Modelling of thermally induced electrical instabilities in intestine using COMSOL Multiphysics

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Nonlinear Physics and Mathematical Modelling Lab.

Overview

1. Biological Problem
2. Mathematical Model
3. COMSOL Multiphysics Implementation
4. Results

1.1 Paralytic Ileus

- **Temporary intestine destabilization:**
  - impairment of the peristaltic motion
  - 24 - 48 h following main abdominal surgeries
- **Multyfactorial disorder:**
  - mechanical stresses
  - anaesthesia drugs
  - steep temperature changes

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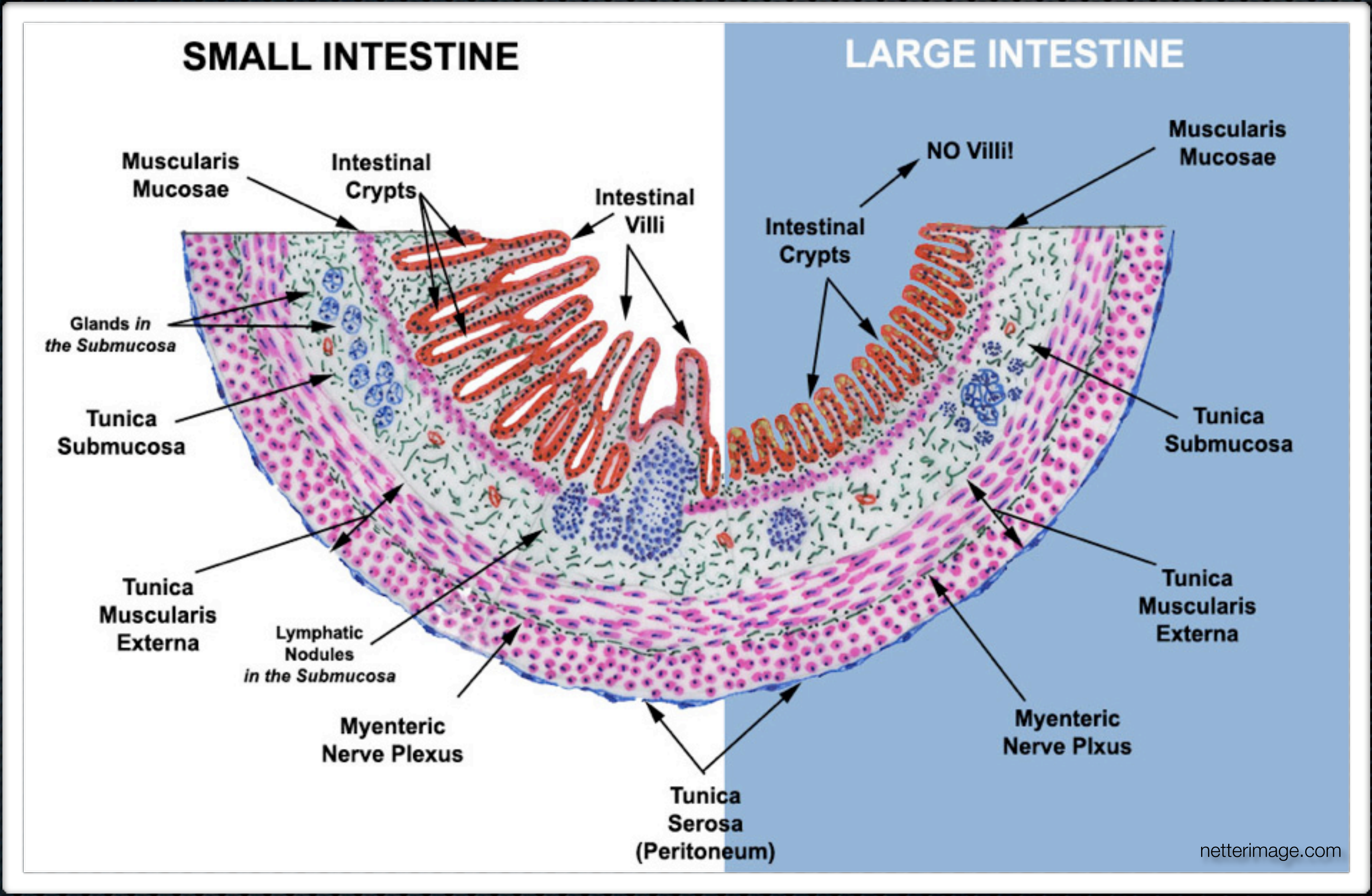
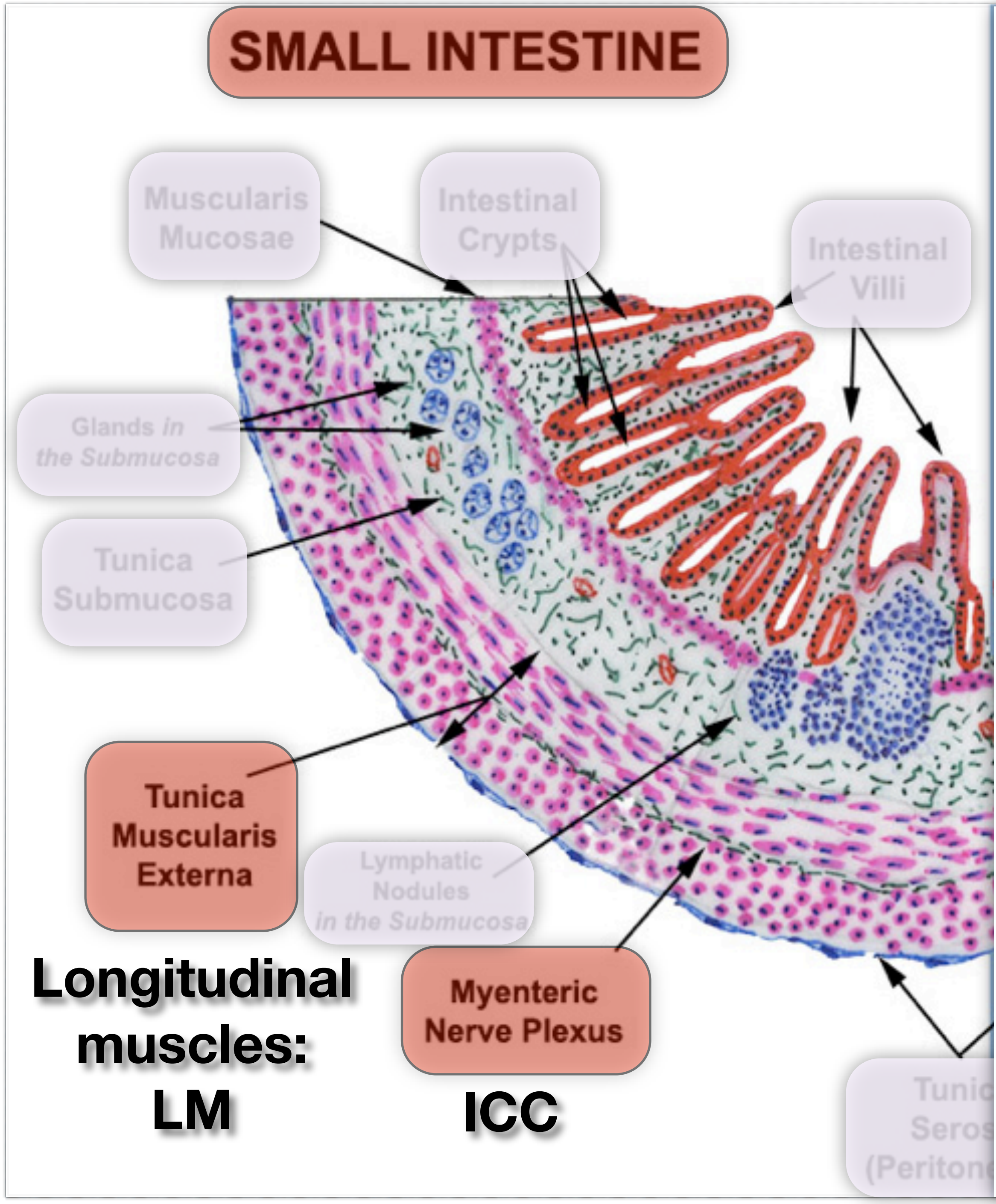
  - **steep temperature changes**

1.2 Intestine Tissue

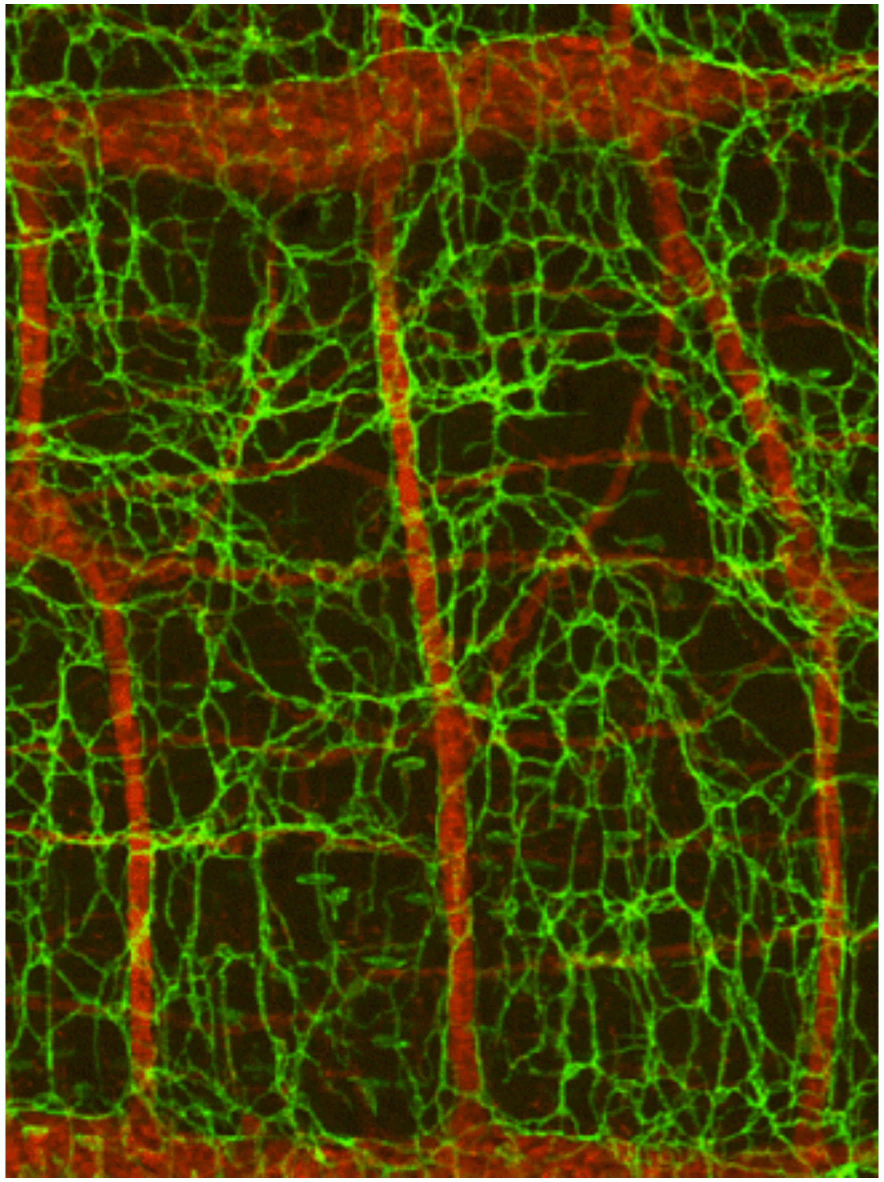


Multilayered structure

1.3 Tissues Modelled



Interstitial cells of Cajal (green)



The Journal of Physiology October 1, 2009, 587 (19)

Multilayered structure

2.1 Math. Modelling

- **Excitable double layered structure: LM, ICC**
  - **Physical distinct but Mathematical coincident**
- **Electric Aliev-Richards-Wikswo model**
  - **Pair of PDEs (4 eqs.)**
  - **FitzHugh-Nagumo scheme**
  - **Reaction-diffusion system**
  - **Homogeneous and Isotropic**
- **Multiphysical Bio-Thermal transport**
  - **thermo-ionic feedback**

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- **Multiphysical Bio-Thermal transport (Pennes)**
  - thermo-ionic feedback

2.2 Equations

ARW-model

$$\partial_t U_l = f(U_l) + D_l \Delta U_l - V_l + F_l(U_l, U_i)$$
$$\partial_t V_l = Q_{10}(T)\,\zeta_l[\delta_l(U_l - \alpha_l) - V_l]$$
$$\partial_t U_i = g(U_i) + D_i \Delta U_i - V_i + F_i(U_l, U_i)$$
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l: **LM**

i: **ICC**

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- Transmembrane potentials
- Slow currents

2.2 Equations

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Nonlinear reaction functions

$$f(U_l) = \gamma_l U_l (U_l - b_l)(1 - U_l)$$
$$F_l(U_l, U_i) = \mu_l D_{li}(U_l - U_i)$$
$$g(U_i) = \gamma_i U_i (U_i - b_i)(1 - U_i)$$
$$F_i(U_l, U_i) = \mu_i D_{il}(U_l - U_i).$$

