

Calculation of the Magnetic Field Intensity in a Rectangular Conductor Carrying Current in Electromagnetism Introductory Courses

Juan C. Olivares-Galván^{1*}, Iván Hernández², Pavlos S. Georgilakis³, Eduardo Campero-Littlewood¹

¹ Universidad Autónoma Metropolitana.

² Centro de Investigación y de Estudios Avanzados del IPN, Unidad Guadalajara, Guadalajara, Jalisco, México.

³ School of Electrical and Computer Engineering, National Technical University of Athens, Athens, Greece.

*Corresponding author: Av. San Pablo No. 180, Col. Reynosa Tamaulipas, Del. Azcapotzalco, 02200, Tel. (+52-55) 5318-9584, México, D.F., e-mail: jolivare_1999@yahoo.com

Abstract: This paper describes a type of didactic material used when teaching electromagnetism. The purpose is to guide the students to verify the results of a Finite Element (FE) simulation using those obtained analytically. This procedure has shown to be of great help during their learning of the FE method. The example in this paper uses a 2D analytical method to estimate the magnetic field generated by the circulation of a direct current through a finite cross-section conductor in free space. The results of the analytical method are compared with those of a FE model using COMSOL Multiphysics. The calculations by both methods give a relative error of about 2%. We have more teaching materials to learn the FE method through simple geometries, which are verifiable analytically. We give a list of electromagnetism exercises for readers interested in this teaching philosophy.

Keywords: Magnetic Field Intensity, Conductor, Electromagnetism, teaching, Finite Element Method (FEM).

1. Introduction

The search for new teaching techniques and procedures to give students more tools to understand electromagnetism has been addressed by many authors [1]-[8]. In this work the magnetic field intensity of a finite cross-section conductor in free space is calculated using a two-dimensional (2D) analytical method using the magnetic vector potential. The results of the analytical method are compared with those obtained from the application of the Finite Element Method (FEM) using Comsol

Multiphysics. The calculations are performed considering direct current. The geometry analyzed consists of a rectangular conductor of finite cross-section, whose dimensions are $a = 2\text{mm}$ and $b = 8\text{mm}$. A direct current of 1 A was applied to the conductor of Figure 1, which produces a current density of $J = 62500\text{ A/m}^2$.

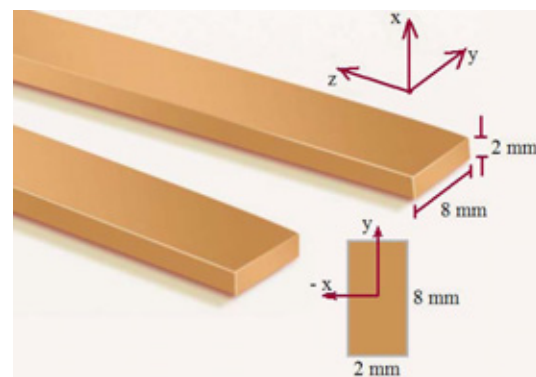


Figure 1: Geometry of the rectangular conductor in free space.

2. Analytical formula to determine the magnetic field intensity using the Struss Simplification.

The vector potential at any point (x, y) distance r from a typical filament with (x', y') is [9]:

$$A = \frac{I\mu_0}{8\pi ab} \int_{-a}^a \int_{-b}^b \log r \, dx' \, dy' \quad (1)$$

Expressing r in terms of the rectangular coordinates gives:

$$A = \frac{I\mu_0}{16\pi ab} \int_{-a-b}^a \int_{-a-b}^b \log[(x'-x)^2 + (y'-y)^2] dx' dy' \quad (2)$$

Using Strutt simplification, (2) becomes,

$$\begin{aligned} A = \frac{I\mu_0}{16\pi ab} & \left\{ (a-x)(b-y) \log[(a-x)^2 + (b-y)^2] + \right. \\ & + (a+x)(b-y) \log[(a+x)^2 + (b-y)^2] + \\ & + (a-x)(b+y) \log[(a-x)^2 + (b+y)^2] + \\ & + (a+x)(b+y) \log[(a+x)^2 + (b+y)^2] + \\ & + (a-x)^2 \left[\tan^{-1} \left(\frac{b-y}{a-x} \right) + \tan^{-1} \left(\frac{b+y}{a-x} \right) \right] + \\ & + (a+x)^2 \left[\tan^{-1} \left(\frac{b-y}{a+x} \right) + \tan^{-1} \left(\frac{b+y}{a+x} \right) \right] + \\ & + (b-x)^2 \left[\tan^{-1} \left(\frac{a-x}{b+y} \right) + \tan^{-1} \left(\frac{a+x}{b+y} \right) \right] + \\ & \left. + (b-y)^2 \left[\tan^{-1} \left(\frac{a-x}{b-y} \right) + \tan^{-1} \left(\frac{a+x}{b-y} \right) \right] \right\} \end{aligned} \quad \dots(3)$$

