Newtonian and Non-Newtonian Blood Flow over a Backward-Facing Step – A Case Study

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Abstract: In this work, the fluid flow over a 2D backward-facing step is analyzed in order to provide a case study for the use of different models for the blood dynamic viscosity in COMSOL Multiphysics. Three non-Newtonian models, as well as the Newtonian model are used to study the shear stresses and the reattachment length as a function of the fluid speed. The non-Newtonian models used in this study are the Carreau model, the power law model, and one of its variants, the Walburn-Schneck model. For the models studied, a transition from a Newtonian to a non-Newtonian behavior is observed as the center line speed is decreased.

Keywords: Non-Newtonian fluid flow; Dynamic viscosity; Blood flow; Power law model; Carreau model.

1. Introduction

The development of atherosclerosis is often associated with local hemodynamics including shear stress and recirculation zones. However, in particular with respect to the atheromatous plaques, there does not seem to be an agreement if they are correlated with high or low wall shear stress [1, 2]. Computational studies of the blood flow have been used to address this problem, but the modeling is not straightforward. This is due to the non-Newtonian nature of the blood at shear rates less than 100 s^{-1}, with the shear-thinning property of viscosity being considered to be the most significant non-Newtonian characteristic of blood [3]. Because the instantaneous shear rate over one cardiac cycle in several arteries varies from 0 to 1000 s^{-1}, studies that use a Newtonian model assuming that shear rates are always greater than 100 s^{-1}, are oversimplifications of the blood flow.

In the first approximation the strain dependence of the blood viscosity can be mimicked using a power law [4]. While this approach gives acceptable results in the limit of low shear rate, the asymptotic decrease in viscosity at high shear rates is not reflective of the real blood behavior. This poses limitations on its range of applicability. One variant of the power-law model is the Walburn-Schnek model [5], which while including the effects of the hematocrit count on the blood dynamic viscosity, still shows an asymptotic decrease at high strains. Nevertheless, the application of these models shows significant differences between the results of computational analysis of the fluid flow modeled with Newtonian and non-Newtonian models, respectively [6]. In order to circumvent the disadvantages of the power law model, alternative approaches such as the Carreau – Yasuda model [4], make use of viscosity functions that have finite values both at low and high shear. Simulations of steady and oscillatory flows using these models [7] also highlight the effect on the fluid flow when using a strain dependent viscosity, in particular with regard to the magnitude of the flow which was found to be smaller in the non-Newtonian case. For a more extensive analysis of the advantages and disadvantages of various models used to model the blood properties, please refer to Ref. 4 and 6.

In this study four blood models, three non-Newtonian (the power law, the Walburn – Schnek, and the Carreau ones) and one Newtonian, were used to evaluate the flow over the simple geometry of a backward-facing step, using COMSOL Multiphysics and its Chemical Engineering Module, in order to identify the flow characteristics of a fluid with properties similar with that of the blood. The reattachment length and the shear stress are determined as a function of speed in the four models.

2. Methods

2.1 Viscosity models

The parameters used for modeling the blood dynamic viscosity η as a function of the strain rate $\dot{\gamma}$ in the four models employed are similar with those used in Ref. 6. The four models are as follows:

1. Newtonian model with:

$$\eta = 0.0035 \text{ Pa} \cdot \text{s} \quad (1)$$

This viscosity value corresponds roughly to a H = 40% hematocrit count.

2. Power law model with:

$$\eta = \eta_0 \dot{\gamma}^{n-1} \quad (2)$$

where η_0 = 0.035 and n = 0.6 [6, 8].

3. Walburn – Schneck model with:

$$\eta = \mathrm{C_1 e^{C_2 H}} \cdot \mathrm{e}^{C_4 \cdot TMPA/H^2} \cdot \dot{\gamma}^{\mathrm{-C_3}H} \quad (3)$$

where C_1 = 0.00797, C_2 = 0.0608, C_3 = 0.00499, C_4 = 14.585 g^{-1}, H = 40% and $TPMA$ = 25.9 g/l [5, 6].