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\end{aligned}
$$

l: LM

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Excitability parameter

$$\zeta_i(z) = 0.032 + 0.05 \exp\left(-\frac{z}{68}\right)$$

2.2 Equations

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Pennes **bio-heat** equation

$$\partial_t T = \Gamma \Delta T + \Omega(T_a - T)$$

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Isotropic thermal diffusion

Homogeneous perfusion

2.2 Equations

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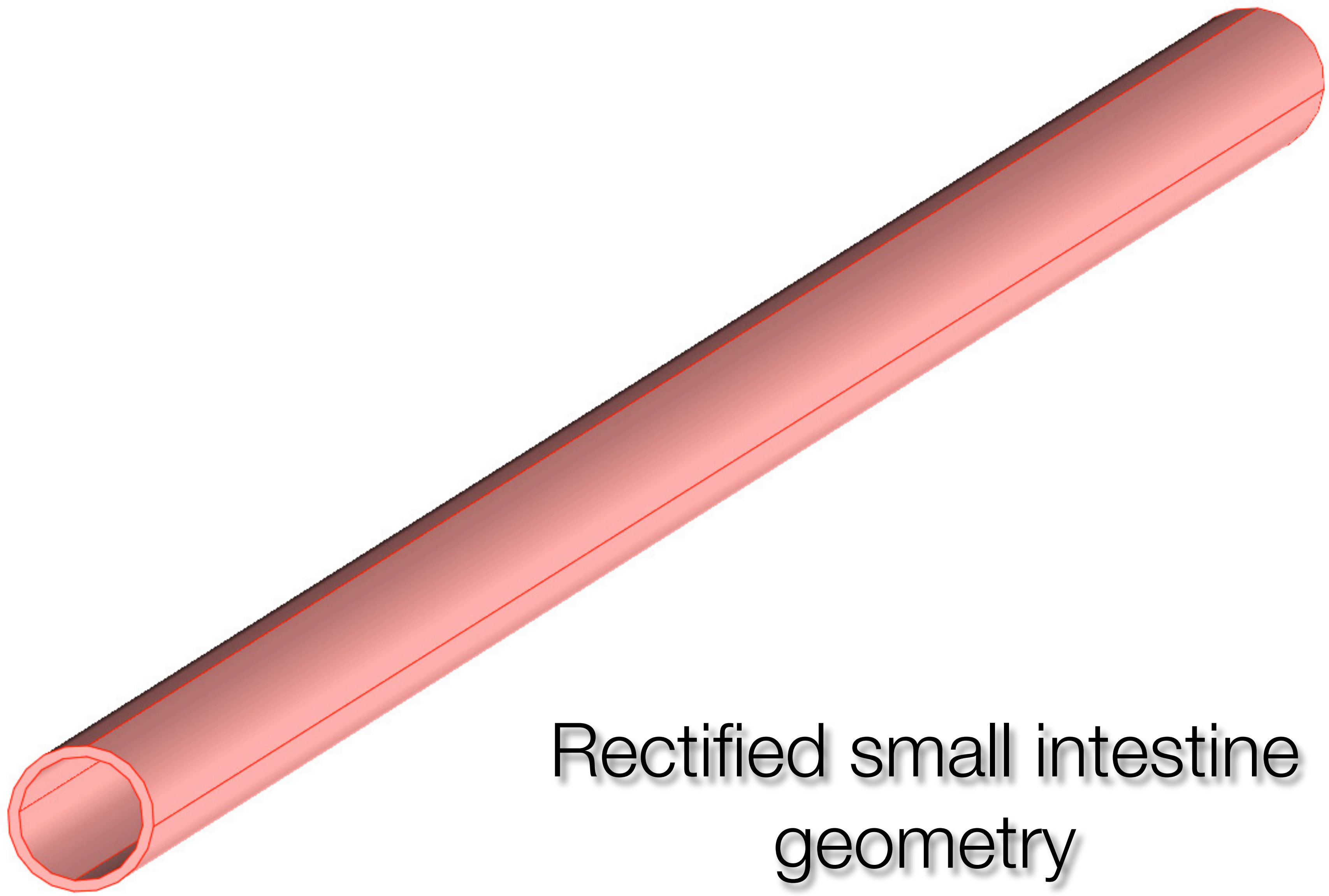
Thermo-ionic **feedback**

$$\boxed{Q_{10} = B \exp\left[\left(T - T_a\right) / 10^{\circ} C\right]}$$

3.0 Geometry Model

- **Cylindrical shape:**

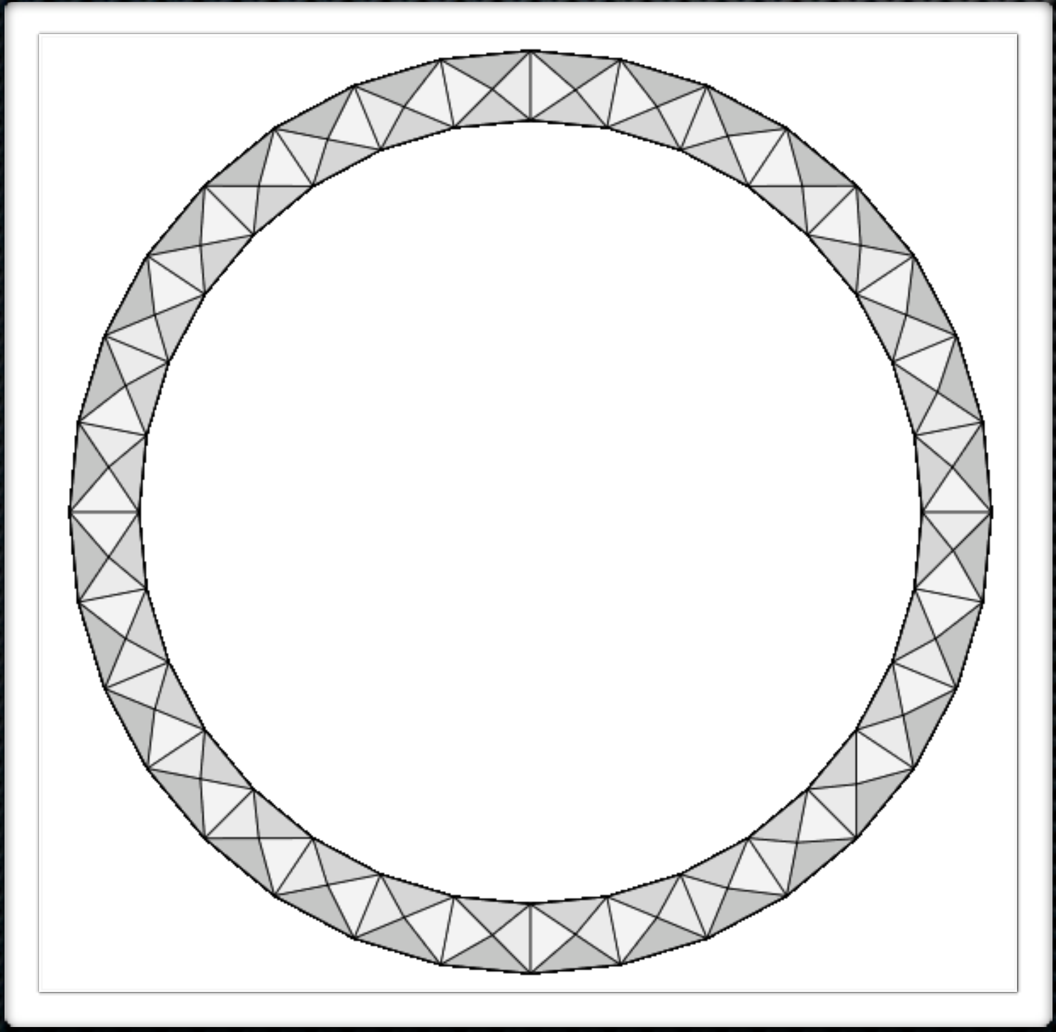
  Ri = 1.7 cm
  Re= 2.0 cm
  L = 240.0 cm

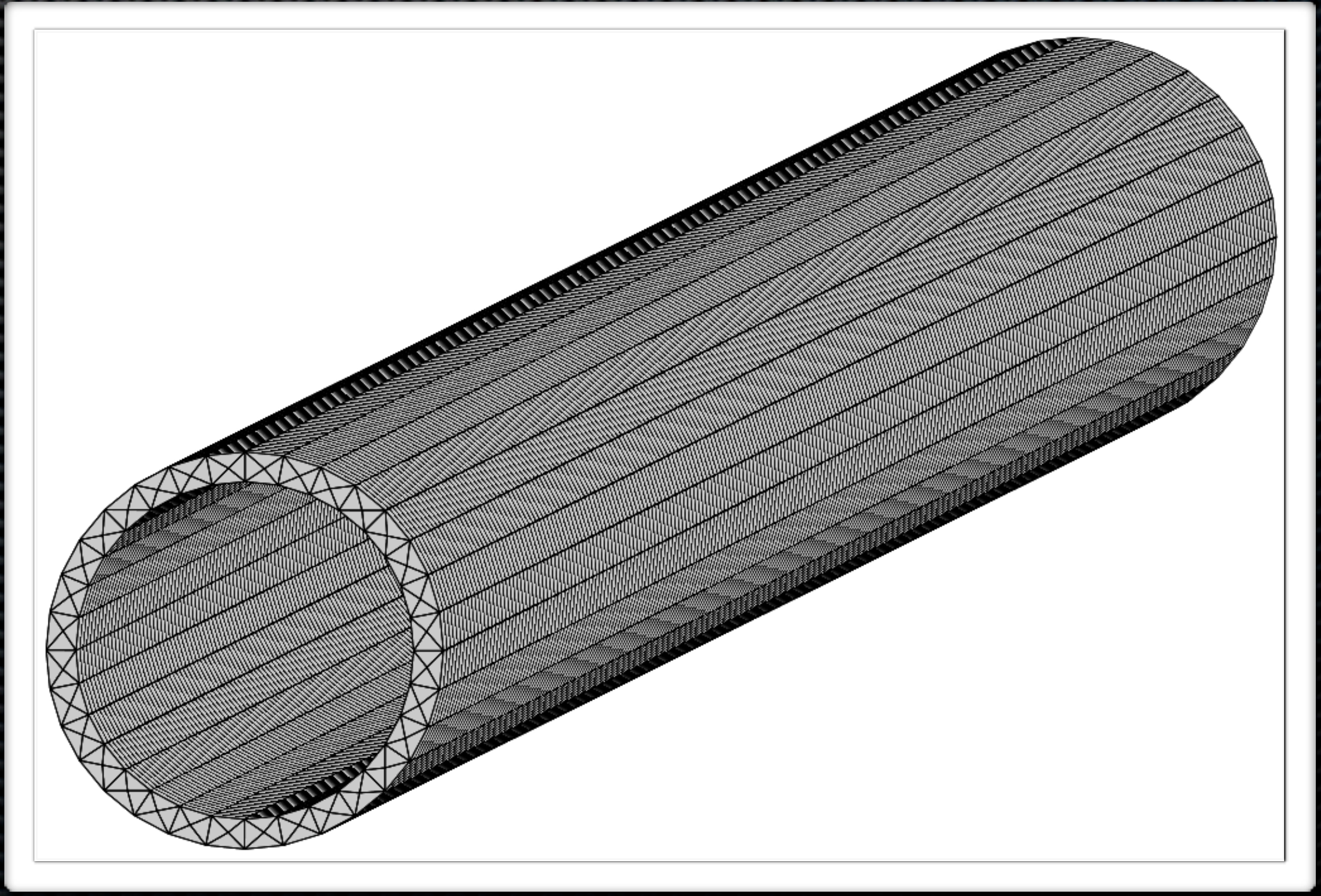


Rectified small intestine geometry

3.0 Geometry Model

- **Tetrahedral swept mesh:**
  3e4 Lagrange Quadratic El. - 8e5 d.o.f.