The field components H_x and H_y are given by,

$$H_x = \frac{1}{\mu_0} \frac{\partial A}{\partial y} \quad (4)$$

$$H_y = -\frac{1}{\mu_0} \frac{\partial A}{\partial x} \quad (5)$$

The simplification in the calculation of H_x and H_y is achieved by using r_1, r_2, r_3, r_4 and $\theta_1, \theta_2, \theta_3, \theta_4$, which are shown in Figure 2.

$$H_x = \frac{1}{8\pi ab} \left[w(\theta_1 - \theta_2) + u(\theta_4 - \theta_3) + v \log \frac{r_2}{r_3} + t \log \frac{r_1}{r_4} \right] \quad \dots(6)$$

$$H_y = \frac{1}{8\pi ab} \left[v(\theta_2 - \theta_3) + t(\theta_1 - \theta_4) + w \log \frac{r_2}{r_1} + u \log \frac{r_3}{r_4} \right] \quad \dots(7)$$

The magnitude of the magnetic field intensity is given by:

$$H_{TOTAL} = \sqrt{H_x^2 + H_y^2} \quad (8)$$

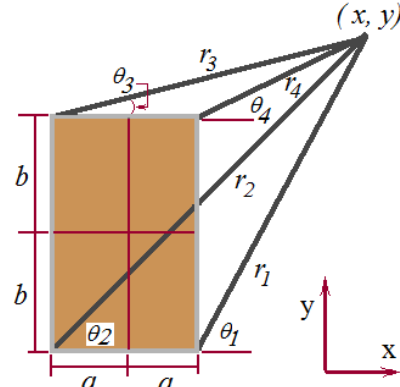


Figure 2: Geometry of the rectangular conductor for the calculation of H_x and H_y

3. Magnetic field intensity obtained using COMSOL Multiphysics.

The students learn to familiarize with the AC/DC module from COMSOL Multiphysics. For this study they select the magnetostatic case and apply the perpendicular current module. COMSOL Multiphysics has a very didactic description of the chosen module. See Figure 3a.

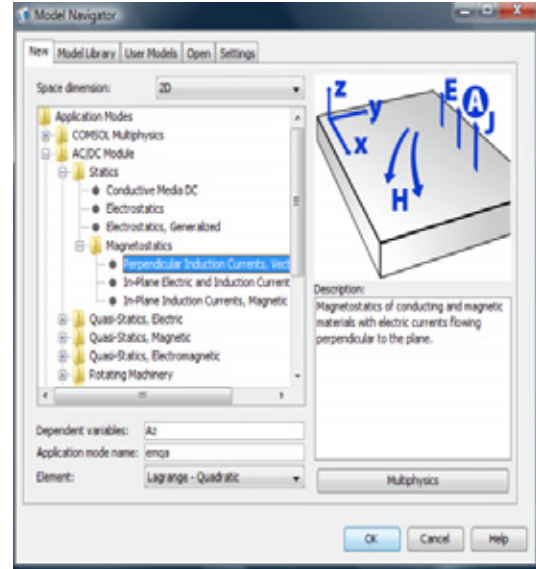


Figure 3a: Selecting the module in COMSOL Multiphysics

A magnetic insulation boundary condition was chosen for the bubble that encloses the conductor $A_z = 0$, as shown in Figure 3b.

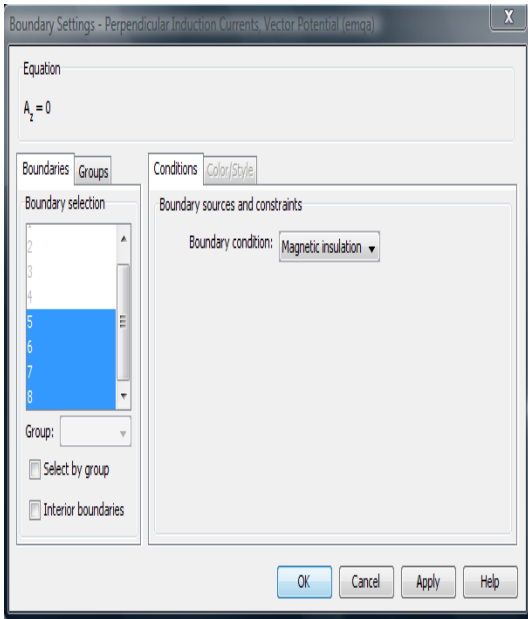


Figure 3b: Boundary Settings.

