Transport Phenomena and Shrinkage Modeling During Convective Drying of Vegetables.

Stefano Curcio[*1] and Maria Aversa[1]
[1]Department of Engineering Modelling – University of Calabria.
*Corresponding author: Ponte P. Bucci – cubo 39/C – Rende (CS) - ITALY, stefano.curcio@unical.it

Abstract: The aim of the present work is the formulation of a theoretical model describing the transport phenomena involved in food drying process. The attention has been focused on the simultaneous transfer of momentum, heat and mass occurring in a convective drier where hot dry air flows, in turbulent conditions, around the food sample. Shrinkage, as well as all the transport phenomena occurring in both air and food domains, have been described. The proposed model does not rely on the specification of interfacial heat and mass transfer coefficients and, therefore, represents a general tool capable of describing the behavior of real driers over a wide range of process and fluid-dynamic conditions. The system of non-linear unsteady-state partial differential equations modelling the process has been solved by means of the Finite Elements Method coupled to the ALE (Arbitrary Lagrangian Eulerian) procedure that, by a proper modification of integration domain, accounts for shrinkage effects. In order to describe shrinkage phenomenon, the above-mentioned transport equations have been coupled with a structural mechanics analysis performed on the food sample.

Keywords: Drying, shrinkage, structural mechanics, Transport Phenomena, ALE.

1. Introduction

In a previous paper (Curcio, Aversa, Calabrò, Iorio 2008) the authors of the present work formulated a theoretical model describing the transport phenomena involved in food drying process. The attention was focused on the simultaneous transfer of momentum, heat and mass occurring in a convective drier where dry and hot air flowed, in turbulent conditions, around a wet and cold food sample with a low porosity. Moisture transport inside food with low porosity, where inner evaporation can be neglected, (May & Perré 2002), was modelled by an effective diffusion coefficient of water in the food, thus not distinguishing between the actual transport of both liquid water and vapour within the food structure. Porous foods are hygroscopic materials that contain both bound and free water (Datta 2007 I), in fact they are characterized by a level of moisture content below which the internal vapour pressure is a function of moisture content and temperature (this is a characteristic of bound water) and is lower than that of pure water. Above this moisture level the vapour pressure is a function of temperature only (free water). During drying, due to the porosity of food matter, water evaporation takes place inside the food as well as at the food external surface. The heat necessary for water evaporation and for food heating is supplied, in convective drying process, by drying air. Water thus exists both as a liquid and as a vapour in food for which porosity cannot be neglected. Actually, to describe the transport of both liquid water and vapour two different mass balance equations are needed since it is necessary to account for the different transport mechanisms, i.e. capillarity and molecular diffusion, taking place in the food matter (Datta 2007 I).

Moreover, although evaporation takes place inside the food, the transfer rates occurring at air/food interface are strongly dependent on the drying air velocity field existing in the drying chamber and, particularly, in the boundary layers developing close to the food surfaces exposed to air. For this reason, to improve the precision of the model, in particular close to the solid surfaces, the k-ω model (Wilcox 1998) has been used in the present paper to calculate drying air velocity field and to describe the momentum transport in turbulent conditions. The k-ω was chosen because of its main features, for instance the higher accuracy in boundary layer modelling (with both adverse and favourable pressure gradient).

The aim of the present work is to adopt a conservation-based approach to develop a multiphase transport model so to describe convective drying process. The model is based on conservation of liquid water, vapour and energy in both air and food domain. Moreover,

the transfer of momentum in air, in turbulent conditions, is also modelled by k-ω model. Water in air has been considered as a vapour only, whereas, in food, a contemporary presence of liquid water and vapour has been considered.
To properly describe shrinkage phenomenon (food volume variation during drying), the above-mentioned transport equations have been coupled with a structural mechanics analysis performed on the food sample.
The system of non-linear unsteady-state partial differential equations were solved by means of the Finite elements method and by Comsol Multiphysics® 3.4.

2. Theory

The food under study was a potato sample, dried in a convective oven as shown in Fig. 1.