4. Carreau model with:

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\lambda\dot{\gamma})^2]^{\frac{n-1}{2}} \quad (4)$$

where the viscosity at high shear rate η_∞ equals the value for the Newtonian model (i.e. 0.0035 Pa s) while the value at zero shear is η_0 = 0.056 Pa s. Also: λ = 3.313s and n = 0.3568 [6, 8].

2.2 Flow field modeling

The geometry investigated consists of a 10 cm long channel with a backward facing step 2 cm from the inlet of the channel. At the step the width of the channel increases from 1 mm to 2 mm, which leads to a recirculation area next to the backstep.

The flow fields for each structure were modeled using the COMSOL Multiphysics 3.5 package and its Chemical Engineering Module. The flow fields are obtained by solving in 2D the Navier – Stokes equations of motion for an incompressible fluid, coupled with the continuity equation:

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)\mathbf{u}\right] = \nabla\left[-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}})\right]$$

$$\nabla \cdot \mathbf{u} = 0 \quad (6)$$

where $\boldsymbol{u}$ is the velocity, ρ is the fluid density (~ 1050 kg/m^3), η is the fluid viscosity which is dependent on the stress rate, t is the time, and p is the pressure. The equations are solved for a steady state flow with no-slip boundary conditions at all the top and bottom boundaries. A parabolic velocity profile is used for the inlet with the velocity at the center line being varied from 0.025 to 0.8 m/s. While the parabolic profile is not a good approximation for the non-Newtonian cases, the ratio between the length and width of the smaller diameter channel is large enough to ensure that the flow field converges to the proper profile well before the step change in the diameter. The boundary condition for the outlet is zero pressure. The equations are solved for a mesh with a typical number of elements of about ~ 7,000, with a minimum element quality of 0.8210. For central inlet speeds less than 0.5 m/s, the Direct (SPOOLES) solver is used. For speeds larger than 0.5 m/s, convergent solutions were found using the iterative GMERS solver. Also, for the higher speeds, the mesh resolution had to be increased in the vicinity of the backward facing step to insure convergence.

3. Results and Discussion

Figures 1a and 1b show the flow fields for the Carreau model for two inlet velocities, i.e. 0.025 m/s and 0.8 m/s. Similar flow fields have also been obtained for the Newtonian, power law, and Walburn – Schneck model at various speeds between 0.025 m/s to 0.8 m/s. Comparison of the velocity profiles at the outlet between the different models shows distinct differences between them. While for all the inlet speeds the Newtonian model exhibits the characteristic parabolic profile, at low speeds the dynamic viscosity models show profiles that are flatter at the center and exhibit larger velocity gradients towards the walls (Figure 1c). This is a direct consequence of the shear thinning property of blood. On the other hand, for the higher inlet speed (0.8 m/s), the velocity profiles at the outlet for the Newtonian and Carreau model are almost

