Poroelasticity

(M. Biot, 1905-1985)

Pores and solid grains = skeleton

- Skeleton of a sandstone with pore spaces in 3D





Deformation of a linear Solid

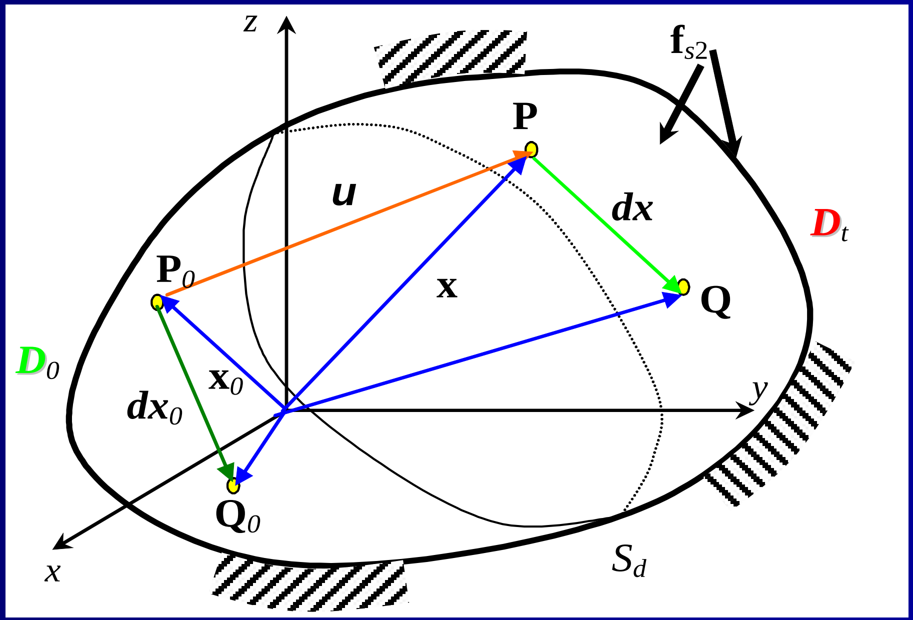
- Displacement vectors and strains:

$$\vec{\mathbf{u}} = \vec{P} - \vec{P}_0 = (u_X, u_Y, u_Z)\,; \quad \vec{\mathbf{u}} = \vec{u}_s - \vec{u}_f$$

$$d\vec{x} - d\vec{x}_0 = d(\vec{x} - \vec{x}_0) = d\vec{\mathbf{u}} = \vec{P}\vec{Q} - \vec{P}_0\vec{Q}_0$$

$$\varepsilon_i = \frac{\partial u_i}{\partial x_i},\ x_i = x, y, z$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



The Deformation of a Solid Rock:

- Main elastic functions and experimental moduli:

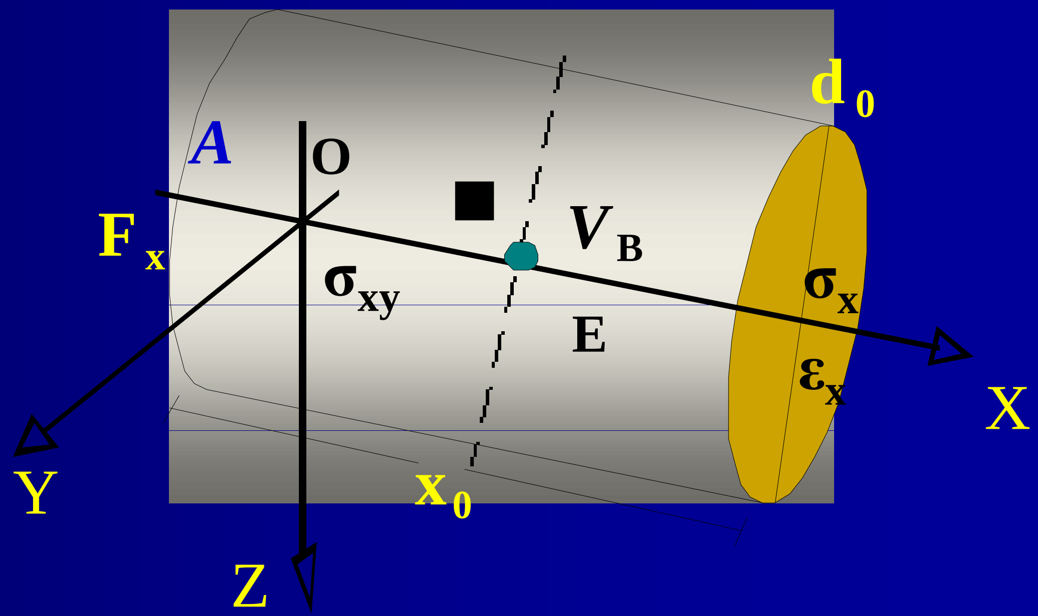
$$\sigma_X = \frac{F_X}{A}, \quad \varepsilon_x = \frac{x - x_0}{x_0}, \quad \varepsilon_B = \frac{\Delta V_B}{V_B}, \quad K_B = \frac{\Delta p}{\varepsilon_B}$$

$$E = \frac{\sigma_x}{\varepsilon_x}, \quad \nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{1}{2}\frac{\sigma_{xy}}{\varepsilon_{xy}}$$