To develop the present transport model, it is necessary to analyse the actual transport phenomena occurring in the both the food and the air domains. Liquid water transport in food system is promoted by pressure and capillary pressure gradients while vapour transfer is promoted by pressure and concentration gradients. For liquid water transport the capillary pressure prevails over the effect of external pressure if the hygroscopic material contains an amount of bound water highly greater than free water: in food system this is true in the majority of the cases and also in the case of potatoes. With respect to vapour transfer as promoted by external pressure, it is generally described by the Darcy's equation that is valid with Reynolds number ranging from 1 to 10. In a typical case of microwave heating of potato samples, however, it was found that the value of Reynolds number was equal to10^{-5} (Datta 2007 I). Generally, in the case of convective drying the inner evaporation is considerably lower than that of microwave process; so, it can be assumed that pressure driven flow is negligible and vapour molecular diffusion can be considered as the prevailing mechanism.

Definitively, capillary flow was used to model the liquid water transfer and molecular diffusion was used to model the vapour flow.

The mass balance referred to the liquid water and vapour in the food sample leads, respectively, to the unsteady-state mass transfer equations (Bird, Stewart, Lightfoot, 1960, Welty, Wicks, Wilson, Rorrer, 2001):

$$\partial C_w \big/ \partial t + \underline{\nabla} \cdot \left(-D_w \underline{\nabla} C_w\right) + \dot{I} = 0 \qquad (1)$$

$$\partial C_v \big/ \partial t + \underline{\nabla} \cdot \left(-D_v \underline{\nabla} C_v\right) - \dot{I} = 0 \qquad (2)$$

where C_w is the water concentration in food, C_v is the vapour concentration in food, D_w is the capillary diffusivity in food and D_v is the diffusion coefficient of vapour in food and $\dot{I}$ is the evaporation rate.

The energy balance in the food material leads, according to the Fourier's law, to the unsteady-state heat transfer equation (Bird, Stewart, Lightfoot, 1960, Welty, Wicks, Wilson, Rorrer, 2001).

$$\rho_s C_{p_s} \partial T \big/ \partial t - \underline{\nabla} \cdot \left(k_{eff} \underline{\nabla} T\right) + \lambda \cdot \dot{I} = 0 \qquad (3)$$

where T is the food temperature, ρ_s is the density of food sample, $C_{p\,s}$ its specific heat, k_{eff} is the food effective thermal conductivity accounting for a combination of different transport mechanisms. The energy balance contains the evaporation term since it is assumed that evaporation phenomena take place not only on the food external surfaces, but also within its structure.

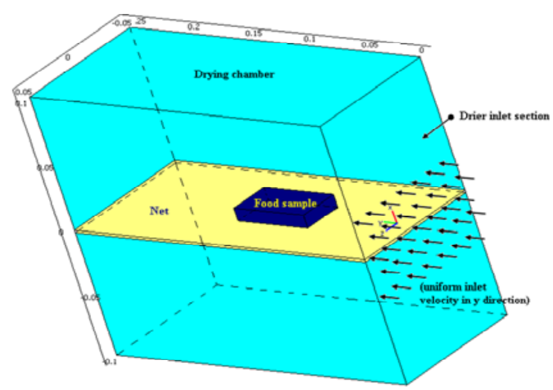


Figure 1. Schematic of the system under consideration.

The non-isothermal turbulent flow of air within the drying chamber was modelled by means of the well-known k-ω model (Curcio, Aversa, Calabrò, Iorio 2008), that is based on two additional semi-empirical transport equations for the variables k and ω i.e. the turbulent kinetic energy and the dissipation for unit of turbulent kinetic energy respectively.