3.1 Boundary Conditions

- **Electric variables**: **Neumann zero flux**

$$\frac{\partial u}{\partial \mathbf{n}} = 0, \qquad \frac{\partial v}{\partial \mathbf{n}} = 0$$

- **Temperature**: **Dirichlet time-dependent**

$$T_{XB} = T_X + (T_a - T_X)\exp(-t/\tau)$$

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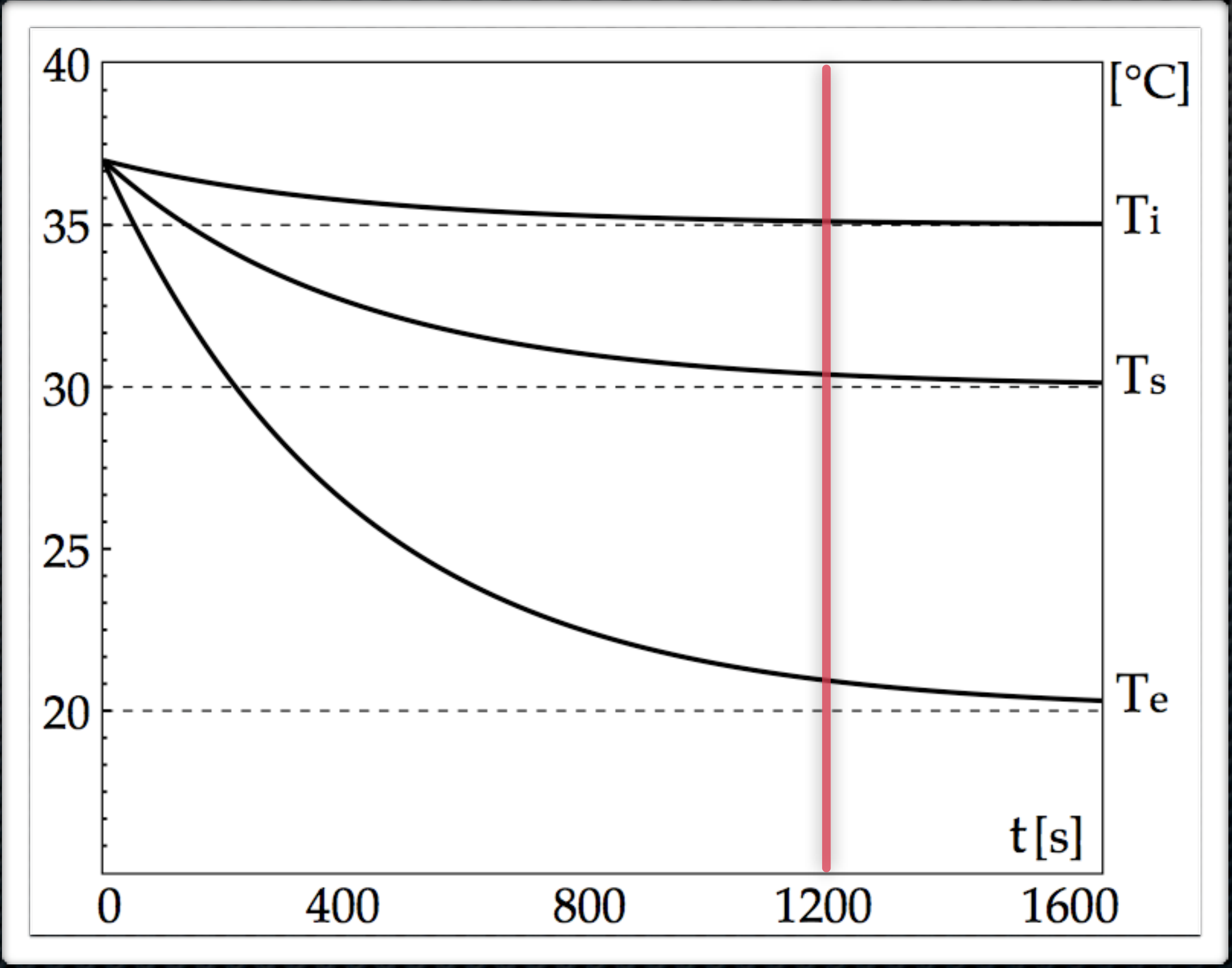
3.1 Boundary Conditions

- **Three thermal punctual regions**

$$T_{XB} = T_X + (T_a - T_X)\exp(-t/\tau)$$

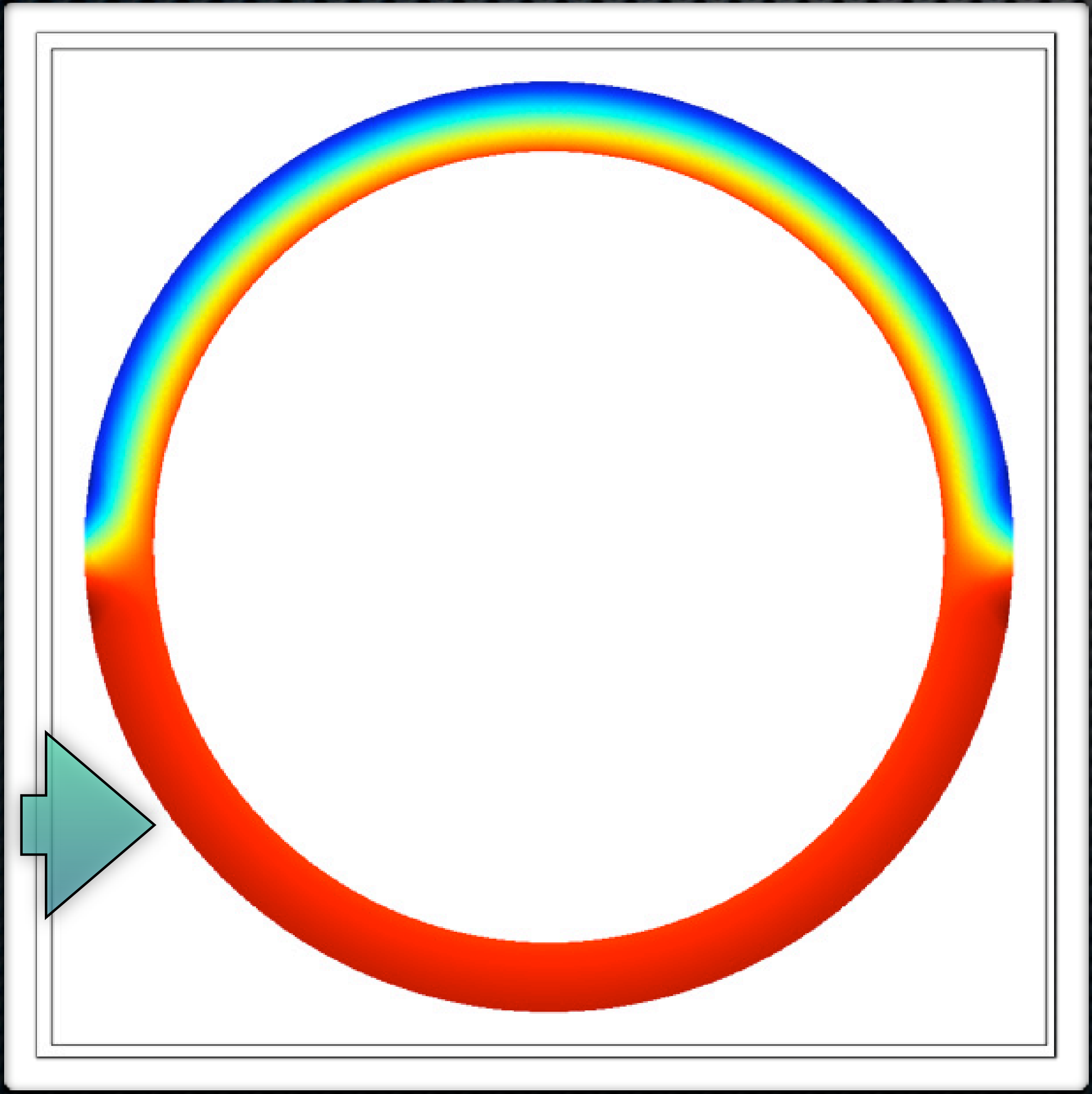
$$\tau = 400\,s$$

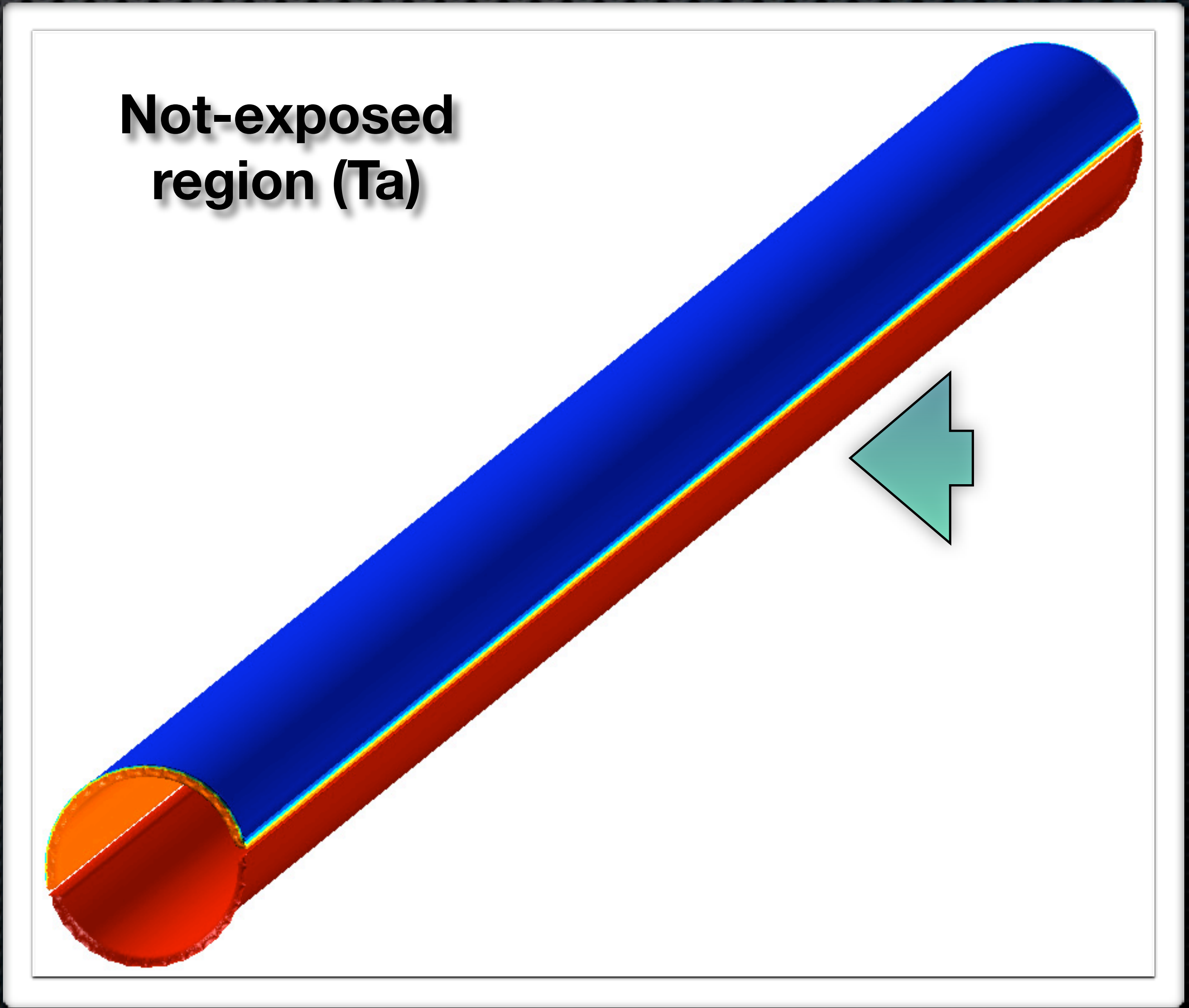


Ta: *reference temp.*
Ti: *internal temp.*
Ts: *sectional temp.*
Te: *external temp.* (exposed tissue)

3.1 Boundary Conditions

- **Three thermal spatial regions:** **21-37 °C**





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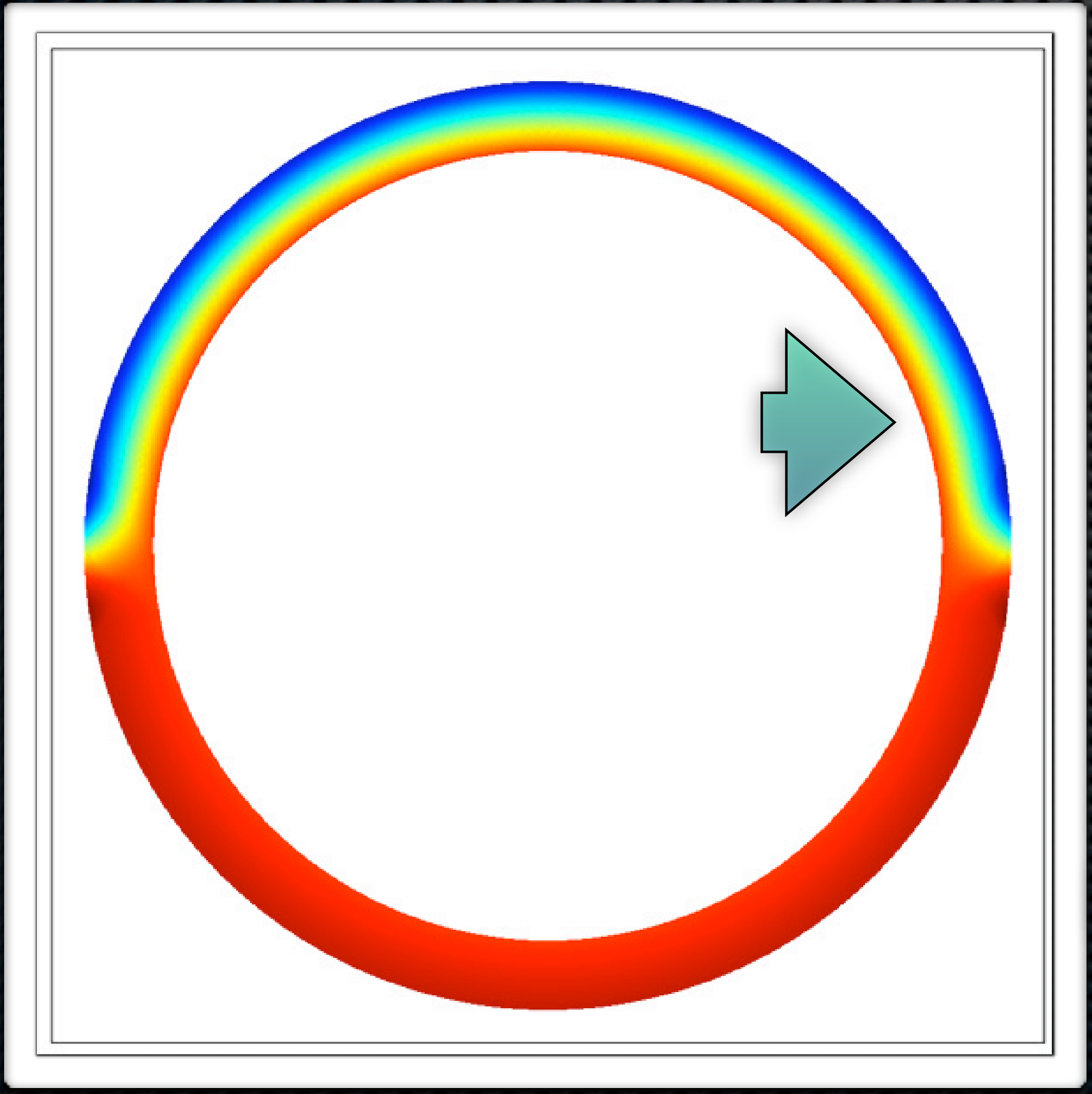