The extremely fine option of the predefined mesh size is used, producing 33640 finite elements. This helps to obtain a detailed and precise solution of the problem. The solution required about 380 MB of ram memory and 1.87s of computing time. The accuracy of the solution by COMSOL Multiphysics is highly dependent of the distribution and size of the elements. We can see in Figure 3c that a fine mesh is required wherever the field has a large variation and a courser mesh is used elsewhere.

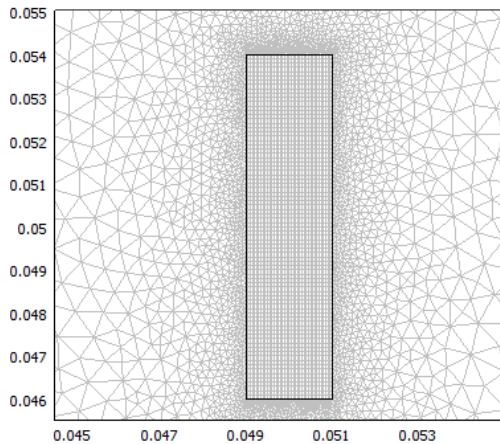


Figure 3c: Mesh generated by COMSOL

Fig. 4 shows the magnetic field intensity along the straight line with origin at $x = 0$ $y = 0$ and

final point $x = 0.05$ and $y = 0$. The curves were obtained with the following methodologies, a) FEM, and b) Analytical formulation using the Strutt simplifications.

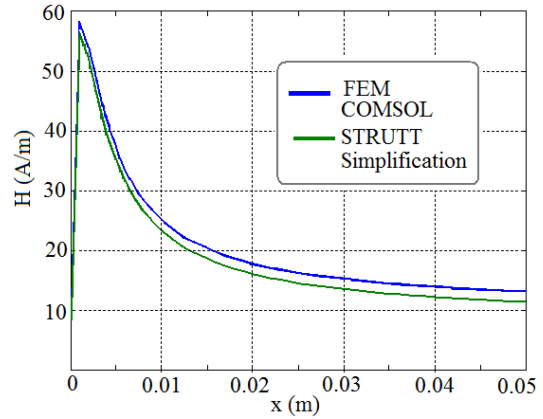


Figure 4: Variation of the magnetic field along the x-axis.

Fig 5 shows the magnetic potential lines inside and outside of the rectangular conductor, obtained using a magnetostatic analysis. Fig. 6 shows the magnetic potential lines generated by the current flowing through the rectangular conductor, indicating some values in specific points.

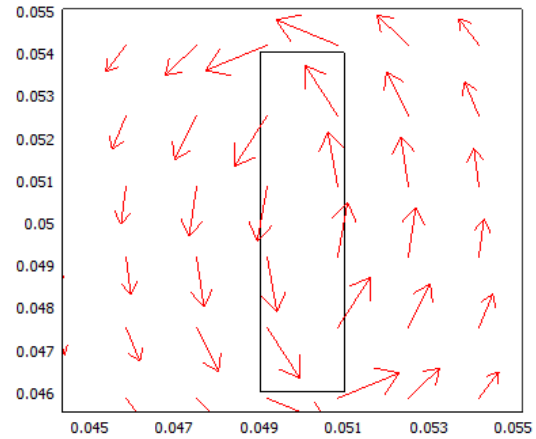


Figure 5: Contour: Magnetic potential; arrow: magnetic field lines inside and outside of the rectangular conductor.

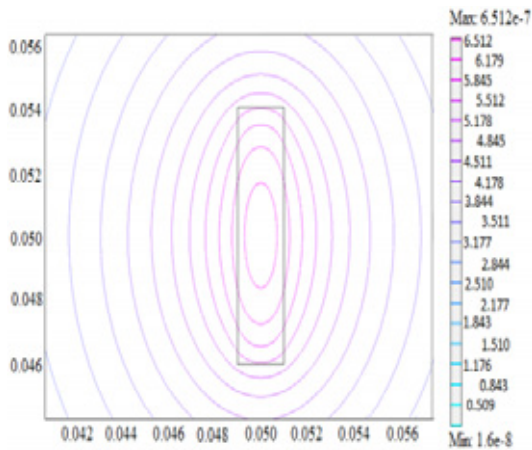


Figure 6: Magnetic potential inside and outside of the rectangular conductor.