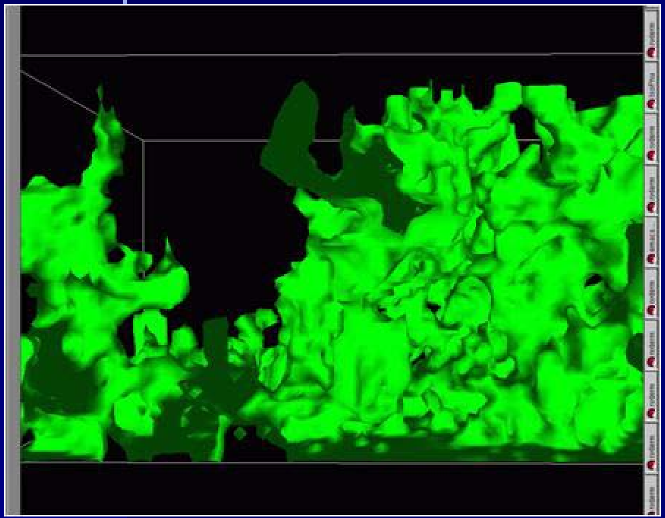
$$\lambda = K_B - \frac{2}{3}G$$

$$C_B = \frac{1}{K_B}$$



Biot Poroelastic Strain:

- Relative displacement is the key parameter in a deformed porous rock:



volumetric dilatations of the solid and fluid phases : $\varepsilon_r = \varepsilon_s - \varepsilon_f$

$$\varepsilon_s = \vec{\nabla} \cdot \vec{u}_s \ , \ \ \varepsilon_f = \vec{\nabla} \cdot \vec{u}_f \ \ \Rightarrow$$

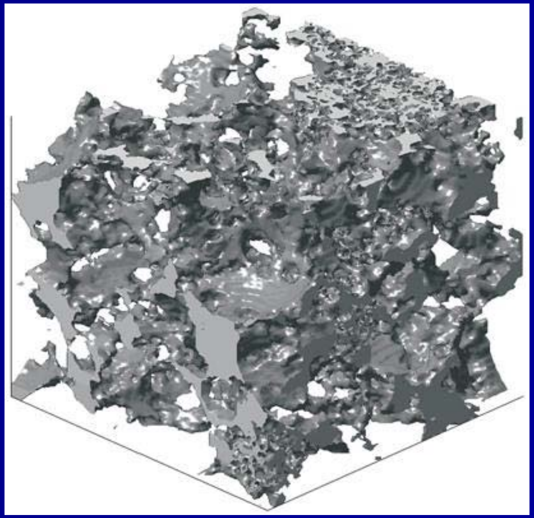
$$\zeta(\vec{u}, t) = \vec{\nabla} \cdot \left(\varphi_f \vec{u} \right)$$

- ζ = increment or decrement of fluid content in the pore during a poroelastic deformation.

$$\Rightarrow \quad \zeta(\vec{u}, t) = \varphi_f \ \varepsilon_r$$

Biot's Poroelastic Theory (1957, 1972):

- Fluids flow through a deformable porous rock,
- the solid skeleton is elastic (SDH),
- The fluid content can change:

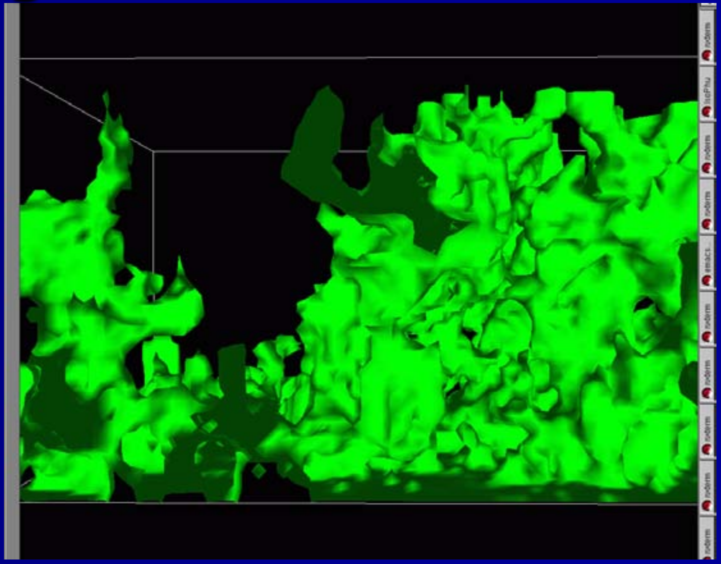


$$\zeta = \frac{m_f - m_0}{\rho_0} = \frac{\rho_f \Delta\varphi + \varphi \Delta\rho_f}{\rho_0}$$

- ζ is a special strain variable representing the change of the fluid content.

Fundamental Poroelastic parameters:

- Linear relationships among the poroelastic variables:
- H^{-1} is a poroelastic expansion coefficient,
- R^{-1} unconstrained specific storage coefficient.



$$\varepsilon_B = \frac{\sigma}{K_B} + \frac{p_f}{H}, \qquad \zeta = \frac{\sigma}{H} + \frac{p_f}{R}$$