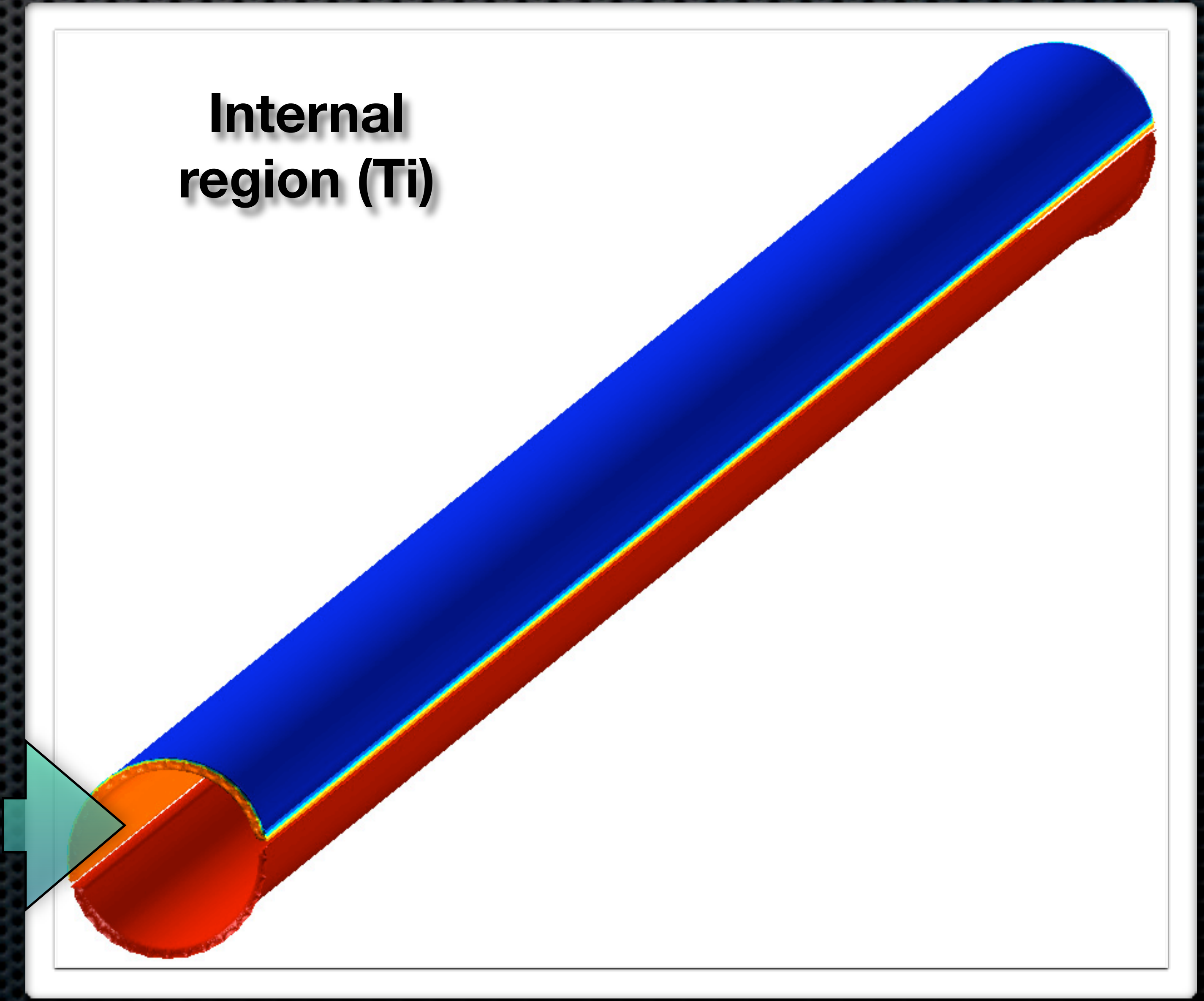

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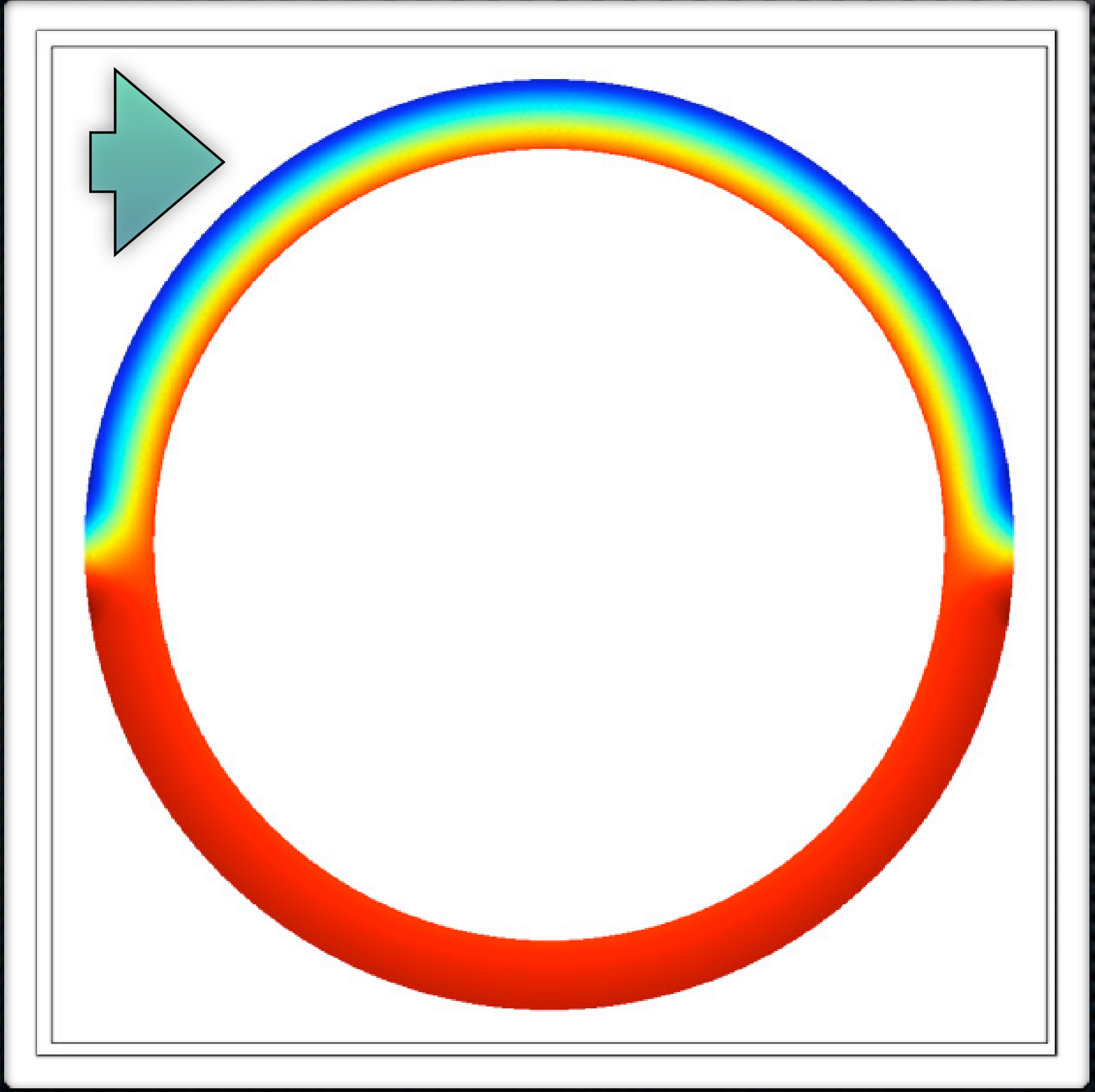