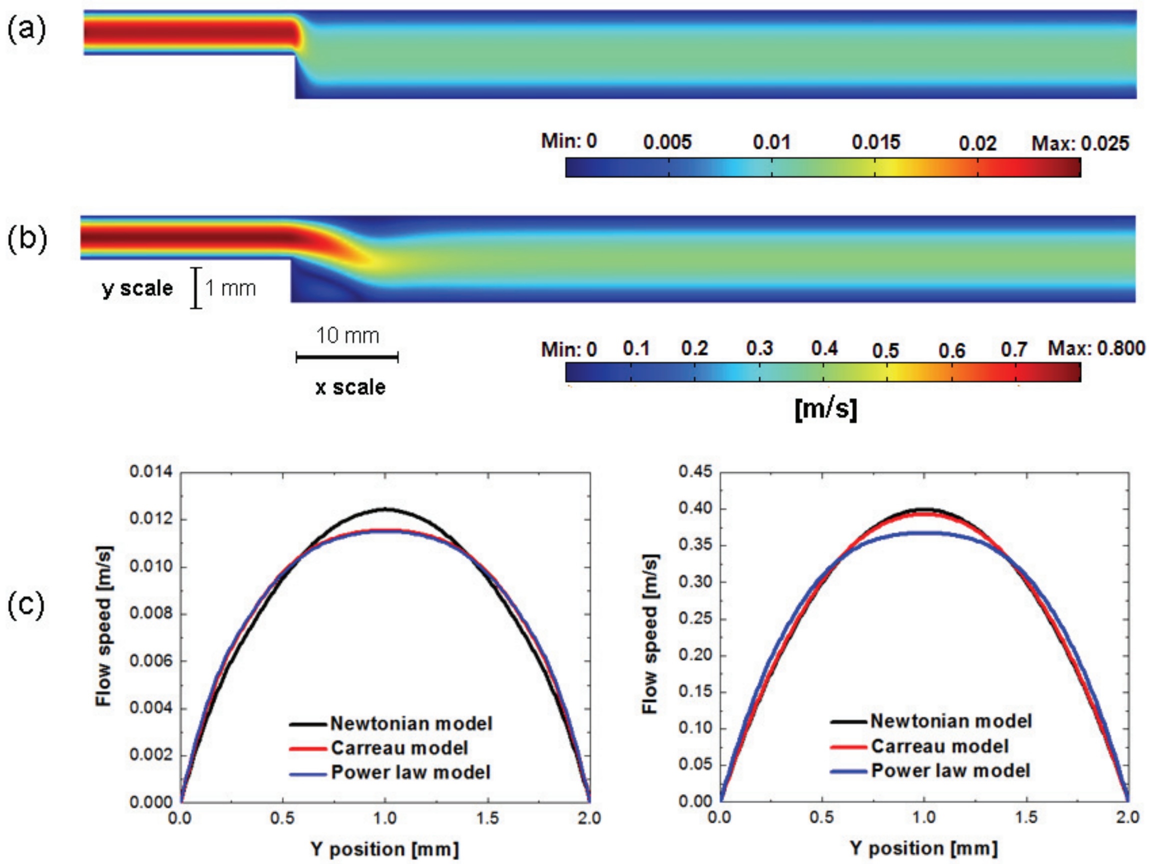

Figure 1. Velocity field for the Carreau model for inlet center line velocities of (a) 0.025 m/s, and (b) 0.8 m/s (For clarity different scales are used for the *x* and *y* axis throughout the paper); (c) Comparison between the velocity profiles at the outlet for the Newtonian, Carreau and Power –law models: (left) inlet speed = 0.025 m/s (the power law profile overlaps almost perfectly with the Carreau profile); (right) – inlet speed = 0.8 m/s.

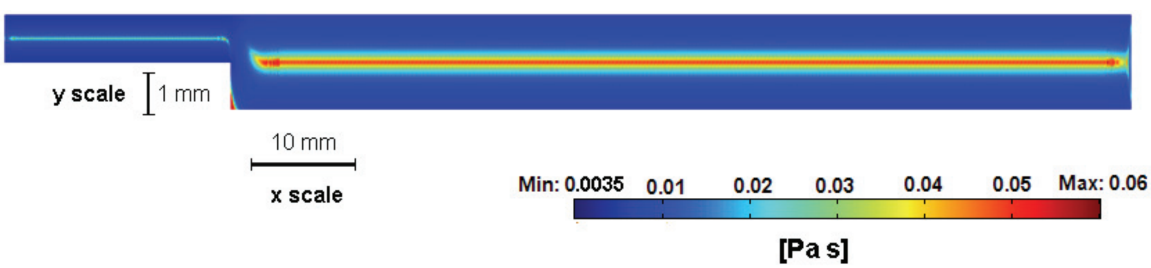

Figure 2. Dynamic viscosity at 0.025 m/s inlet center line velocity for the Carreau model (the minimum 0.0035 Pa s in viscosity corresponds to the viscosity in the Newtonian model).

indistinguishable, which is a consequence of the convergence of the Carreau model to the constant viscosity Newtonian one. In contrast, the power law and the Walburn - Schneck models maintain the profile characteristic to the shear thinning due to their decreasing viscosity with strain.