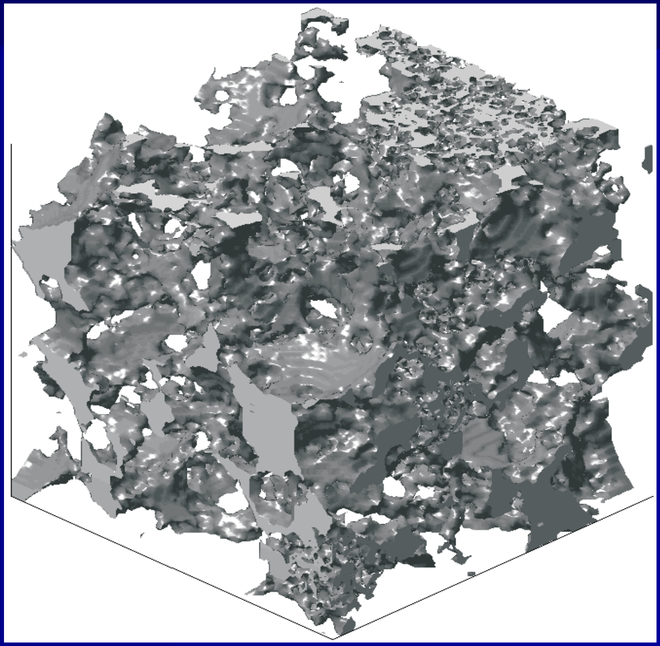
$$C_B = \frac{1}{K_B} = \left(\frac{\Delta \varepsilon_B}{\Delta \sigma} \right)_{p_f}$$

$$\frac{1}{H} = \left(\frac{\Delta \varepsilon_B}{\Delta p_f} \right)_{\sigma} = \left(\frac{\Delta \zeta}{\Delta \sigma} \right)_{p_f} \qquad \frac{1}{R} = \left(\frac{\Delta \zeta}{\Delta p_f} \right)_{\sigma}$$

Other Poroelastic coefficients:

- The constrained specific storage (fluid elastic properties)

$$\frac{1}{M} = \left(\frac{\Delta \zeta}{\Delta p_f} \right)_{\varepsilon_B} = \frac{1}{R} - \frac{K_B}{H^2}$$



- coupling of deformations between the solid grains and the fluid:

$$C = \frac{K_B R H}{H^2 - K_B R} = \frac{K_B}{H} M$$

- The Biot – Willis and the Skempton coefficients:

$$b = \frac{C}{M} = \frac{K_B}{H}, \quad B = -\left(\frac{\Delta p_f}{\Delta \sigma} \right)_{\zeta} = \frac{R}{H}$$

Poroelastic Hookean rocks:

- Strains and stresses are linearly dependent:

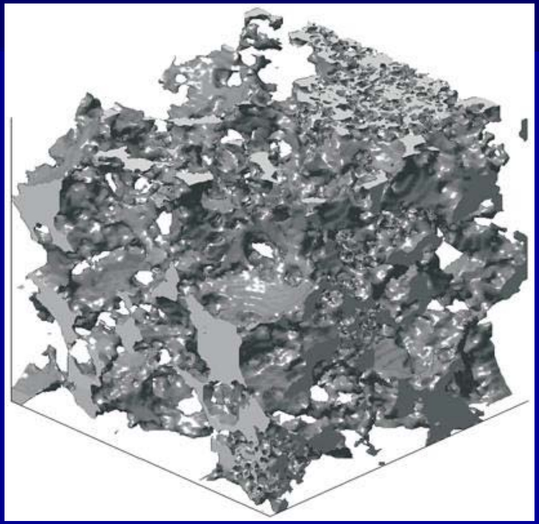
$$\vec{\sigma}_{\mathrm{T}} = C_{\mathrm{B}} \cdot \vec{\varepsilon}_{\mathrm{T}} \quad \Leftrightarrow \quad \vec{\varepsilon}_{\mathrm{T}} = C_{\mathrm{B}}^{-1} \cdot \vec{\sigma}_{\mathrm{T}}$$

$$\begin{pmatrix} \sigma_x^s \\ \sigma_y^s \\ \sigma_z^s \\ \sigma_{xy}^s \\ \sigma_{xz}^s \\ \sigma_{yz}^s \\ \sigma_f \end{pmatrix} = \begin{pmatrix} \lambda+2G & \lambda & \lambda & 0 & 0 & 0 & -C \\ \lambda & \lambda+2G & \lambda & 0 & 0 & 0 & -C \\ \lambda & \lambda & \lambda+2G & 0 & 0 & 0 & -C \\ 0 & 0 & 0 & 2G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G & 0 \\ -C & -C & -C & 0 & 0 & 0 & M \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_x^s \\ \varepsilon_y^s \\ \varepsilon_z^s \\ \varepsilon_{xy}^s \\ \varepsilon_{xz}^s \\ \varepsilon_{yz}^s \\ \zeta \end{pmatrix}$$

- *C* holds the coupling between solid and fluid.
- *M* depends on the elastic properties of the fluid.

Terzaghi effective stresses in porous rocks:

- (τ_{ij}) supports one portion of the total applied tensions and the fluid in the pores ($b\,p_f$) supports the other part:



$$\tau_{ij} = \lambda\, e_B\, \delta_{ij} + 2G\varepsilon_{ij} \quad \Rightarrow$$

$$\boxed{\sigma_{ij} = \tau_{ij} - b\,p_f\,\delta_{ij}}$$

$$\begin{pmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_f \end{pmatrix} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} & 0 \\ \tau_{xy} & \tau_{yy} & \tau_{yz} & 0 \\ \tau_{xz} & \tau_{yz} & \tau_{zz} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} p_f$$

FORMULATION OF THE THERMOPOROELASTIC MODEL USING THE GIBBS POTENTIAL

A powerful technique to construct the thermoporoelastic equations:

- The Gibbs free enthalpy of the skeleton and the energy dissipation function:

$$G_S = H_S - TS \Rightarrow g_S = h_S - TS_S \rightarrow \left[\frac{J}{m^3} = Pa \right]$$

$$\frac{d\Psi_s}{dt} = \sigma_{ij} \frac{d\varepsilon_{ij}}{dt} - S_s \frac{dT}{dt} - \varphi \frac{dp}{dt} - \frac{dg_s}{dt} \geq 0$$