The unsteady-state momentum balance coupled to the continuity equation written for the drying air leaded to (Bird, Stewart, Lightfoot, 1960, Verboven, Scheerlinck, De Baerdemaeker, Nicolaï, 2001):

$$\partial \rho_a / \partial t + \underline{\nabla} \cdot \rho_a \underline{u} = 0 \qquad (4)$$

$$\rho_a \, \partial \underline{u} / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} \underline{u} = \underline{\nabla} \cdot [\; -p\underline{I} + (\eta_a + \eta_t)\left(\underline{\nabla u} + \underline{(\nabla u)}^T - (2/3)(\underline{\nabla} \cdot \underline{u})\underline{I}\right) - (2/3)\rho_a k \underline{I} \;] \qquad (5)$$

$$\rho_a \, \partial k / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} k = \underline{\nabla} \cdot [(\eta_a + \sigma_k \eta_t)(\underline{\nabla} k)] + \eta_t P(\underline{u}) - (2\rho_a k / 3)(\underline{\nabla} \cdot \underline{u}) - \beta_k \rho_a k \omega \qquad (6)$$

$$\rho_a \, \partial \omega / \partial t + \rho_a \underline{u} \cdot \underline{\nabla} \omega = \underline{\nabla} \cdot [(\eta_a + \sigma_\omega \eta_t)(\underline{\nabla} \omega)] + (\alpha\omega / k)[\eta_t P(\underline{u}) - (2\rho_a k / 3)(\underline{\nabla} \cdot \underline{u})] - \beta \rho_a \omega^2 / k \qquad (7)$$

where ρ_a is the air density, η_a is its viscosity, both expressed in terms of the local values of temperature and of water content, p is the pressure within the drying chamber, $\underline{u}$ is the velocity vector. β_k, σ_k , σ_ω ,α, β are constants (Wilcox 1998). The term, P($\underline{u}$), contains the contribution of the shear stresses:

$$P(\underline{u}) = \underline{\nabla u} : \left(\underline{\nabla u} + \underline{(\nabla u)}^T\right) - \left(2/3\right)\left(\underline{\nabla} \cdot \underline{u}\right)^2 \qquad (8)$$

The following definition for η_t, i.e. the turbulent viscosity introduced in the above Eqs. 4-7, holds:

$$\eta_t = \rho_a k / \omega \qquad (9)$$

The energy balance in the drying air, accounting for both convective and conductive contributions, leaded to (Bird, Stewart, Lightfoot, 1960, Welty, Wicks, Wilson, Rorrer, 2001):

$$\rho_a C_{pa} \, \partial T_2 / \partial t - \underline{\nabla} \cdot \left(k_a \underline{\nabla} T_2\right) + \rho_a C_{pa} \underline{u} \cdot \underline{\nabla} T_2 = 0 \qquad (10)$$

where T_2 is the air temperature, C_{pa} is its specific heat and k_a its thermal conductivity.

The mass balance in the drying air, referred to the water vapour and accounting for both convective and diffusive contributions, leaded to (Bird, Stewart, Lightfoot, 1960, Welty, Wicks, Wilson, Rorrer, 2001):

$$\partial C_2 / \partial t + \underline{\nabla} \cdot \left(-D_a \underline{\nabla} C_2\right) + \underline{u} \bullet \underline{\nabla} C_2 = 0 \qquad (11)$$

where C_2 is the water concentration in the air and D_a is the diffusion coefficient of water in air.

The transport and physical properties of potato has been obtained by Srikiatden, Roberts (2008), Zhang, Datta (2004) and Maroulis, Saravacos, Krokida, Panagiotou (2002).

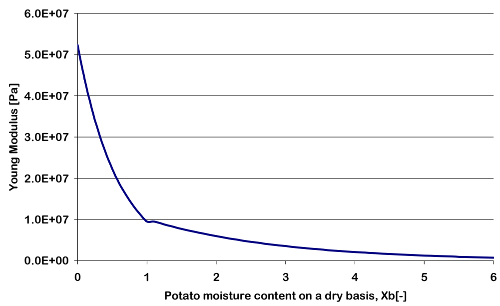
As far as the boundary conditions were concerned, the continuity of both heat and mass fluxes and of temperature was formulated at the food/air interfaces. Moreover an equilibrium relationship between the liquid water and vapour inside food was used (Smith, Van Ness, Abbot, 1987):