The above conclusion correlates well with the dynamic viscosity profiles for all the non-Newtonian models investigated which show decreased viscosity at the center of the channels and higher viscosity close to the walls where the stresses are larger (Figure 2). As the velocity is increased, the average viscosity for all non-Newtonian models becomes closer to the value for the Newtonian case. The differences are smaller for the Carreau model given the fact that this model limits the decrease in the viscosity as the strain is increased. This is not the case for the power-law models.

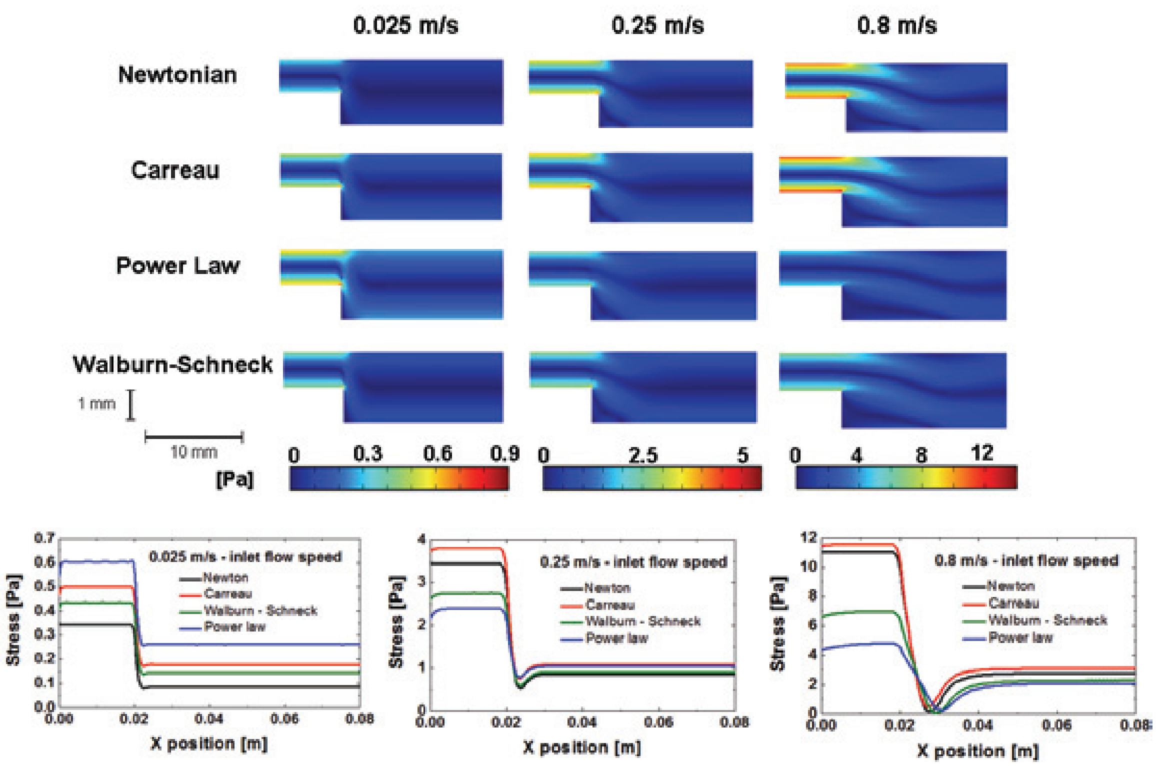

Figure 3.(Top) Close up on the stress profiles obtained at three inlet speeds 0.025 m/s, 0.25 m/s and 0.8 m/s in the four models used. (Bottom) The position dependence of the stress along the top boundary of the channel.