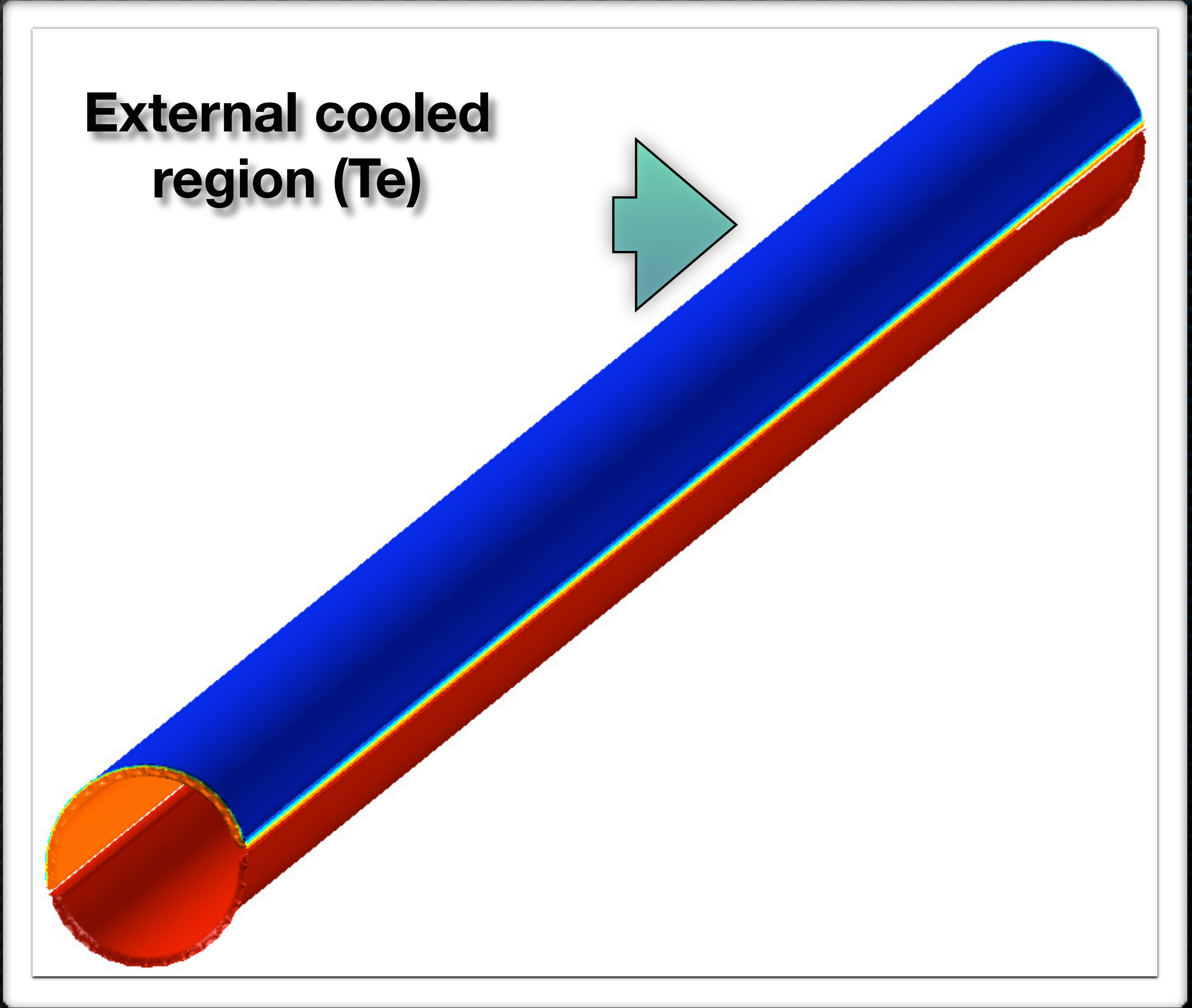

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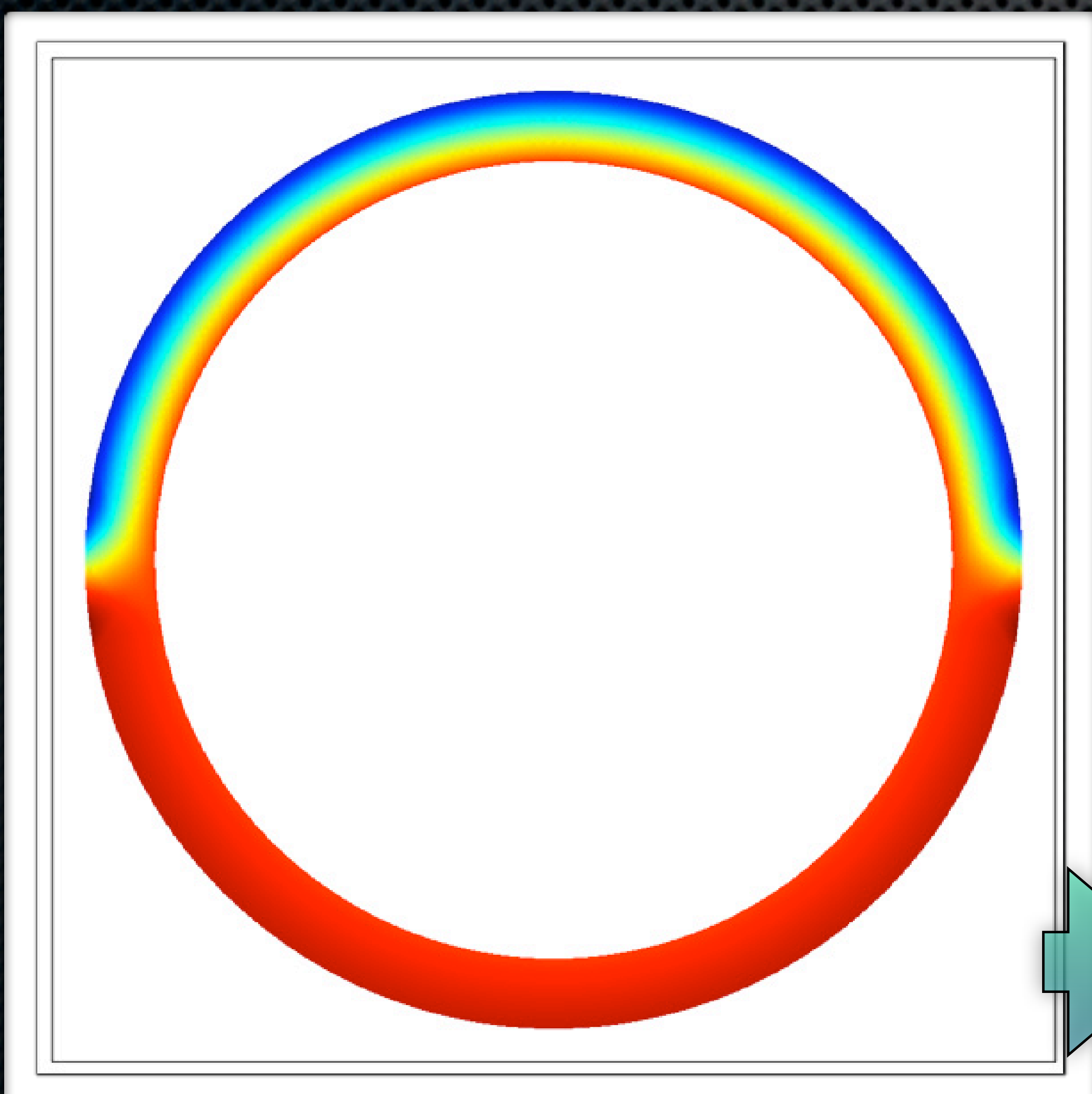
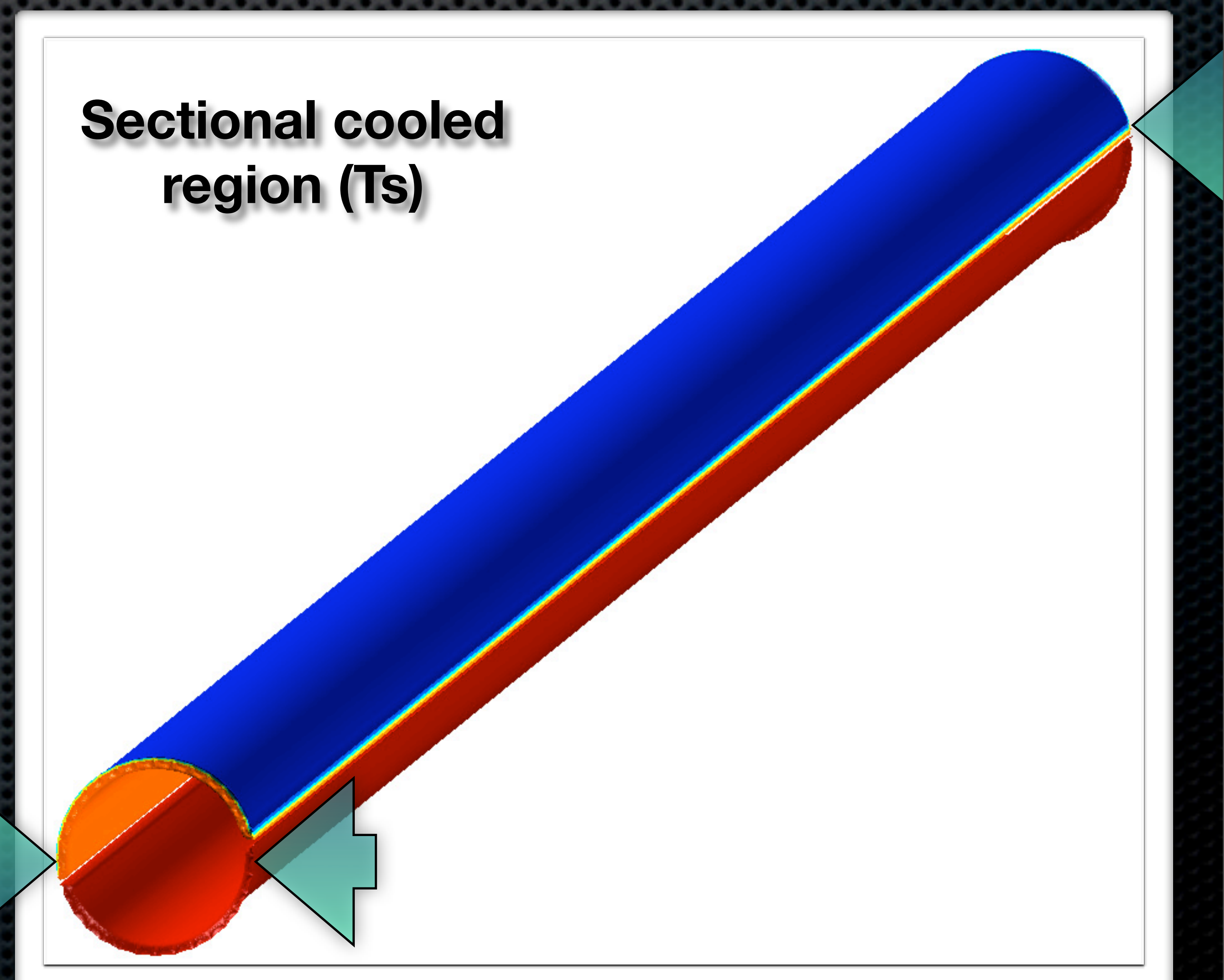


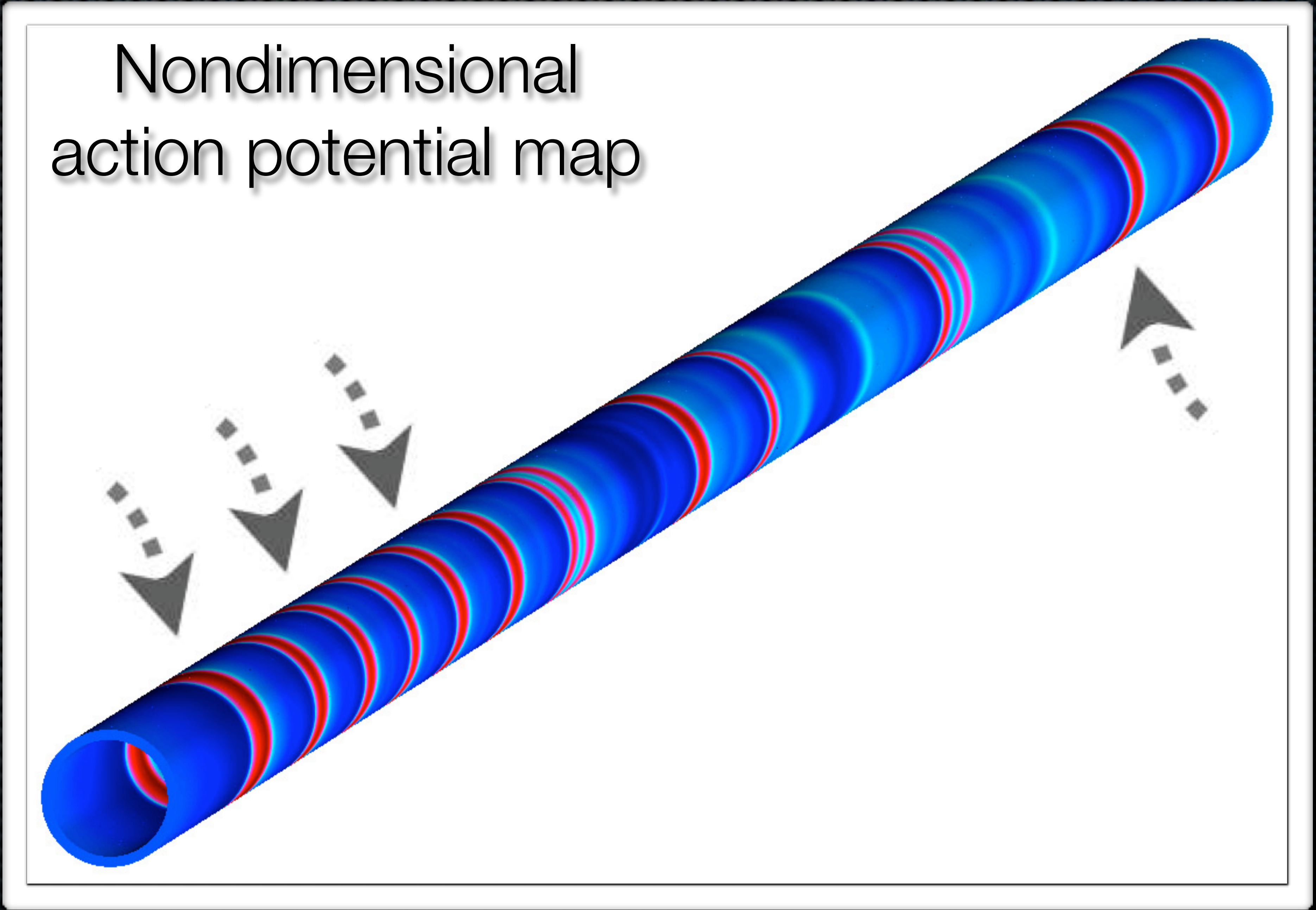


3.2 Initial Conditions

- **Constant-homogeneous conditions:**
  - **auto-excitatory electric model**
  - **boundary time-dependent thermal conditions**
- **Simulations**
  - $T = 3^*\tau = 1200$ s (after electric stabilization)
  - Rel. Err. = 1e-4, Abs. Err. = 1e-5, Direct PARDISO

4.1 Results

- **Electrical activity on normothermal tissue**
  - **Spikes**
  - **Longitudinal propagation**
  - **Phase break (space-dep excitability)**