$$\gamma_w \, x_w \, f_w = \hat{\phi}_w \, y_w \, p \qquad (12)$$

Where γ_w, the activity coefficient of water and f_w , the fugacity of water, refer to the liquid phase, $\hat{\phi}_w$, the fugacity coefficient of water, refers – instead – to the vapour phase; x_w and y_w are the molar fractions of water in food and in air, respectively, p is the pressure within the drying chamber. At low pressures, vapour phase usually approximates ideal gases and allows simplifying eq. 12 to:

$$\gamma_w \, x_w \, P_w^{sat} = y_w \, p \qquad (13)$$

where P_w^{sat} is the vapour pressure of water. It should be observed that in hygroscopic materials, like most of the foods, γ_w accounts for the effects related to the amount of physically bound water so it is usually expressed as a function of both food moisture content and of its temperature (Datta 2007 Part II, Ruiz-López, Córdova, Rodríguez-Jimenes, García-Alvarado, 2004). Once the activity coefficient is known for the particular food under examination, eq. 13 permits calculating the molar fraction of water occurring in the vapour phase.



Young Modulus [Pa]	$5.23\ 10^7 \exp(-1.704\ X_b)$	$X_b <= 1.0$	$1.691\ 10^7 \exp(-0.522\ X_b)$	$X_b > 1.0$
Yield Stress [kN/m²]	$7.26\ 10^7 \exp(-5.04\ X_b)$	$X_b <= 0.75$	$1.453\ 10^6 \exp(-0.164\ X_b)$	$X_b <= 0.75$

(Yang and Sakai, 2001)

Figure 2. Young Modulus and Yield Stress during potatoes drying.

At the oven outlet section, conduction and diffusion phenomena were neglected with respect to convection (Danckwerts conditions). As far as the boundary conditions referred to the momentum balance were concerned, a two-velocity scale logarithmic wall function was used on the solid surfaces, (Lacasse, Turgeon and Pelletier, 2004).

Food shrinkage was modelled defining the local total strains $\{d\varepsilon\}$ as a function of changes in mechanical strains $\{d\varepsilon_s\}$ (constrained deformation due to food mechanical properties) and in shrinkage strains $\{d\varepsilon_0\}$ (the sum of a free deformation due to moisture loss):

$$\{d\varepsilon\} = \{d\varepsilon_s\} + \{d\varepsilon_0\} \tag{14}$$

Total strain $\{d\varepsilon\}$ is actually a function of total displacement $\{dU\}$

$$\{d\varepsilon\} = [A]\{dU\} \tag{15}$$

Changes in stresses $\{d\sigma\}$ are function of changes in mechanical strains $\{d\varepsilon_s\}$

$$\{d\sigma\} = [D]\{d\varepsilon_s\} \tag{16}$$

Where [D] is the stress-strain matrix containing the Young Modulus. Yang and Sakai (2001) reported its value during potatoes drying (Fig. 2).

To express the free drying shrinkage strains $\{d\varepsilon_0\}$, it was assumed that the free deformation due to moisture loss was proportional to the water content variation, through a constant (the hydrous compressibility factor). The constant was estimated from the experimental data showing drying shrinkage vs. weight loss.

Virtual work principle was formulated to obtain the equilibrium equation. By assuming that zero body and surface forces are applied to food, it can be written:

$$\int_V \delta\{d\varepsilon\}^T \{d\sigma\} dV = 0 \tag{17}$$

As far as the boundary conditions are concerned, one side of the food rests on the drier net (fixed position) whereas the other three are free to move.

Since physical and transport properties of both air and food are expressed in terms of the local values of temperature and moisture content, the transport equations for both air and food, together with the virtual work principle represent a system of unsteady-state, non-linear, partial differential equations that can be solved only by means of a numerical method. Equations were written referring to a time dependent deformed mesh that accounted for food volume variation due to water transport.

An Arbitrary Lagrangian-Eulerian (ALE) description, implemented by Comsol Multiphysics, was adopted. It is worthwhile to remark that the ALE method can be considered as “intermediate” between the Lagrangian and Eulerian approaches since it combines the best features of both of them and allows describing moving boundaries without the need for the mesh movement to follow the material.