- Assuming that the energy dissipation function is zero:

$$\frac{d\Psi_s}{dt} = 0$$

The thermoporoelastic matrix equations, include the thermal tensions in the stress:

$$\begin{pmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} & 0 \\ \sigma_{xy} & \sigma_y & \sigma_{yz} & 0 \\ \sigma_{xz} & \sigma_{yz} & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_f \end{pmatrix} = \varepsilon_B \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & -C \end{pmatrix} + 2G \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} & 0 \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} & 0 \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-(p-p_0)\begin{pmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & -M\dfrac{\zeta-\zeta_0}{p-p_0} \end{pmatrix} - K_B\left(T-T_0\right)\begin{pmatrix} \gamma_B & 0 & 0 & 0 \\ 0 & \gamma_B & 0 & 0 \\ 0 & 0 & \gamma_B & 0 \\ 0 & 0 & 0 & \dfrac{M\varphi}{K_B}\left(\gamma_\varphi - \gamma_f\right) \end{pmatrix}$$

FORMULATION OF 3D THERMOPOROELASTICITY IN TERMS OF QUATERNIONS

Poroelasticity and quaternions:

- Relationships between normal stresses and strains in isothermal processes:

$$\sigma_{ij} = \lambda_U \, \varepsilon_B \delta_{ij} - C\, b\, \delta_{ij} - b\, p_f \delta_{ij} + 2G\, \varepsilon_{ij}$$

$$\sigma_{ii} = \quad q_0 \vec{h} \quad + \; q_1 \vec{i} \quad + \quad q_2 \vec{j} \quad + \; q_3 \vec{k}$$

- Relationships between normal stresses and strains in thermoporoelasticity:

$$\sigma_{ij} - \sigma_{ij}^0 = \lambda\, \varepsilon_B \delta_{ij} - b\left(p - p_0\right)\delta_{ij} - K_B \gamma_B \left(T - T_0\right)\delta_{ij} + 2G\, \varepsilon_{ij}$$

$$\sigma_{ii} - \sigma_{ii}^0 = \left(q_0 \vec{h}\right) \; + \; \left(q_1 \vec{i}\right) \quad + \quad \left(q_2 \vec{j}\right) \; + \; \left(q_3 \vec{k}\right)$$

APPLICATIONS to a POROELASTIC AQUIFER

Table 1: Biot poroelastic parameters.

- Calculated from data of the Los Humeros geothermal field.
- $T = 290°C$, $p_f = 110$ bar, $\rho_f = 739$ kg/m^3 , $\sigma = 320$ bar.

b → (ad)	a_2 (10^{-2})	C (GPa)	M (GPa)	ζ (10^{-5})
0.202	1.14	1.98	9.81	-5.69
0.228	1.23	1.77	7.75	-8.90
0.340	2.13	1.19	3.51	-44.4
0.383	2.20	1.06	2.76	-68.3
0.430	3.00	0.94	2.19	-108

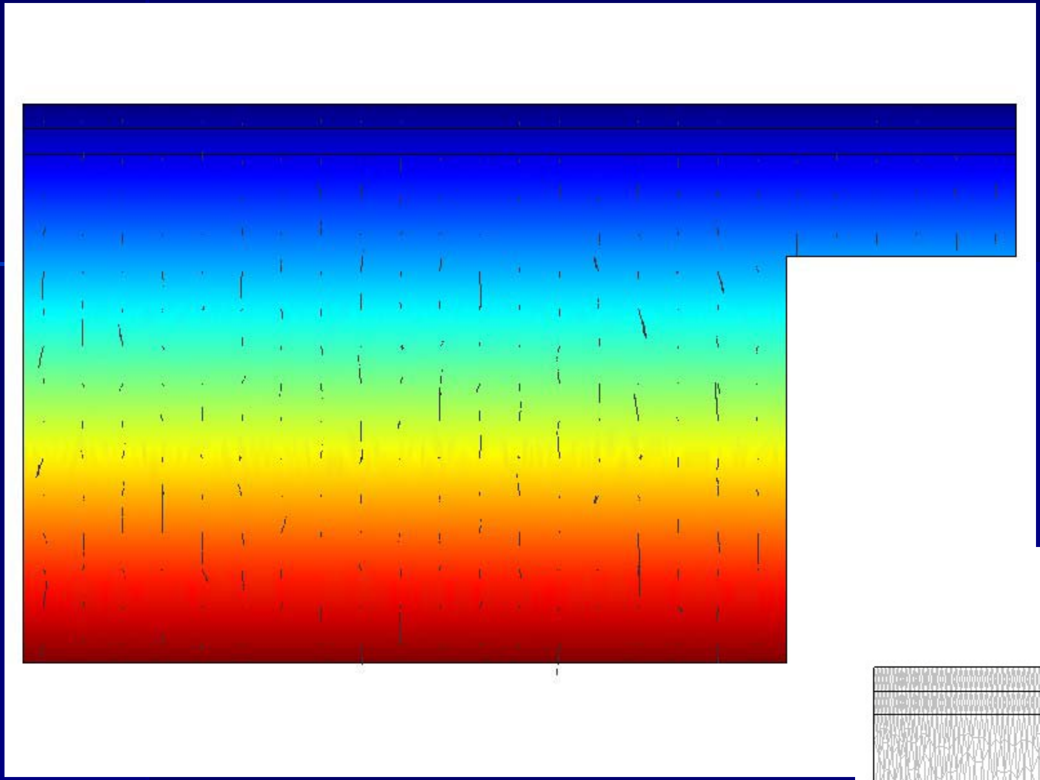
The bulk modulus of water in Porous Flow Coupling