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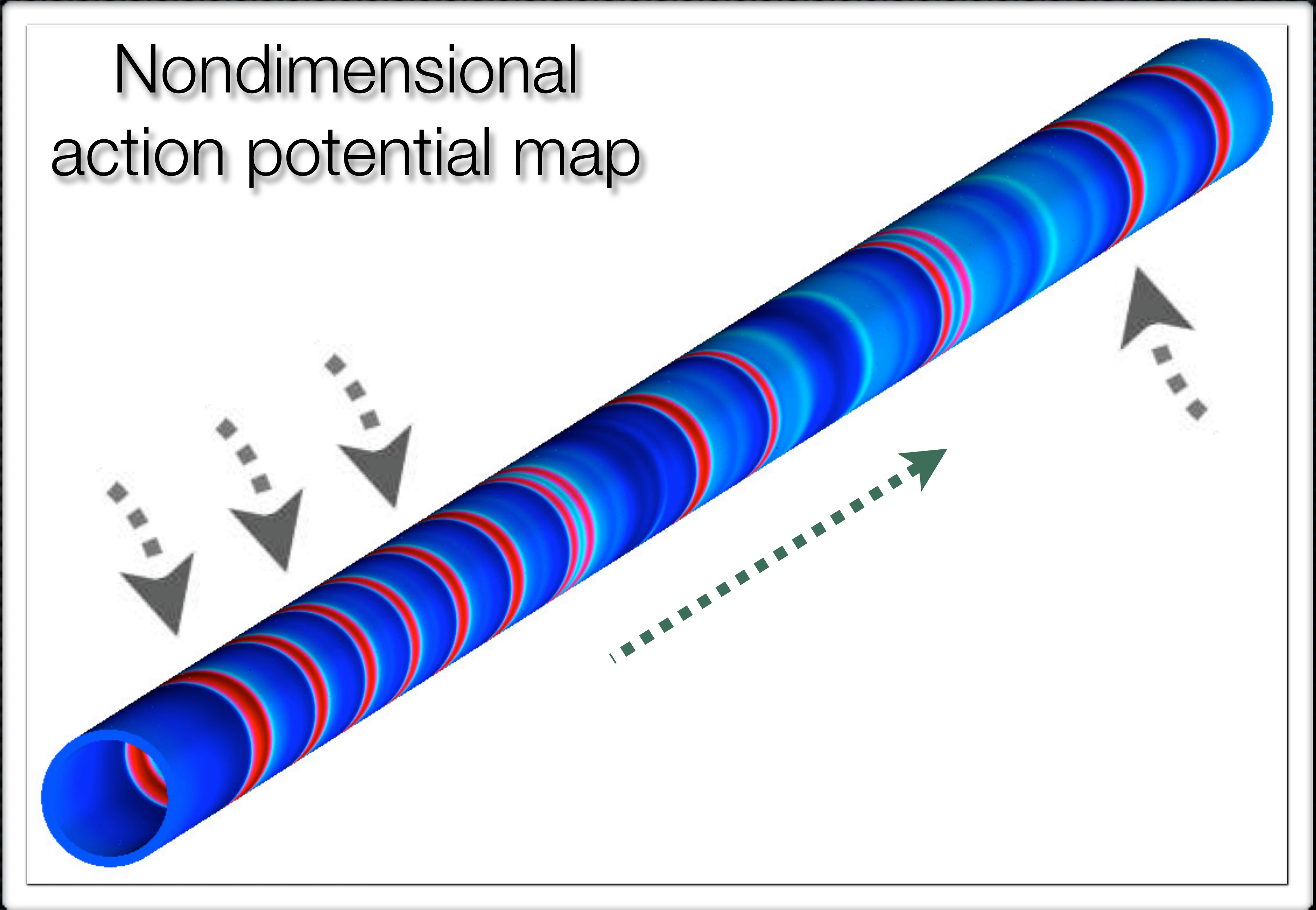

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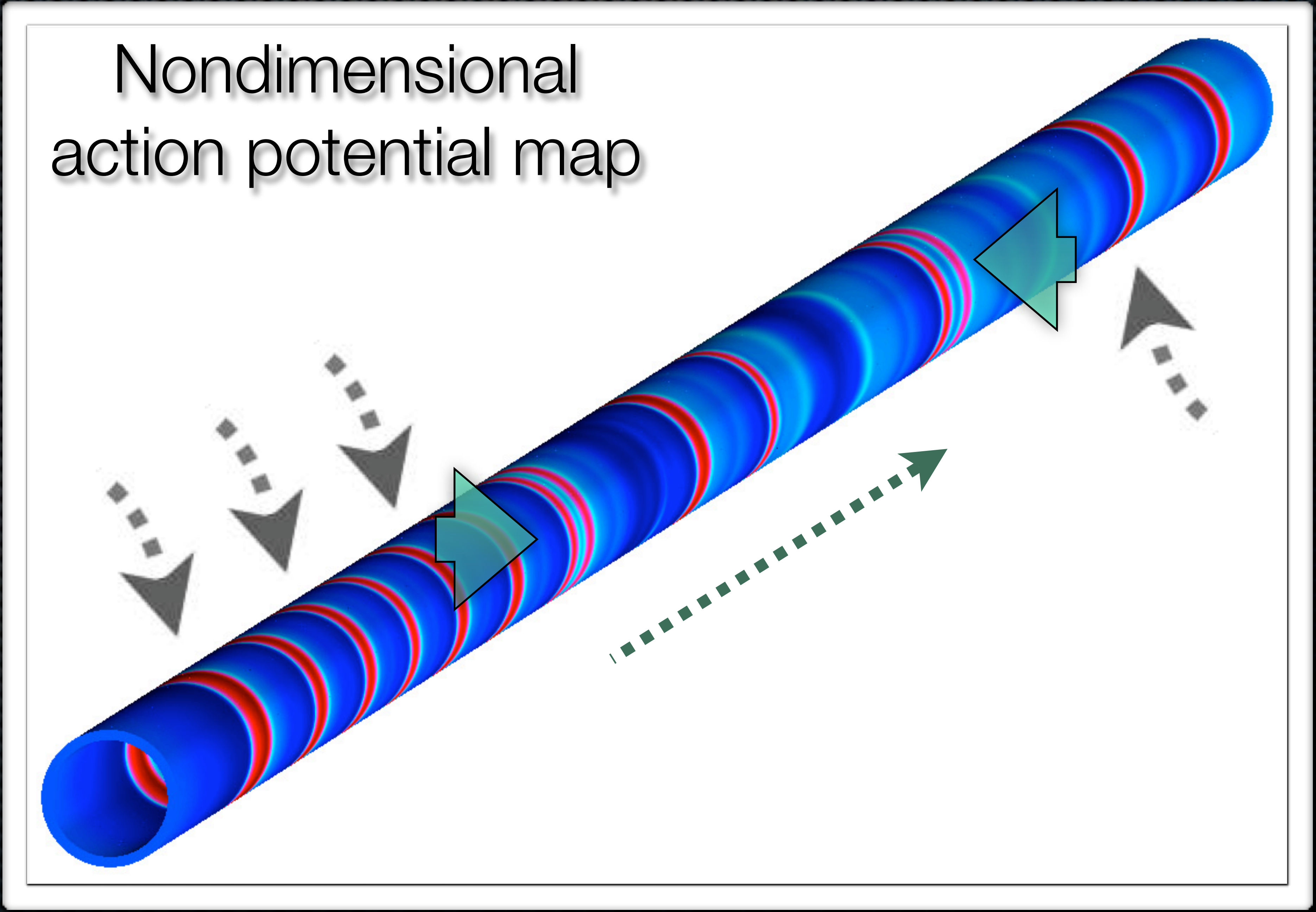
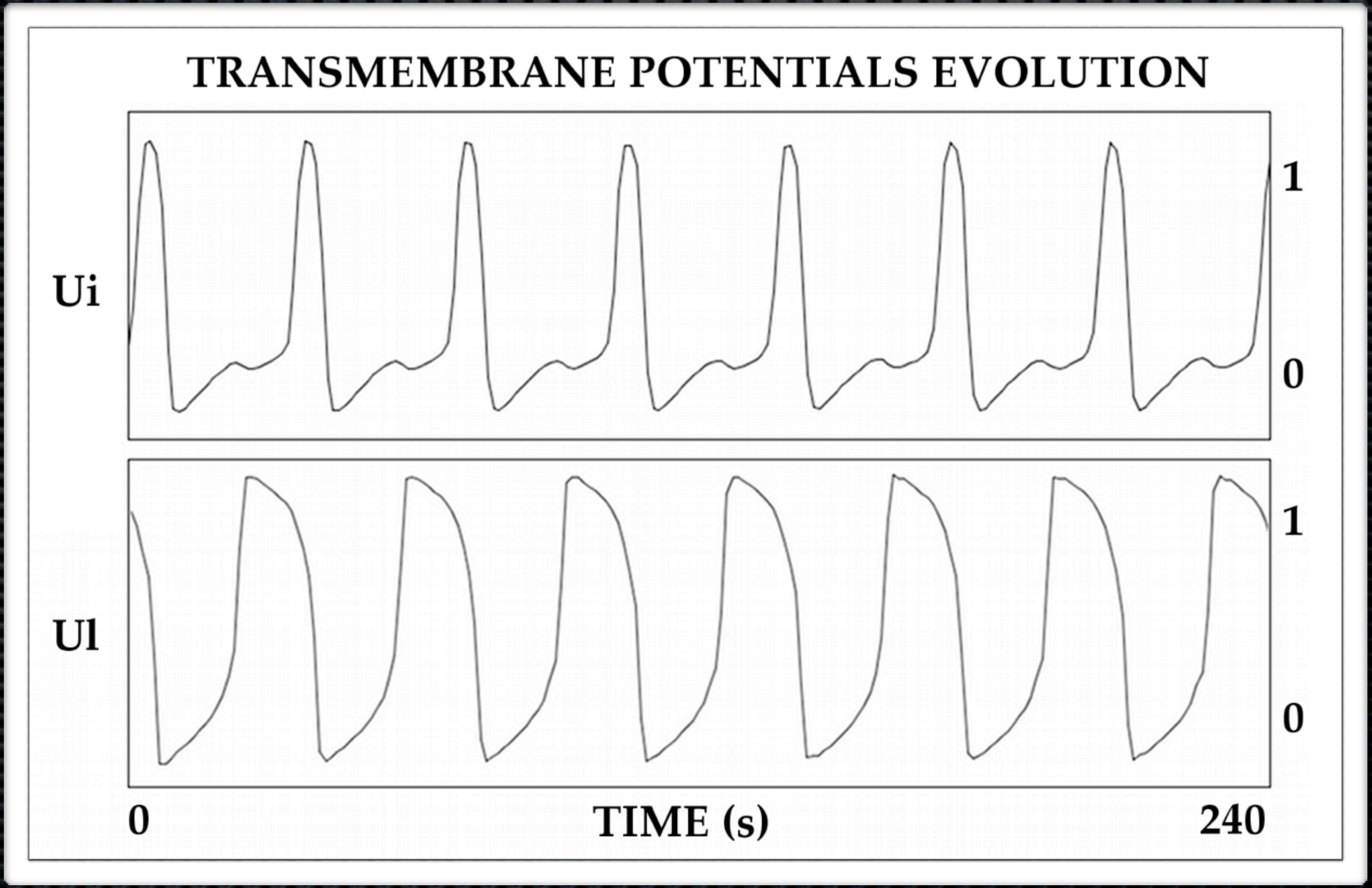