The motion of the deformed mesh was modeled using Laplace smoothing.

The boundary conditions control the displacement of the moving mesh with respect to the initial geometry. At the boundaries of food sample, this displacement is actually that calculated by solving the structural mechanic problem. At the exterior boundaries of the fluid domain, it is set to zero in all directions.

3. Use of COMSOL Multiphysics

The system of unsteady, non-linear PDEs resulting from the present study was solved by the Finite Elements Method using Comsol Multiphysics 3.4. Both food and air domains were discretized into a total number of 9195 triangular finite elements leading to about 93000 degrees of freedom (Fig. 3). In particular, the mesh consisted of 3151 elements (with a minimum element quality of 0.7929) and 6044 elements (with a minimum element quality of 0.7955) within food and air domains, respectively. The considered mesh provided a satisfactory spatial resolution for the system under study. The solution was, in fact, independent of the grid size, even with further refinements. Lagrange finite elements of order two were chosen for the components of air velocity vector u, for the turbulent kinetic energy, the dissipation for unit of turbulent kinetic energy and for the pressure distribution within the drying chamber but, also for water concentration and temperature in, both, air and food. The time-dependent problem was solved by an implicit time-stepping scheme, leading to a non-linear system of equations for each time step. Newton's method was used to solve each non-linear system of equations, whereas a direct linear system solver was adopted to solve the resulting systems of linear equations. The relative and absolute tolerances were set to 0.0005 and 0.00005, respectively. On a dual-core computer running under Linux, a typical drying time of 5 hours was typically simulated in about 50 minutes.

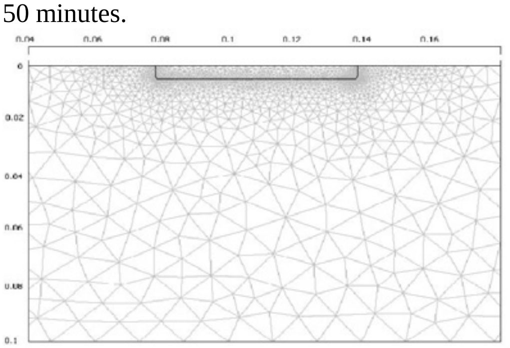


Figure 3. Domain discretization.

4. Experimental

Potato samples were air-dried by air in a convective oven. The potatoes were cut in slab shape. Each sample was dried in lab-scale convective oven (Memmert Universal Oven model UFP 400) monitoring, with respect to time, food weight, by a precision balance (Mettler AE 160, accuracy of ±10-4g) and food dimensions by a vernier caliper.

To perform the present experimental analysis identical potato sample, having an initial side of 30mm and an initial thickness of 15mm, were used. The lab-scale oven allowed monitoring, by a Dostmann electronic Precision Measuring Instrument P 655, air temperature and its humidity (by a rh 071073 probe) and air velocity, by a H 113828 probe. Air velocity was equal to 2.2 m/s and air temperature, Ta, was chosen equal to 50˚C and to 70˚C. Each food sample was placed on a wide-mesh perforated tray.

Experimental data were used also to obtain some important parameters regarding food shrinkage modelling. Food weight and food dimensions were, indeed, used to evaluate food moisture content and food volume respectively which have been plotted in figures 4 and 5: A linear fitting procedures of the aforementioned experimental data allowed estimating the hydrous compressibility factor that, as shown in figures 4 and 5, were equal to $7.44\ 10^{-4}$ and to $8.24\ 10^{-4}$, in the case of a drying temperature of 50°C and 70°C respectively.