We can see in Figure 7 that there is an accumulation of magnetic field on the four corners of the rectangular conductor. The magnetic field decreases as we move away from the rectangular conductor. Figure 8 shows magnetic field intensity (Surface) and magnetic flux density (height) of rectangular conductor. The maximum value of observed magnetic field is of the order of 60 A / m.

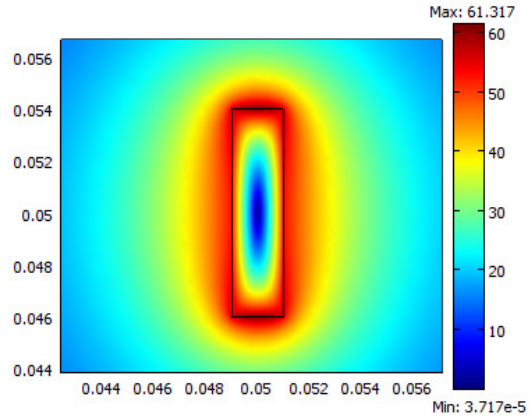


Figure 7: Magnetic field intensity of rectangular conductor in 2D

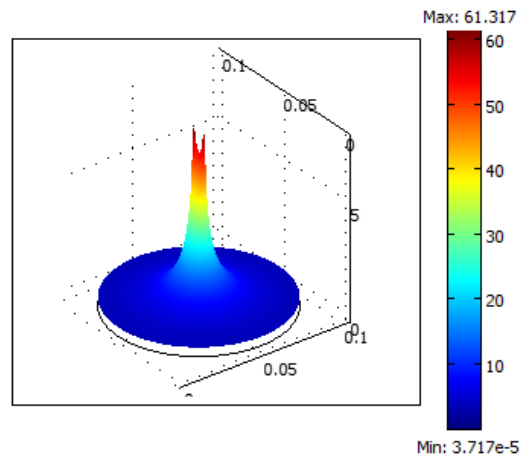


Figure 8: Magnetic field intensity of rectangular conductor in 2D

4. Other study cases for electromagnetism courses

Figures 9 and 10 show more study cases for electromagnetism courses, and equations (9) and (10) calculate the corresponding electromagnetic forces between the two conductor bodies.

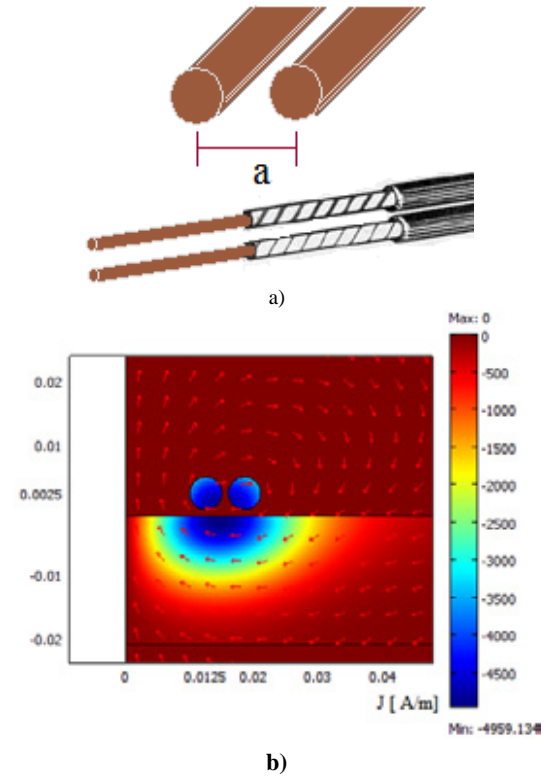


Figure 9. a) Configuration of two cylindrical conductors, b) current density in both conductors.

$$F_y = \frac{\mu_o \mu_r I_1 I_2}{2\pi a} \quad (9)$$

$$F_y = \frac{\mu_o \mu_r I_1 I_2}{2\pi c} \ln \left[\frac{(c^2 + E^2)^{1/2}}{E} \right] \quad (10)$$

