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LAMINAR THERMAL MIXING IN PLANE SHEAR FLOW

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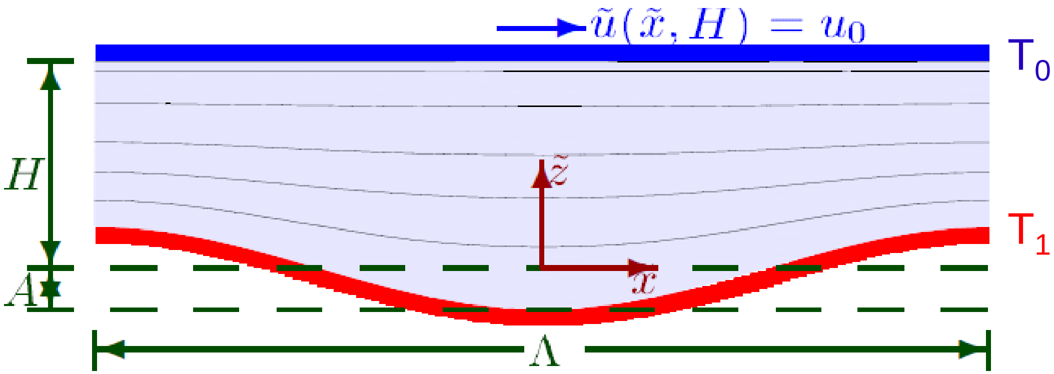
- Two-dimensional
- Steady
- Periodic BC
- Sinusoidally corrugated substrate
- Constant λ, c_p and ρ

Parameters:

- Wavelength: Λ
- Amplitude: A
- Mean gap width: H
- Lid velocity: u_0
- Temperature: $T_1 > T_0$
- Viscosity: $\eta(T) = \eta_0 \left(1 - \eta^* T\right)$



Non-dimensional model equations

Simplifications: No buoyancy and no dissipation

Continuity equation: $u_x + w_z = 0$

Navier-Stokes equations:

$$\mathrm{Re}\left[uu_x + wu_z\right] = -p_x + 2\partial_x\left[\left(1-\eta^* T\right)u_x\right] + \partial_z\left[\left(1-\eta^* T\right)\left(u_z + w_x\right)\right]$$

$$\mathrm{Re}\left[uw_x + ww_z\right] = -p_z + 2\partial_z\left[\left(1-\eta^* T\right)w_z\right] + \partial_x\left[\left(1-\eta^* T\right)\left(u_z + w_x\right)\right]$$

Temperature equation: $\mathrm{Pe}\left[uT_x + wT_z\right] - \left[T_{xx} + T_{zz}\right] = 0$

Dimensionless numbers:

$$\mathrm{Re} = \frac{\rho_0 u_0 \Lambda}{2\pi\eta_0} \qquad \mathrm{Pe} = \frac{\Lambda u_0 \rho_0 c_p}{2\pi\lambda}$$

Methods of solution

- Semi-analytical approach
  - Based on a variational formulation
  - Stokes flow limit Re → 0
  - neglect of thermoviscosity coupling term

- Finite element methods

  Comsol® Multiphysics modules
  - Incompressible Navier-Stokes
  - Heat transfer/convection

Unilaterally coupled set of equations:

- Hydrodynamic field:

$$\begin{aligned} u_x + w_z &= 0 \\ -p_x + u_{xx} + u_{zz} &= 0 \\ -p_z + w_{xx} + w_{zz} &= 0 \end{aligned}$$

$$\begin{aligned} u(x,-a\cos x) &= 0 \\ w(x,-a\cos x) &= 0 \\ u(x,h) &= 1 \\ w(x,h) &= 0 \end{aligned}$$