- Calculated from data of the Los Humeros, Mexico geothermal field:

- T= 290°C, p_f = 110 bar, K_w = 0.42 GPa

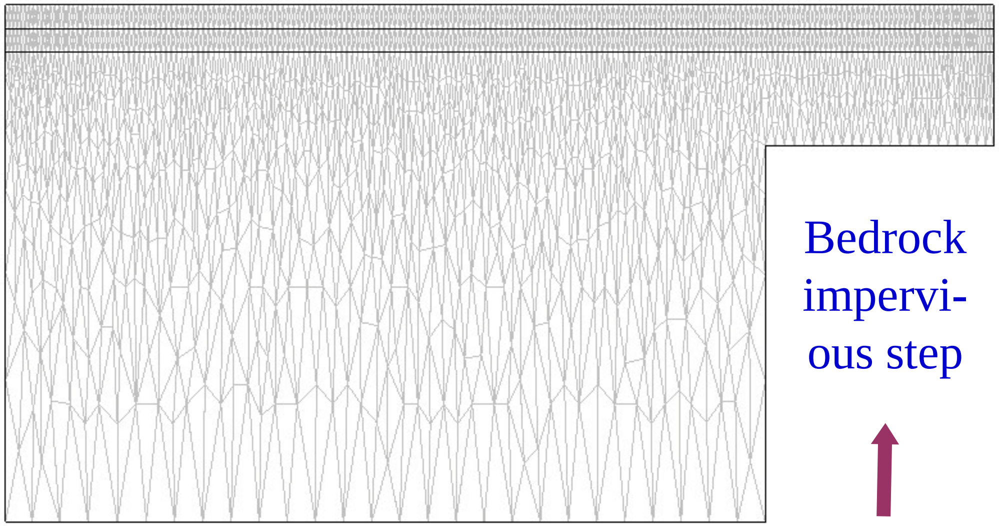
- T= 50°C, p_f = 50 bar, K_w = 2.1 GPa

Poroelastic deformation of an aquifer (Leake & Hsieh)



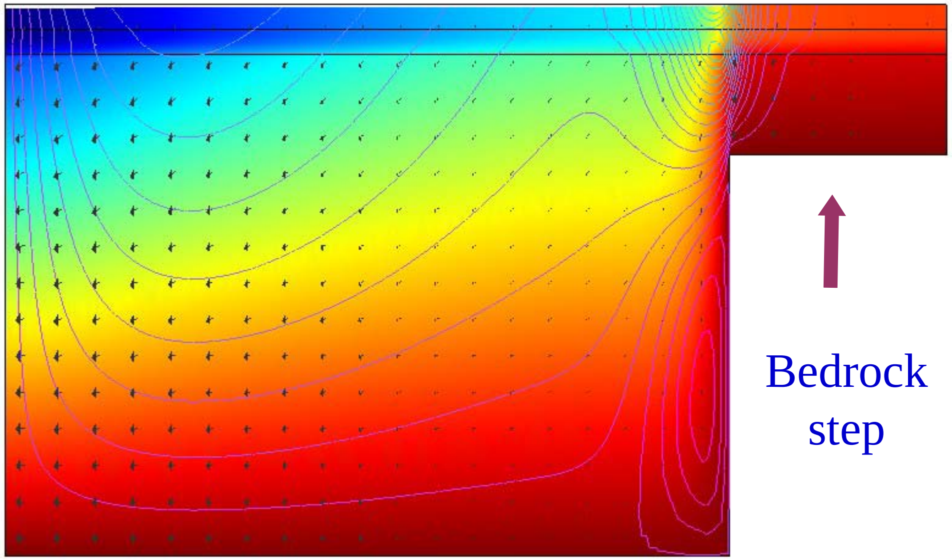
- 1) This example describes the impact of pumping for a basin filled with sediments draping an impervious fault block. The basin is composed of 3 layers having a total depth of 500 m and is 5000 m long.

- The corresponding FE mesh for this example has 3409 elements excluding the bedrock step. The top two layers are 20 m thick each. The rock is Hookean, poroelastic and isothermal.



Poroelastic deformation of an aquifer (Leake & Hsieh,1997)