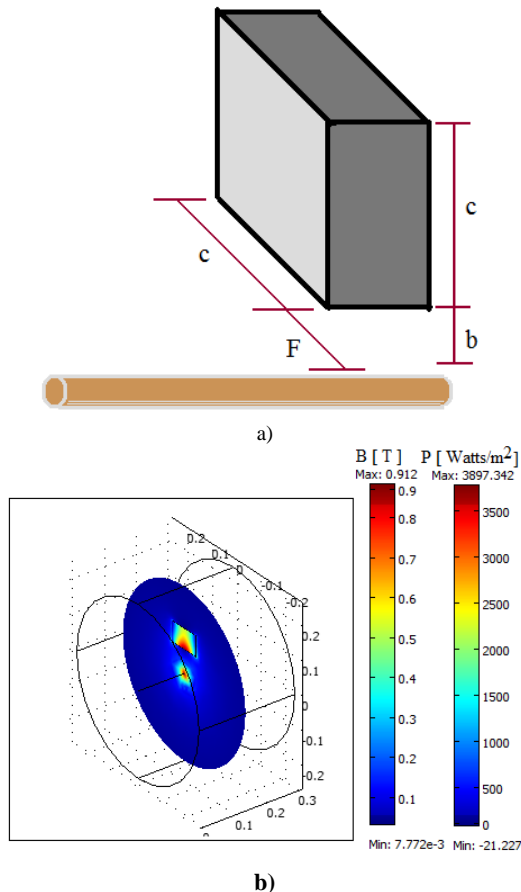


Figure 10. a) Configuration of a conducting plate and a cylindrical conductor, b) Magnetic flux density and loss density in this configuration.

The appendix questionnaire will be used in future electromagnetism courses to have an idea of the impact in students when using COMSOL Multiphysics.

5. Conclusions

This paper uses two methodologies to calculate the magnetic field intensity in a rectangular conductor carrying dc: a) using an analytical expression which requires the magnetic vector potential and also requires the Strutt

simplification, b) using FEM. Both results were compared and an error of around 2% was found. In the calculation of analytical expressions the maple software has been very useful. The objective of simulating simple geometries for electromagnetism courses is that electrical engineering students have an analytical model that allows them to verify their results obtained with the FEM.

6. References

- [1] P. Dular, C. Geuzaine, A. Genon, W. Legros, An Evolutive Software Environment for Teaching the Finite Element Method in Electromagnetism, *IEEE Transactions on Magnetics*, **Vol. 15**, No. 3, pp. 1682-1685, (1999).
- [2] I. Galilia, and D. Kaplan, Changing approach to teaching electromagnetism in a conceptually oriented introductory physics course, *American Association of Physics Teachers*, pp. 657-667, (1997).
- [3] Y. Judy Dori, J. Belcher, How does technology-enabled active learning affect undergraduate students' understanding of electromagnetism concepts?, *The Journal of the Learning Sciences*, **Vol. 14**, No. 2, pp. 243-279, (2005).
- [4] K. F. Warnick, R. H. Selfridge, and D. V. Arnold, Teaching Electromagnetic Field Theory Using Differential Forms, *IEEE Transactions on Education*, **Vol. 40**, No. 1, pp. 53-68, (1997).
- [5] J. C. Olivares G., M. Escalante A., R. Escarela P., E. Campero L., J. L. Hernández A., Irvin López G., Crossword puzzles for learning electromagnetism, *Rev. Eureka Enseñ. Divul. Cien.*, **5**(3), pp. 334-346, (2008).
- [6] F. J. Rosenbaum, Teaching Electromagnetics Around the World: A Survey, *IEEE Transactions on Education*, **Vol. 33**, No. 1, (1990).
- [7] A. Veglis, C. A. Barbargires, y A. Pombortsis, An Interactive Laboratory Exercise For Teaching Computer Science Students Network Performance Evaluation Using Mathcad And MathConnexTM, *29th ASEE/IEEE Frontiers in Education Conference*, (1999).
- [8] P. S. Georgilakis, *Spotlight on Modern Transformer Design*, Springer, London, UK, (2009).

[9] K. J. Binns, *The analytical and numerical solutions of electric and magnetic fields*, John Wiley & Sons, New York, pp. 70-72, (1995).

7. Appendix

The following questions will be answered using Likert scale of 5 points, the value of 5 is if you agree entirely, 4 if you agree partially, 3 if you are undecided, 2 if you are in partial disagreement and 1 if you are in complete disagreement.

- a) Comsol Multiphysics was of great help to understand electromagnetism.
- b) The use of Comsol Multiphysics gave me an incentive to learn electromagnetism.
- c) The use of Comsol Multiphysics made the class more attractive.
- d) The use of Comsol Multiphysics reinforces my skills and knowledge in electromagnetism.
- h) The feedback from the professor was important.
- j) The use of a software in this class made a significant difference on what I learned compare with other courses.
- k) I will recommend this course to other students