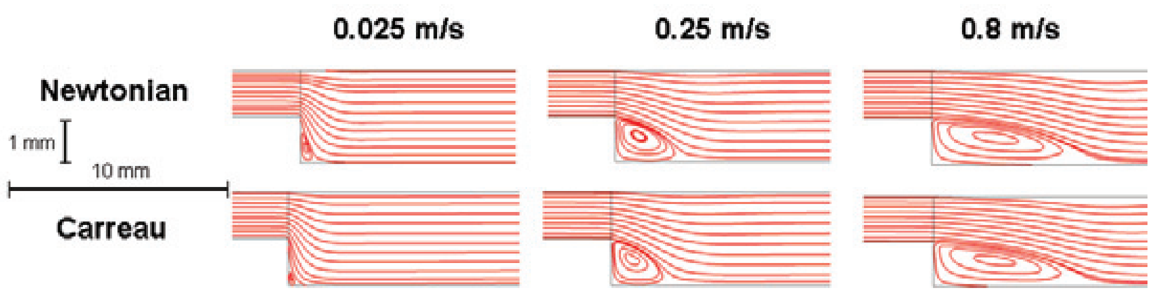

Figure 4. Streamline plots for the Newtonian and Carreau models at different inlet speeds.

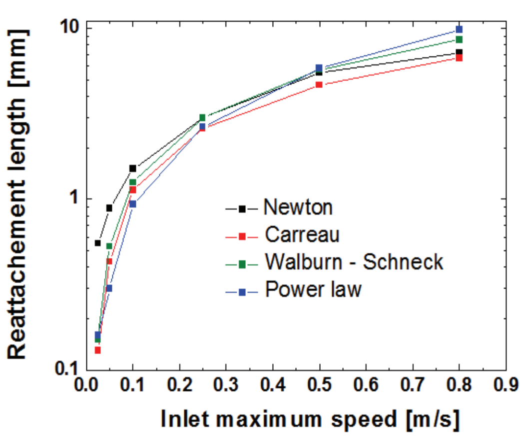

Figure 5. Reattachment length vs. inlet speed.

The complex profile of the dynamic viscosity impacts both the distribution of the shear stress in the structure as well as the structure of the recirculation area near the backward facing step. Surface images of the stress profiles (Fig. 3) show that at low speeds the Newtonian model consistently underestimates the value of the shear stress close to the walls. A line cut along the top wall of the structure shows that this is true not only in the narrow channel and close the backward facing steps where the speed gradients are large, but also in the wider channel. As the inlet speed is increased all the models give similar results for the stress values into the wider channel and close to the backward facing step. This corresponds to the transition of the fluid flow from a shear dependent viscosity to a constant viscosity, as the shear rate is increased (for example for the Carreau model this corresponds with shear rates above ~ 250 s^{-1}). The stresses in the narrower channel are underestimated by the power law based models which at very high stress rates continue to predict smaller viscosity rates than the ones corresponding to the real blood behavior.

The transition between the non-Newtonian and Newtonian regimes is also observed in the dependence of the reattachment length associated with the recirculation area formed behind the backward facing step on the fluid speed. Figures 4 and 5 show that as the fluid speed is increased the reattachment length for all the models increases. The normalized differences are smaller at the higher speeds where the effects associated with the strain dependent viscosity are reduced.

4. Conclusions

The COMSOL Multiphysics package and the Chemical Engineering module were used successfully for the calculation and visualization of the flow fields of a fluid with similar properties to those of blood. For the simple geometry of a backward facing step, the solution to the Navier – Stokes equations for a non-Newtonian fluid is obtained using different models for the dynamic viscosity dependence on the strain at different inlet fluid speeds. The steady state solution indicates that the non-Newtonian character of the blood is particularly important when analyzing flow characteristics such as shear stress and reattachment length at low speeds. At large flow speeds, the differences between the different models are reduced, especially for models such as the Carreau model that limit the change in viscosity at large strains.

5. References

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