- Temperature field

$$\mathrm{Pe}\left[uT_x + wT_z\right] - T_{xx} - T_{zz} = 0$$

$$\begin{aligned} T(x,-a\cos x) &= 1 \\ T(x,h) &= 0 \end{aligned}$$

a: non-dimensional amplitude

h: non-dimensional gap-width

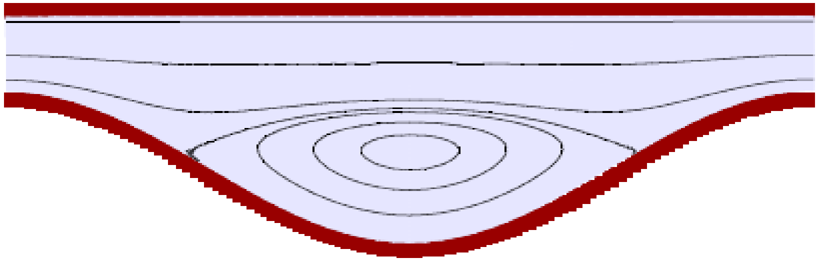
Decoupling:

1) Isothermal flow 2) Heat conduction with convection

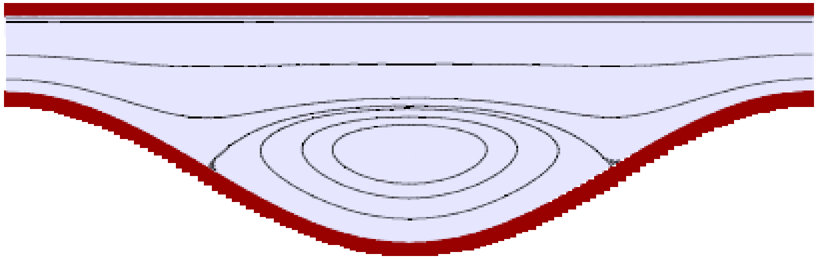
Lubrication approximation

⇨ Analytic solution for the assumption $a << 1$

Finite element solution



Lubrication approximation



⇨ Reasonable results even for moderate amplitudes $a < 1$

Stokes flow - Heat conduction & convection

Variational formulation: $\delta I = 0$ with non-local functional:

$$I := \int_{-\pi}^{\pi} \int_{-a\cos x}^{h} \left[-\mathrm{Pe}\, T(x,z) \left(u \cdot \nabla \right) T(-x,z) + \nabla T(x,z) \cdot \nabla T(-x,z) \right] \mathrm{d}z\mathrm{d}x$$

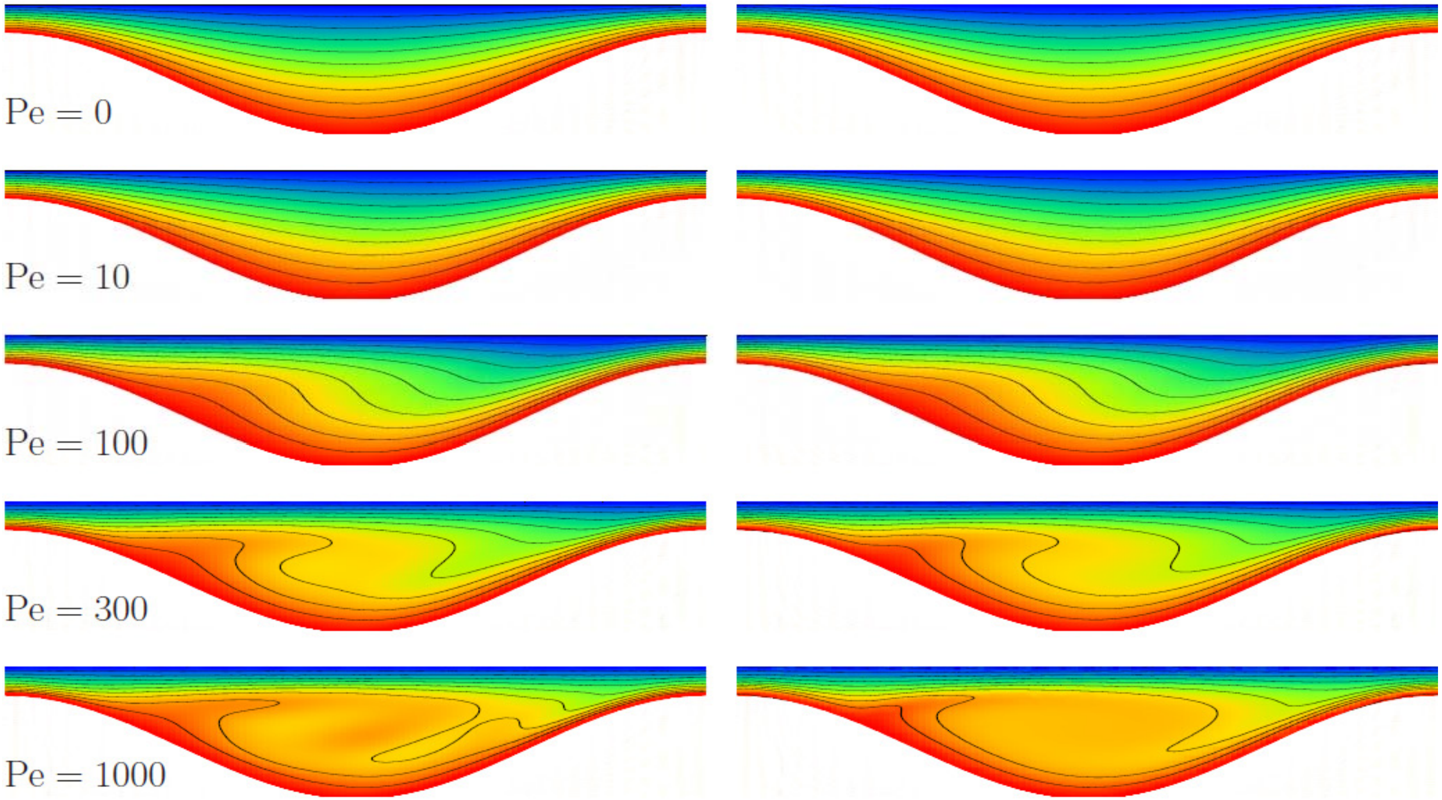
Reproduces the heat conduction equation with convection:

$$\mathrm{Pe}\left[uT_x + wT_z \right] - T_{xx} - T_{zz} = 0$$

⇨ Solution by Ritz‘s direct method with appropriate base functions.

Stokes flow - Temperature field

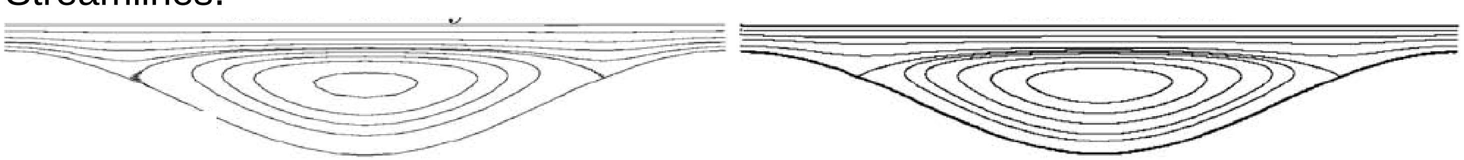
Semi-analytical solution | Finite element solution



$a = 1/2$

$h = 3/4$

Streamlines:



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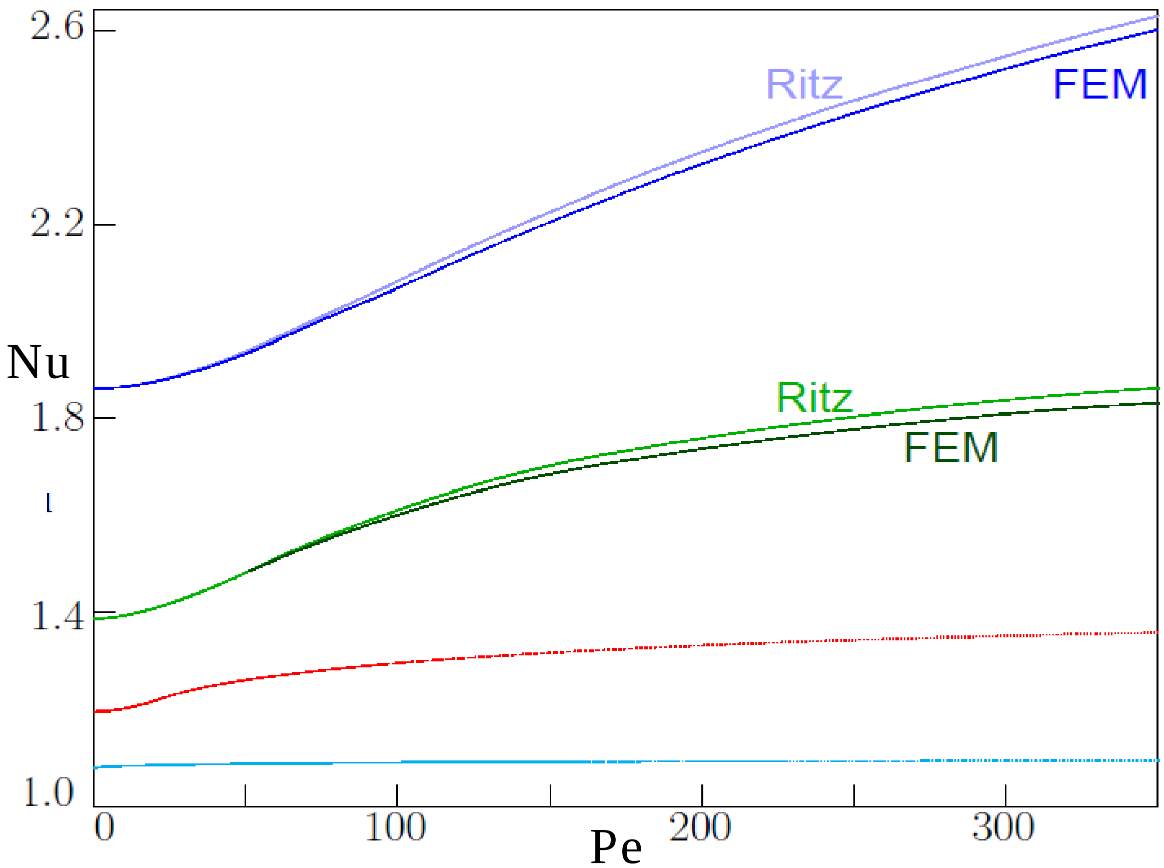
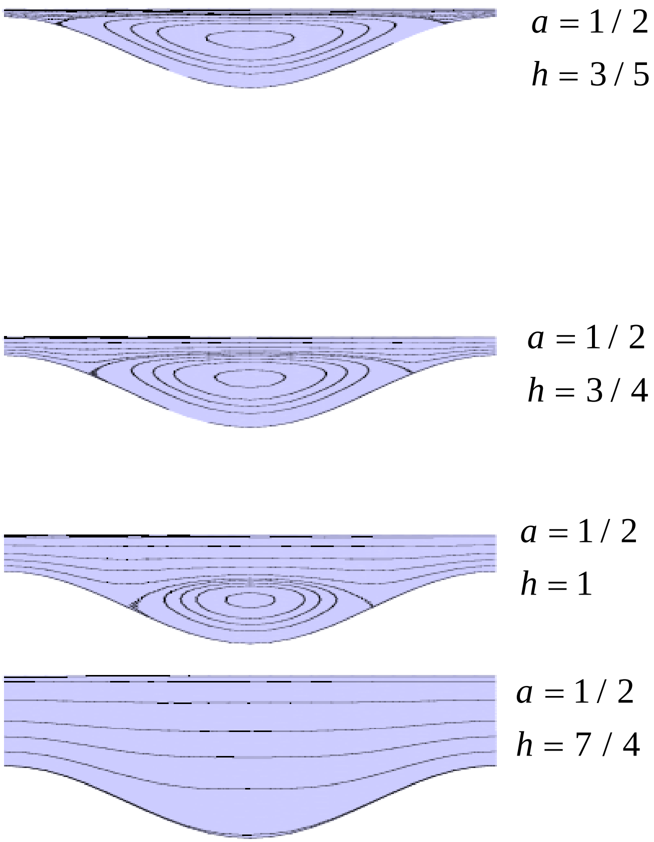
Nusselt number $\mathrm{Nu} = \frac{h\dot{Q}}{\Lambda\left(T_2 - T_1\right)}$ vs. Peclét number $\mathrm{Pe} = \frac{\rho c_p u_0 \lambda}{2\pi\Lambda}$