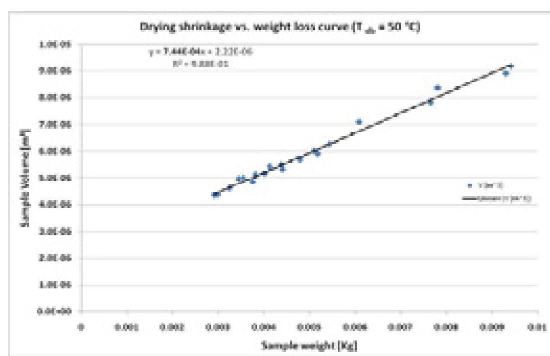


Figure 4. Estimation of hydrous compressibility factor from experimental data at 50°C.

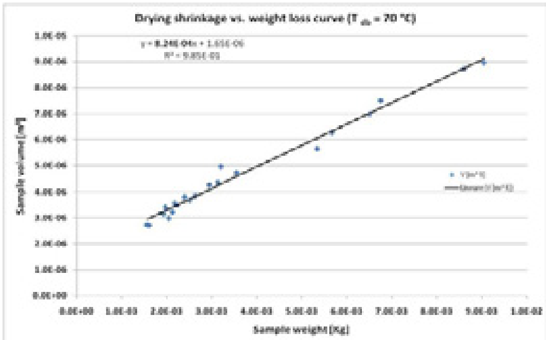


Figure 5. Estimation of hydrous compressibility factor from experimental data at 70°C.

5. Results

The proposed model was able to provide worthwhile information about water, temperature and velocity profiles in both the considered domains. This is mainly interesting in the case of food matter where, the local value of water content and temperature is an index of the conditions that give rise to deterioration reaction due to micro-organism activity. Some of the most representative results are shown in the following Fig. 6, where the time evolution of potato moisture content (on a wet basis) is presented for a potato sample whose shape and dimension change with respect to time because of the shrinkage.

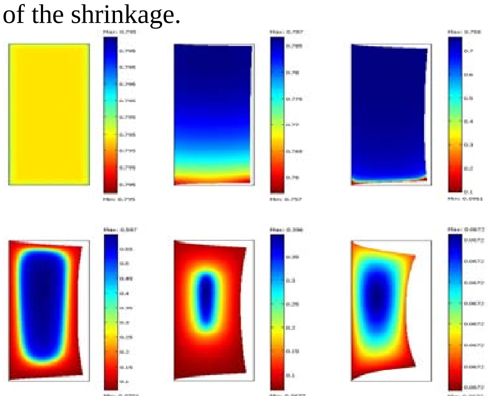


Figure 6. Time evolution of potato moisture content (on a wet basis) during drying accounting for shrinkage effect. (Air temperature of 50°C, air velocity of 2.2m/s)

Comparison between measured and calculated food dimensions in terms of total volume and food thickness are shown in figures 7 and 8 respectively, showing a remarkable agreement between model predictions and experimental data in all the considered cases.

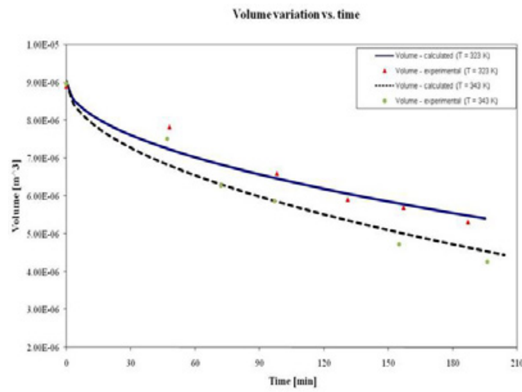


Figure 7. Comparison between experimental and model results in terms of total volume evolution of potato samples during drying.

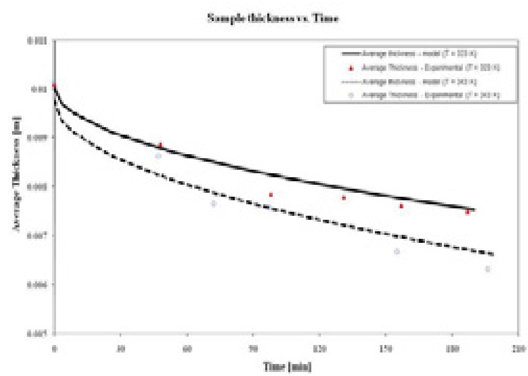


Figure 8. Comparison between experimental and model results in terms of slab thickness evolution of potato samples during drying.