4.1 Results

- **Electrical action potential time evolution**

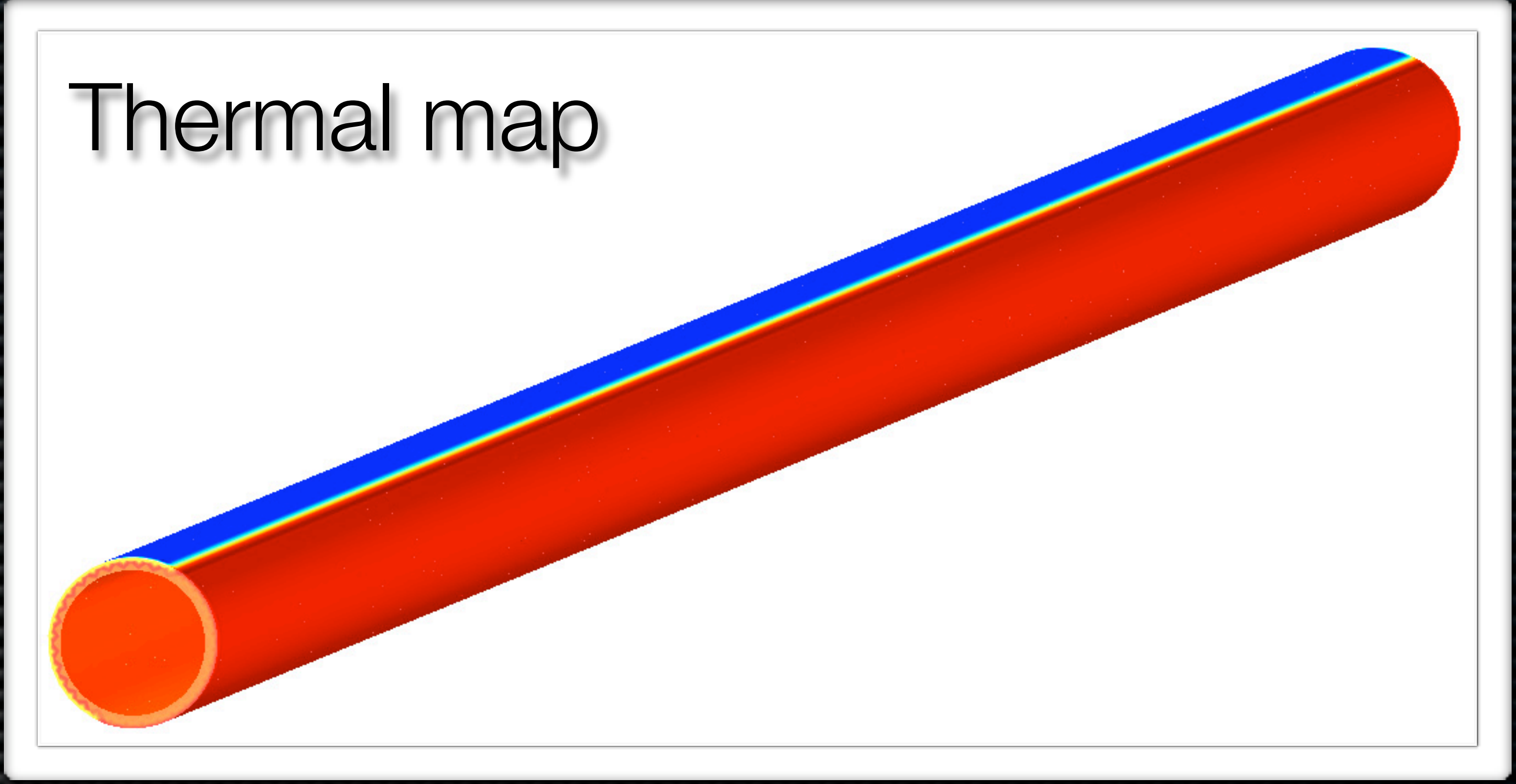


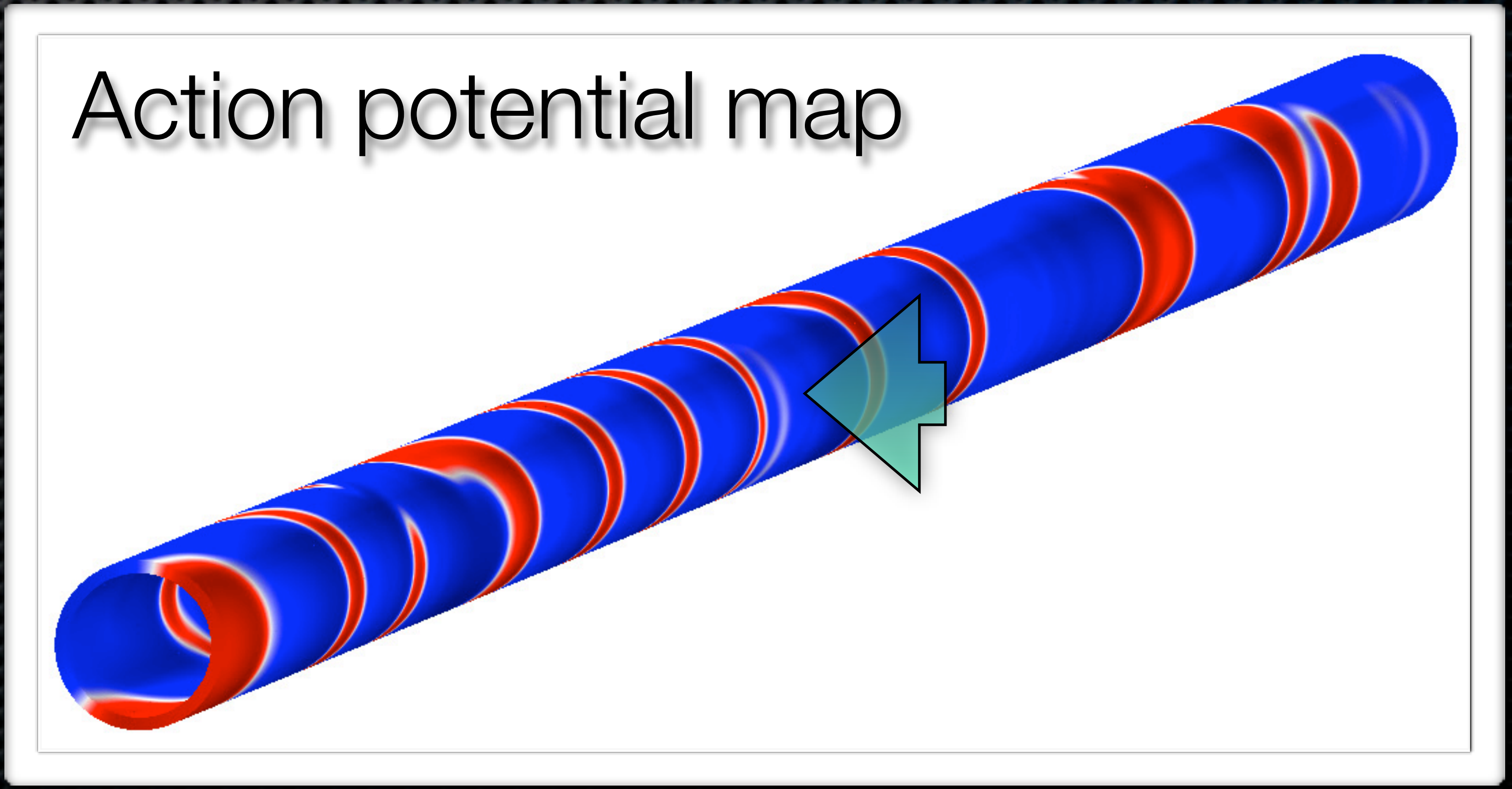
4.2 Results

- **Thermal inhomogeneities induce electrical pertubations**

4.2 Results

- **Electrical instabilities**
- **Distorted action potentials**
- **Turbulent patterns and Spiralling behaviours**





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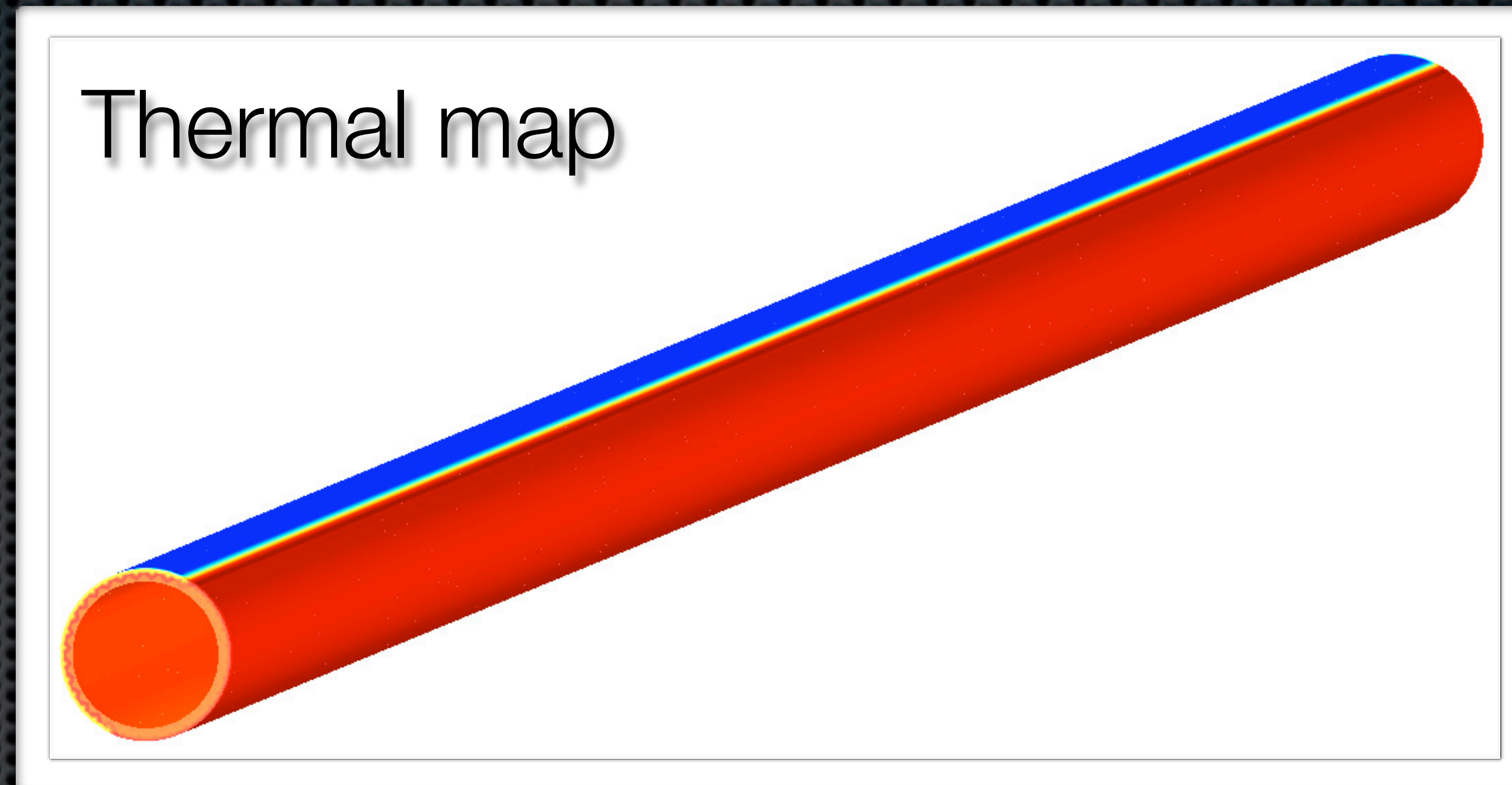