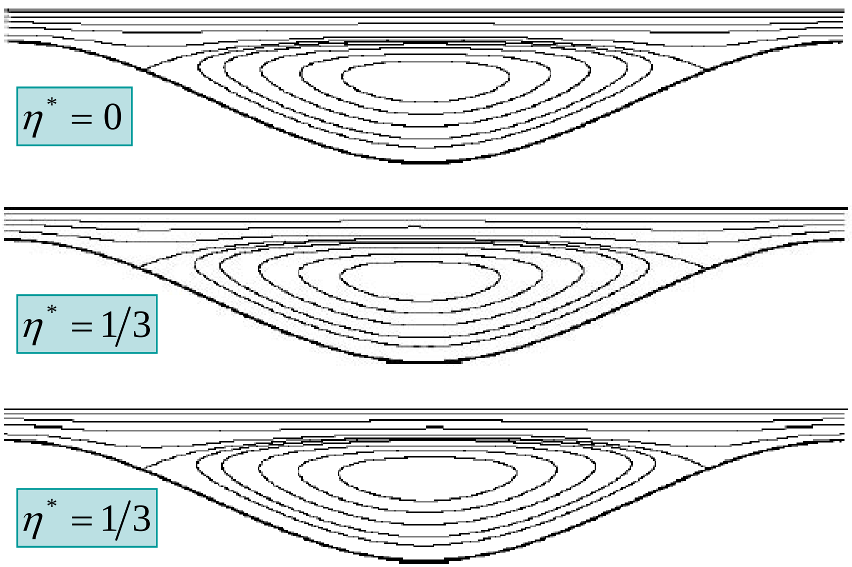




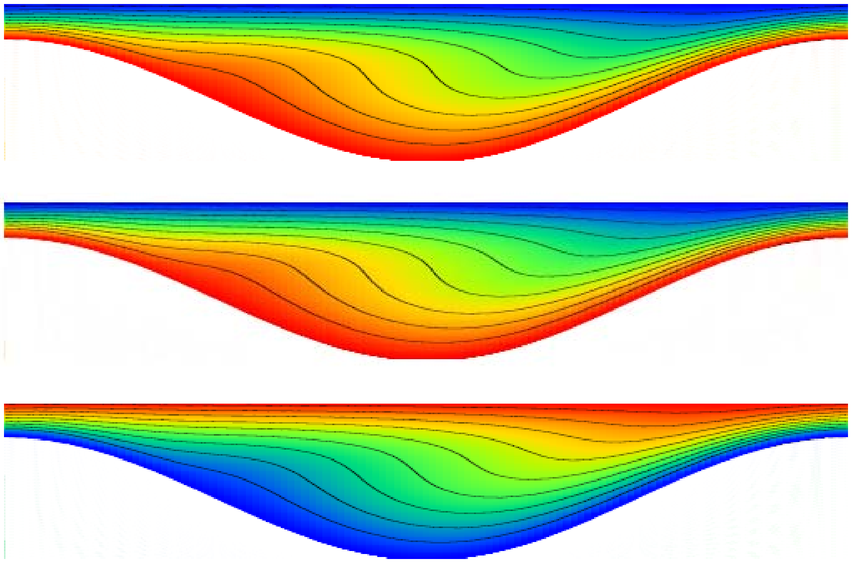
Stokes flow - Thermal feedback

Thermoviscosity: $\eta(T) = \eta_0 \left(1 - \eta^* T\right)$

Hydrodynamic streamlines:



Temperature field:

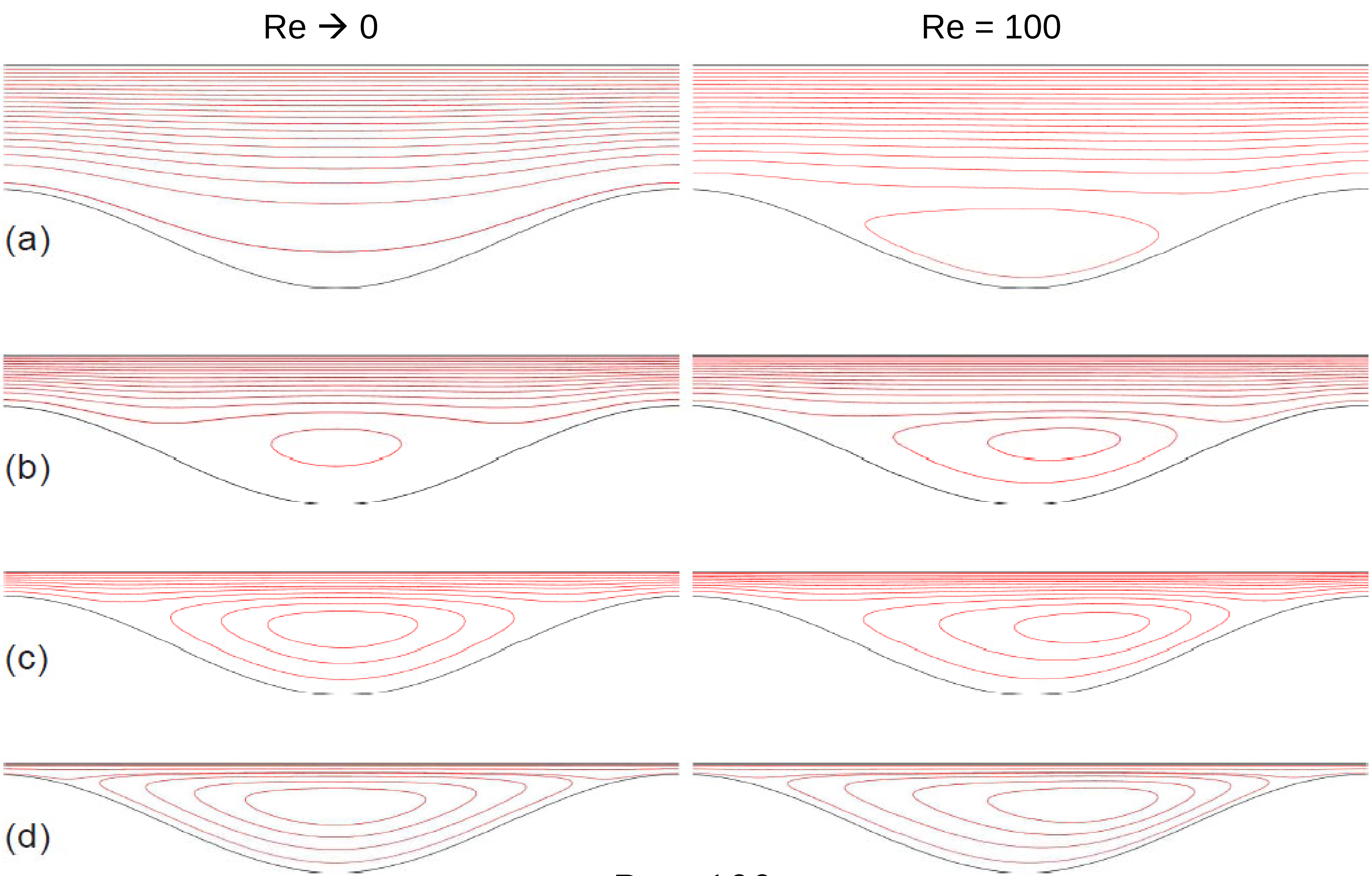


$a = 1/2$

$h = 3/4$

$\mathrm{Pe} = 100$

Inertial effects – Flow structure



(a) $a = 1/2$, $h = 7/4$

(b) $a = 1/2$, $h = 1$

(c) $a = 1/2$, $h = 3/4$

(d) $a = 1/2$, $h = 3/5$

$\mathrm{Pe} = 100$

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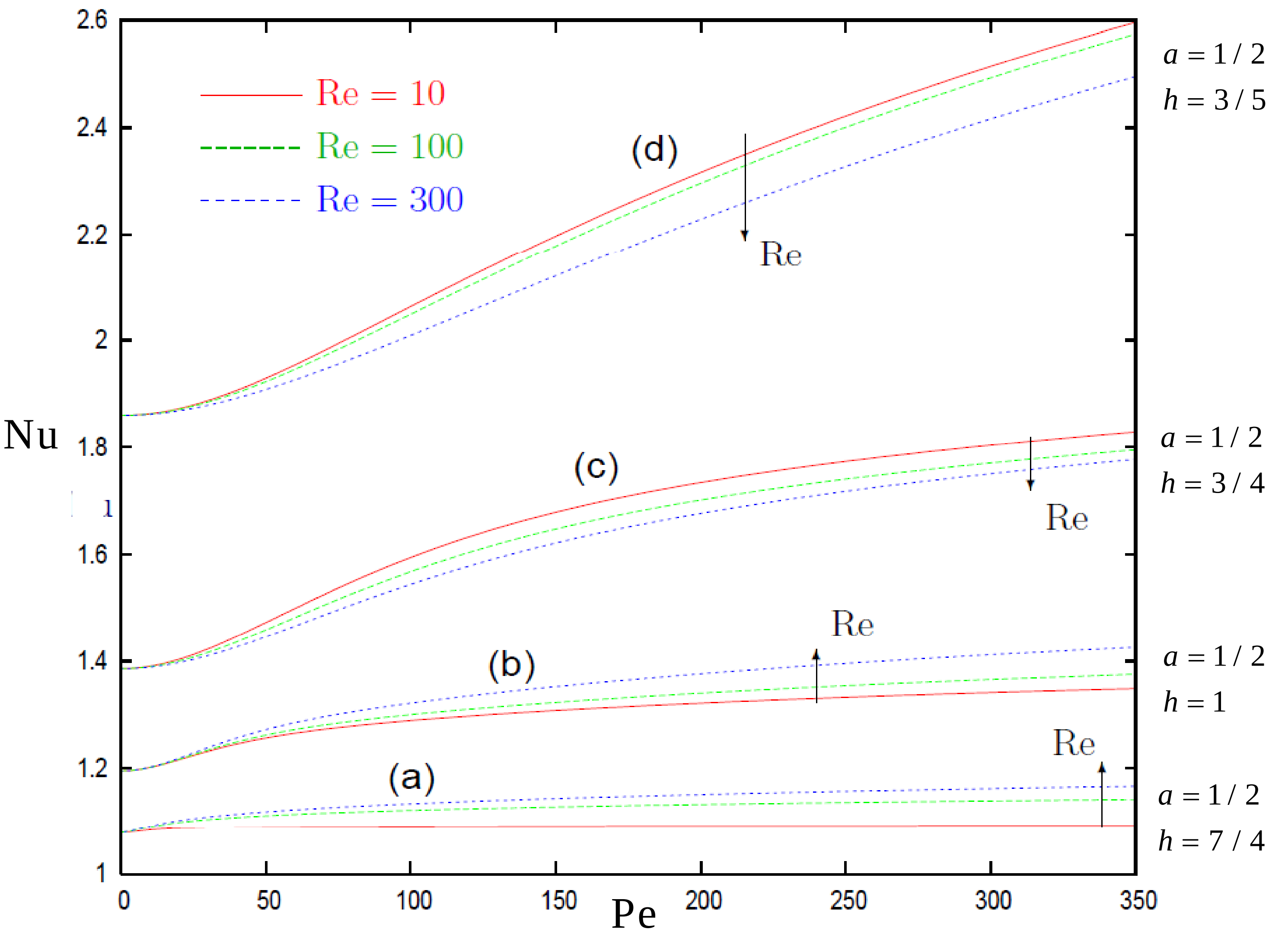
Inertial Effects – Global Heat Transport



Conclusions

- Stokes flow limit: Re → 0
  - Global Heat transfer: Comparison between semi-analytical and finite element solution
  - Influence of thermoviscosity

- Inertial effects on global heat transfer
  - Competing kinematical and inertial effects.

Available literature:

M. Scholle, A. Haas, R.W. Hewson, H.M. Thompson, P.H. Gaskell, N. Aksel, *The effect of locally induced flow structure on global heat transfer for plane laminar shear flow*. Int. J. Heat & Fluid Flow **30**, 175-185 (2009).

M. Scholle, A. Haas, M.C.T. Wilson, H.M. Thompson, P.H. Gaskell, N. Aksel, *Eddy genesis and manipulation in plane shear flow*. Phys. Fluids **21**, 073602 (2009).