5. Conclusions

The transport phenomena involved in food drying process have been analyzed. A general predictive model, i.e. not based on any semi-empirical correlation for estimating heat and mass fluxes at food-air interface, has been formulated.

The proposed model also predicted the spatial moisture profiles at all times, thus allowing detecting the regions within the food core, where high values of moisture content can promote microbial spoilage.

The model has been also improved to predict food shrinkage by coupling transport equations to virtual work principle, written with reference to a deformed mesh whose movement has been described by ALE method. The obtained results are promising. It is intended to improve the transport model by considering also the influence of convection inside the food. Also shrinkage description needs to be improved, for instance by formulating a different assumption relating free deformation due to moisture loss to water content variation, or by taking into account the influence of body and surface forces applied to food.

6. References

Bird, R.B., Stewart, W.E., Lightfoot, E.N., *Transport Phenomena*. John Wiley & Sons, London, UK, (1960).

Curcio S., Aversa M., Calabro' V., Iorio G., Simulation of food drying: FEM analysis and experimental validation, *Journal of Food Engineering*, **Vol 87** (4), pages 541-553 (2008).

Datta A.K., Porous media approaches to studying simultaneous heat and mass transfer in food processes. I: Problem formulations. *Journal of Food Engineering*, **Vol 80** (1), pages 80-95 (2007).

Datta A.K., Porous media approaches to studying simultaneous heat and mass transfer in food processes. II: Property data and representative results, *Journal of Food Engineering*, **Vol 80** (1), pages 96-110 (2007).

Lacasse D., Turgeon É. and Pelletier D., On the judicious use of the k-e psilon model, wall functions and adaptivity, *International Journal of Thermal Sciences*, **Vol 43** (10), pages 925-938 (2004).

Maroulis Z.B., Saravacos G.D., Krokida M.K., Panagiotou N.M., Thermal conductivity prediction for foodstuffs: effect of moisture content and temperature, *Int J Food Propert* , **Vol ,5** pages231–45 (2002).

May, B.K., Perrè, P., The importance of considering exchange surface area reduction to exhibit a constant drying flux period in foodstuffs, *Journal of Food Engineering*, **Vol 54**, pages 271-282 (2002).

Ruiz-López I.I., Córdova A.V., Rodríguez-Jimenes G.C, García-Alvarado M.A., Moisture and Temperature Evolution during Food Drying: Effect of Variable Properties, *Journal of Food Engineering*, **Vol 63** , pages 117-124 (2004).

Smith, J.M., Van Ness, H.C., Abbott, M.M. *Chemical Engineering Thermodynamics* (IV edition). New York, McGraw-Hill, (1987).

Srikiatden J., Roberts J.S., Predicting moisture profiles in potato and carrot during convective hot air drying using isothermally measured effective diffusivity, *Journal of Food Engineering* **Vol 84,** pages 516–525 (2008)

Verboven P., Scheerlinck N., De Baerdemaeker J., Nicolaï B. M., Sensitivity of the Food Centre Temperature with Respect to the Air Velocity and the Turbulence Kinetic Energy, *Journal of Food Engineering* **Vol 48**, pages 53-60 (2001).

Welty, J., Wicks, C., Wilson, R., Rorrer, G., *Fundamentals of Momentum, Heat, and Mass Transfer*. New York: John Wiley and Sons (2001).

Wilcox, D. C., *Turbulence Modeling for CFD, DCW Industries*, Inc., La Canada, CA (1998).

Yang, H., Sakai, N. and Watanabe, M., Drying model with non-isotropic shrinkage deformation undergoing simultaneous heat and mass transfer, *Drying Technology*, **Vol 19** (7), pages 1441-1460 (2001).

Zhang, J. and Datta, A. K., Some considerations in modeling of moisture transport in heating of hygroscopic materials, *Drying Technology*, **Vol 22** (8), pages 1983- 2008 (2004).