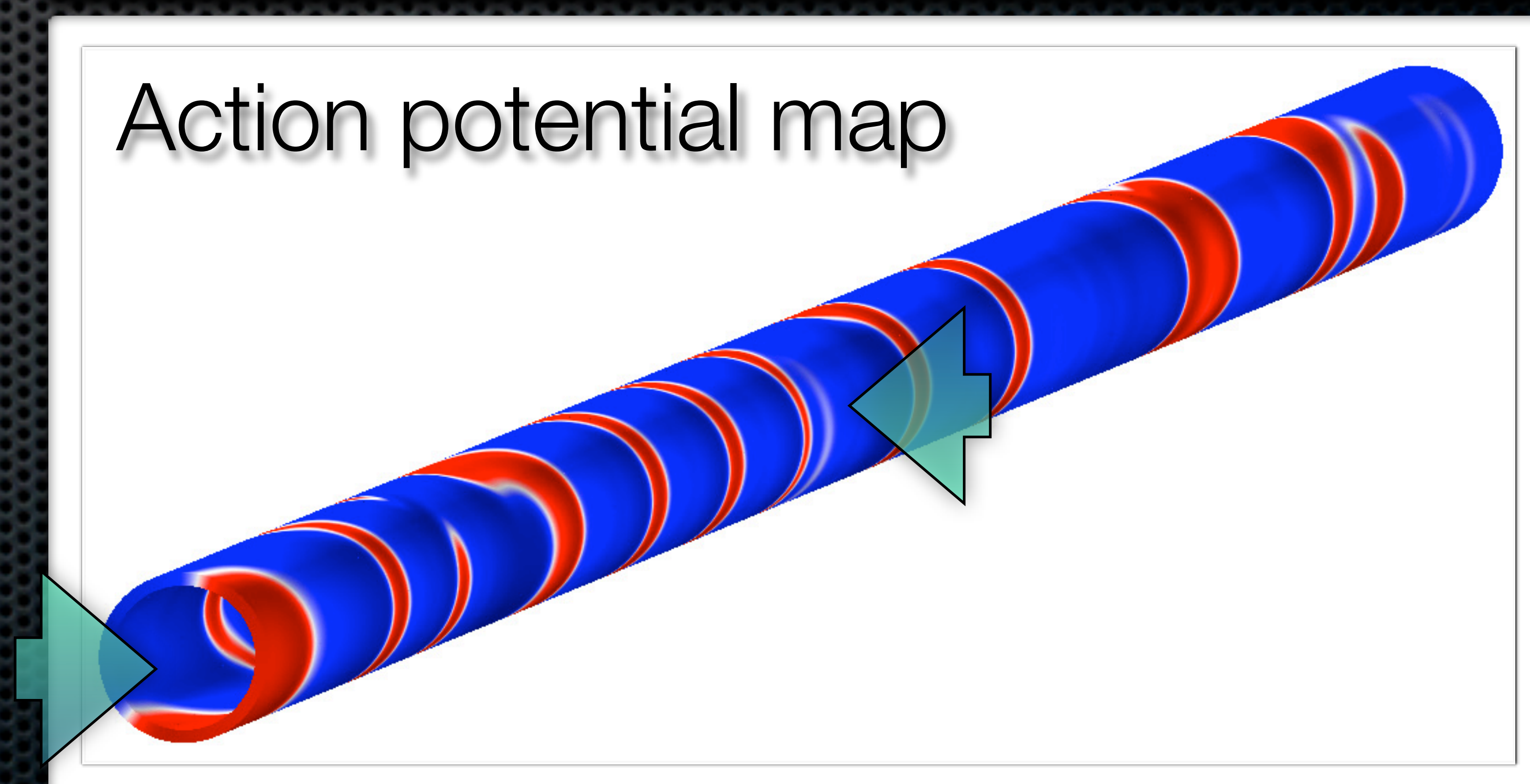

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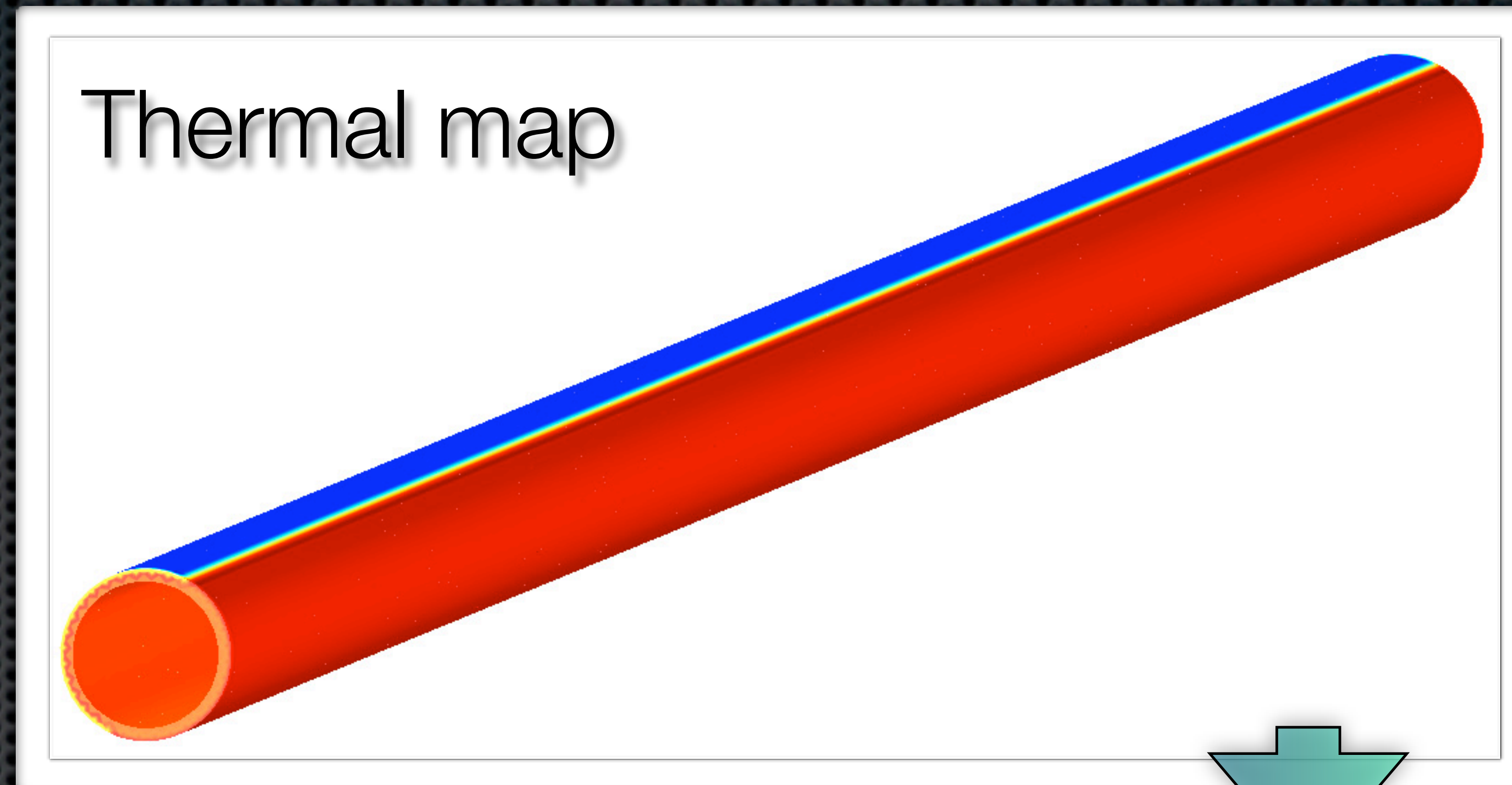

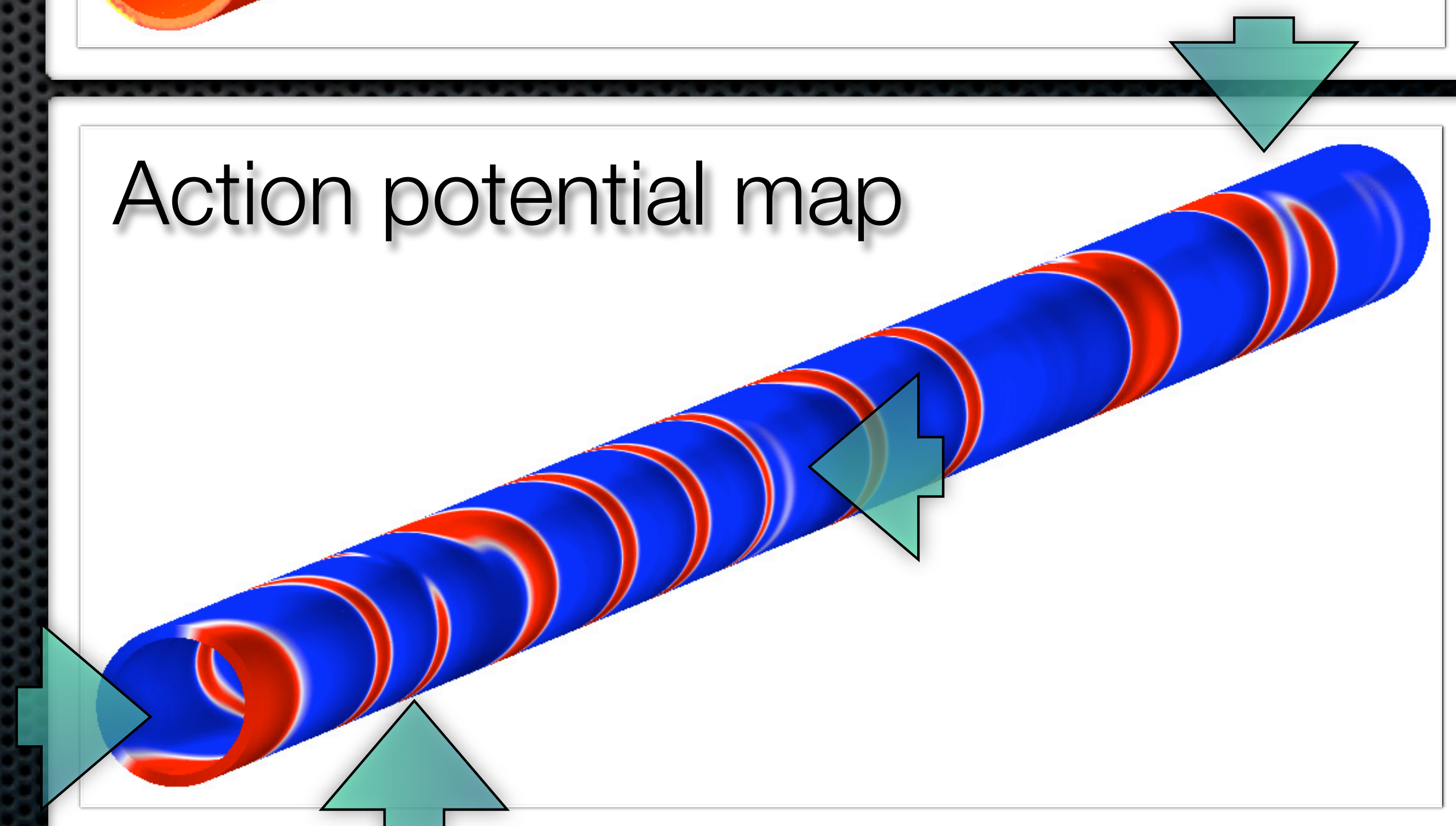

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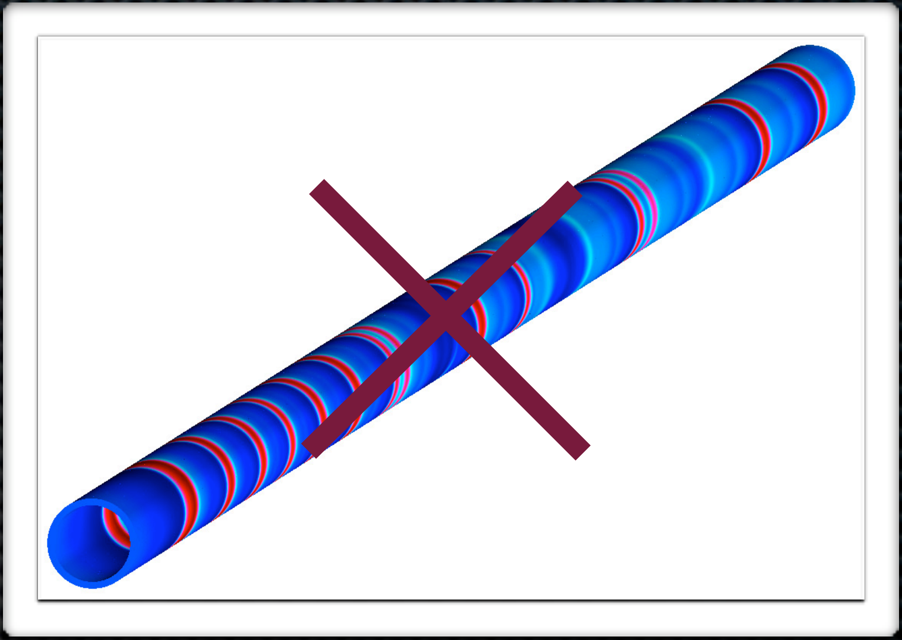
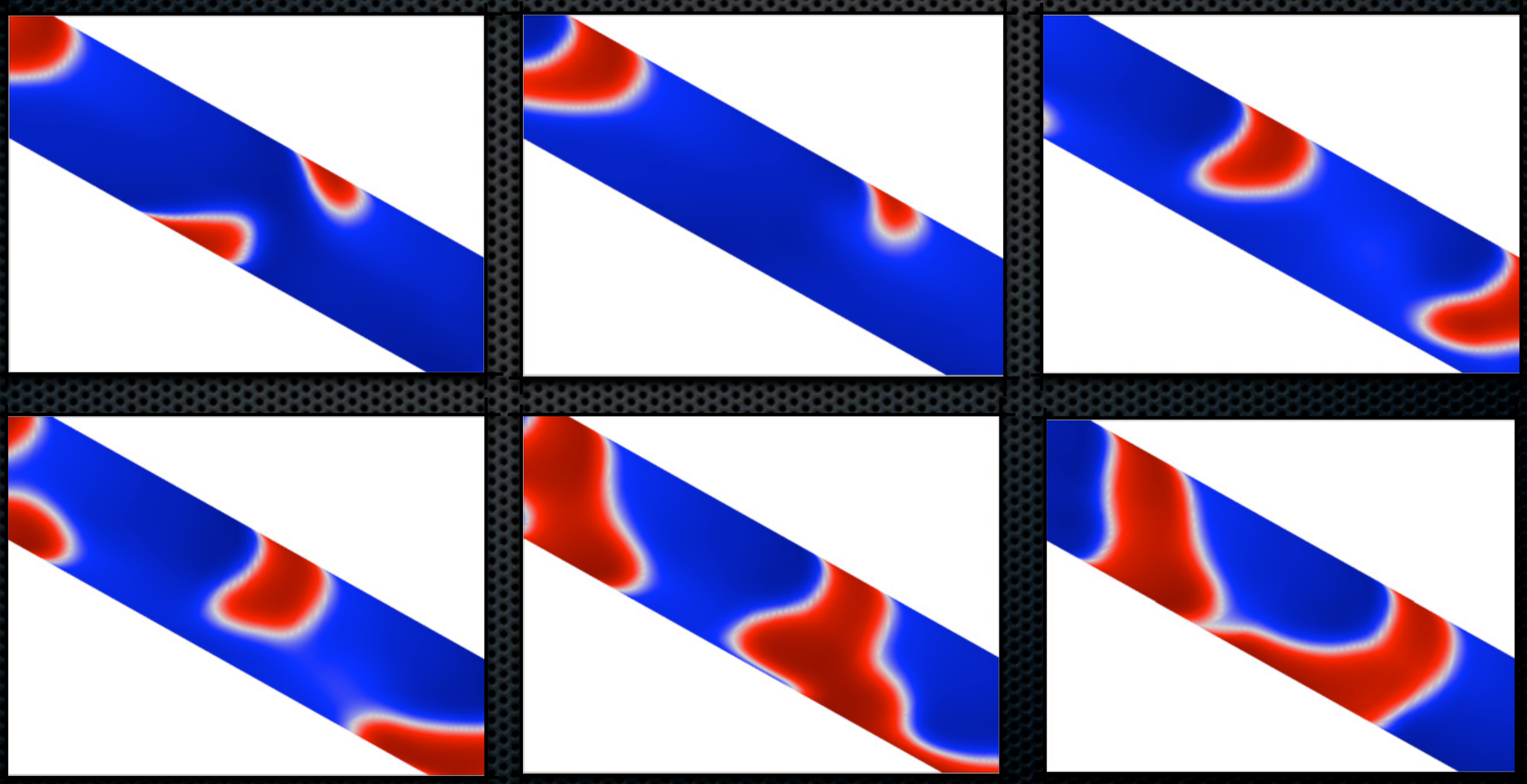


- Zoom...time series



Remark

- **Multiphysical implementation**

- **New insight into biological problems**

Thank you for your attention