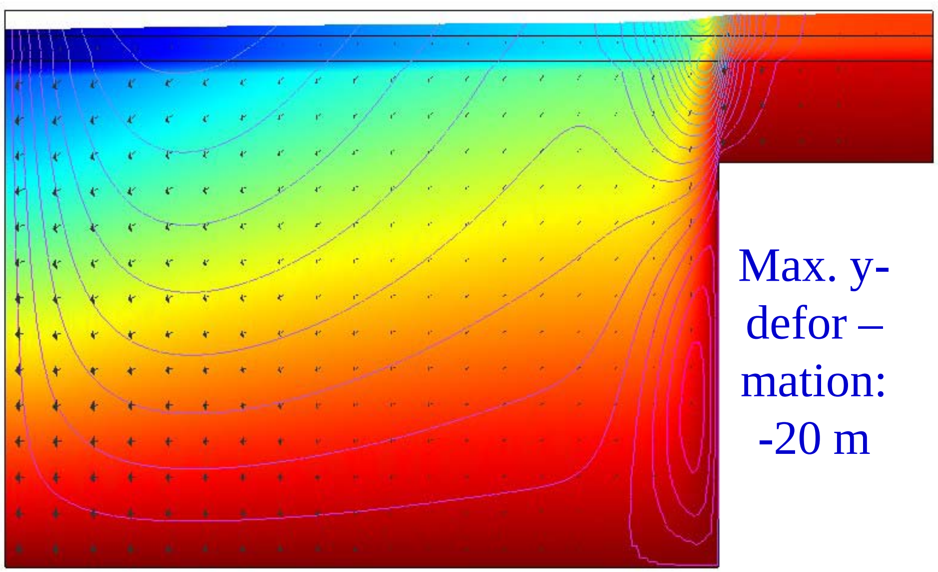
time: 365 days, y - x displacements



1) Max. vertical deformation:
- 5.5 m

Max. X - deformation: - 0.02 m

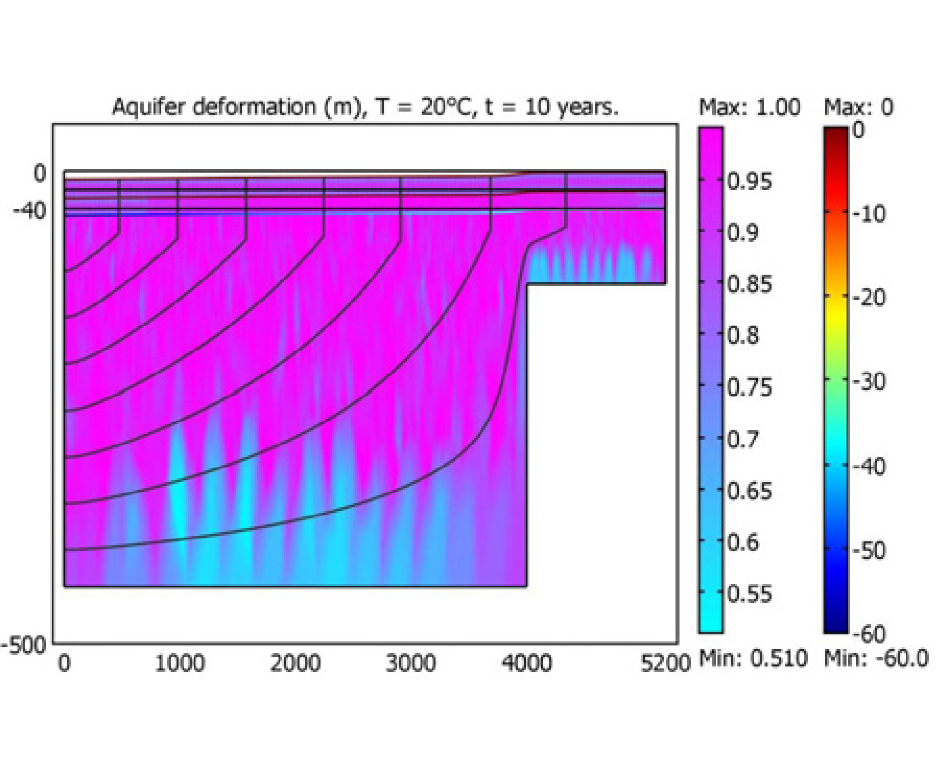
time: 1800 days, y - x displacements

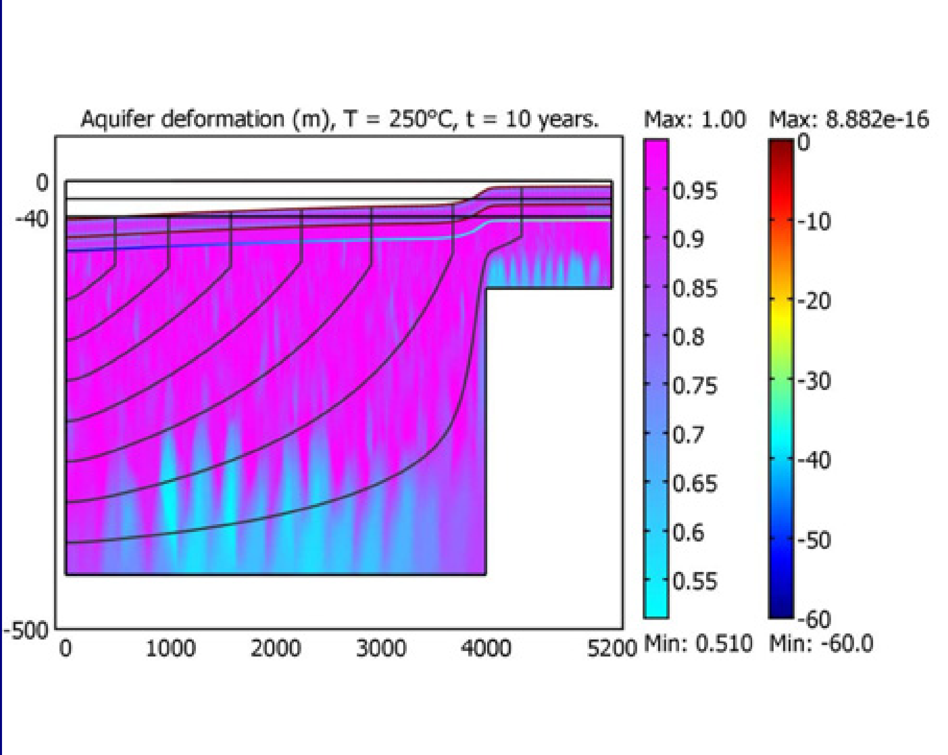


Max. y-defor – mation: -20 m

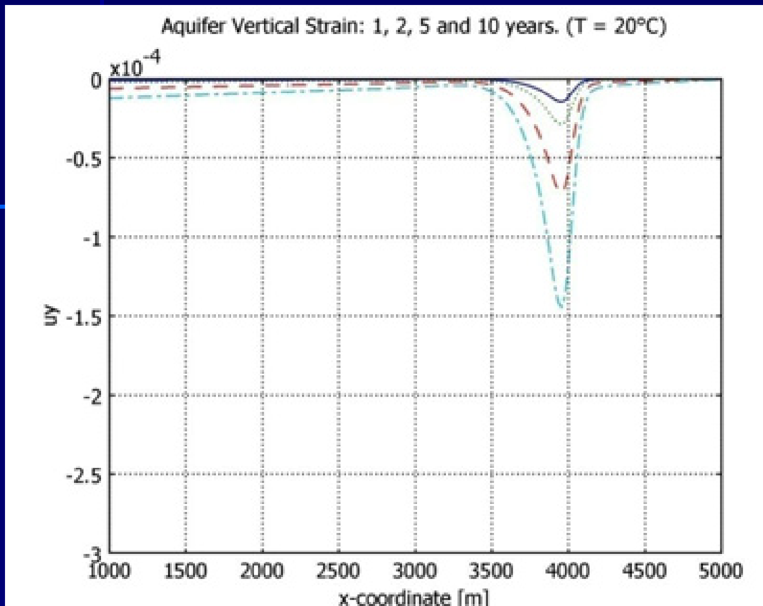
2) Max. vertical deformation:
- 20 m

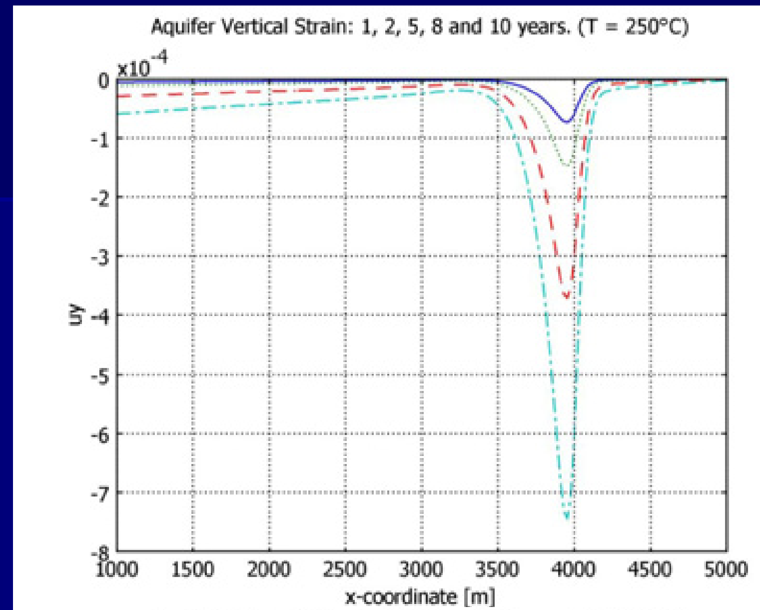
Influence of Temperature

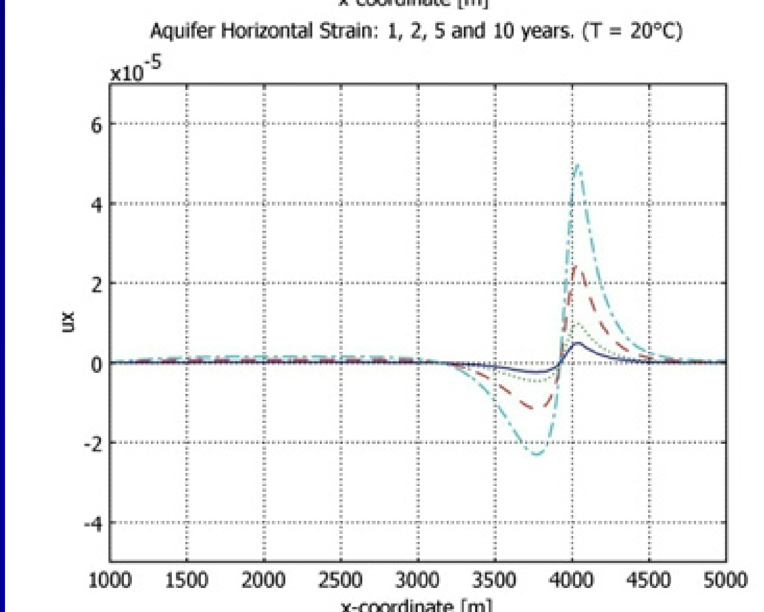


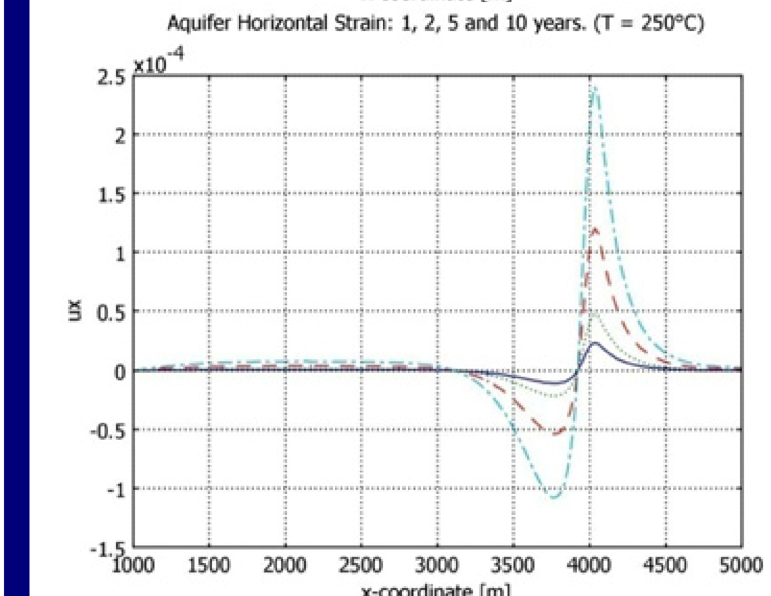


Influence of Temperature









C O N C L U S I O N S

Geomechanics and Porous Flow Coupling

✓ Displacements Influence Porous Flow

❖ All crustal rocks forming reservoirs are poroelastic and the fluid presence inside the pores affects their geomechanical properties.

❖ Fluid Pressures Influence Displacements

❖ Water bulk modulus affect poroelastic coefficients, expansivity of rocks. These effects can produce severe structural damages in porous rocks subjected to strong temperature gradients, as happens during the injection of cold fluids.

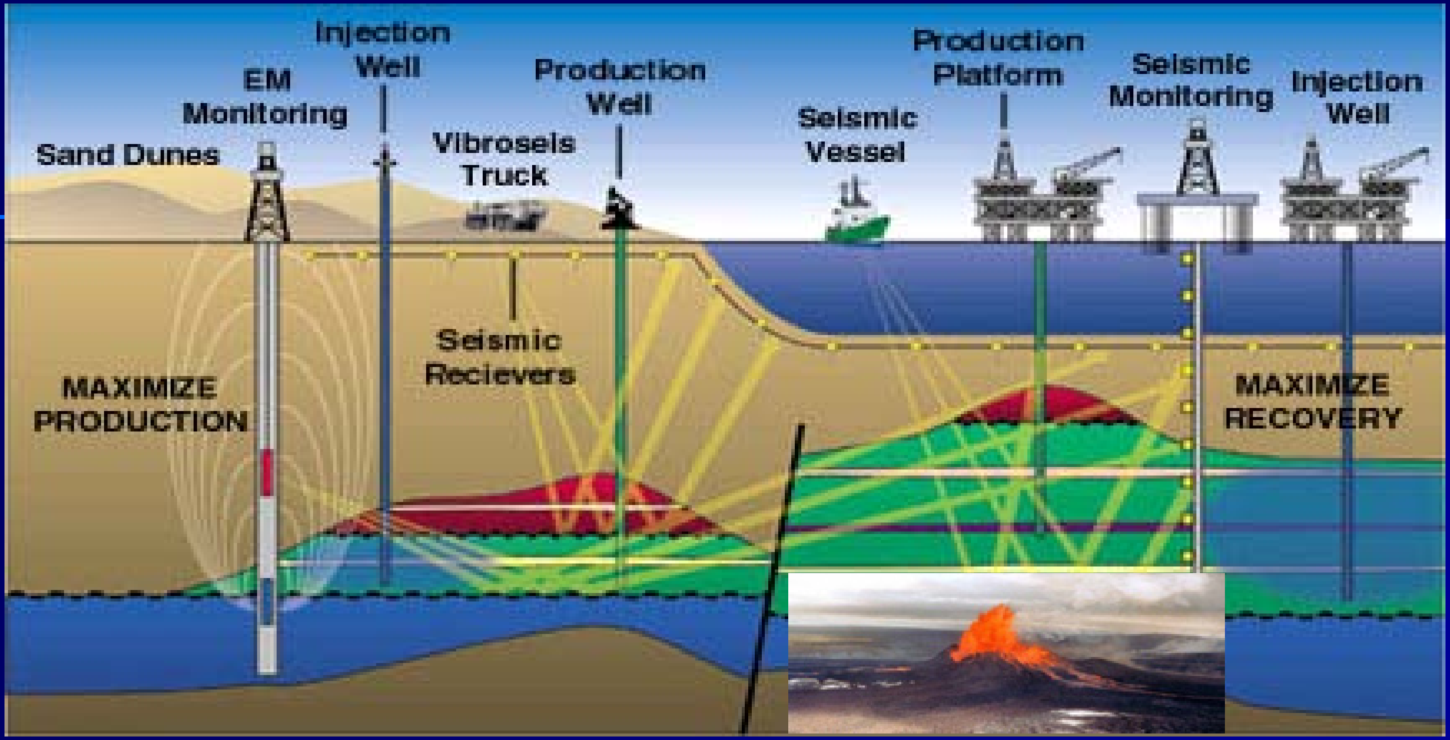
Geomechanics (motivation)

- Applications
  - **Land subsidence**
  - **Wellbore instability**
  - **Reservoir failure**
  - **Hydraulic fracturing**
  - **Reduction of rock permeability**
  - **Change in fault sealing properties**

Goal: "Find a stable and accurate algorithm to extend our existing fluid-flow model to a tightly coupled fluid-flow and geomechanics model with as few changes as possible."

M. Wheeler **(Center of Subsurface Modeling, Texas U.)**

Optimized Geothermal Fields



Detect and track changes in data during production.
Invert data for reservoir properties.
Detect and track reservoir changes.

Assimilate data & reservoir properties into
the evolving reservoir model.
Use simulation and optimization to guide future production.