Model of Heat and Mass Transfer with Moving Boundary during Roasting of Meat in Convection-Oven

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Abstract: A 2D mathematical model of coupled heat and mass transfer describing oven roasting of meat was formulated from first principles. The current formulation of model equations incorporates the effect of shrinkage phenomena and water holding capacity. The model equations are based on conservation of mass and energy. The pressure driven transport of water in meat is expressed using Darcy's equation. The arbitrary Lagrangian–Eulerian (ALE) method was implemented to capture the moving boundary (product-air interface) during the roasting process. The model equations were solved using the Finite Element Method (Multiphysics® version 3.5). The state variables (temperature and water content) were predicted. The effect shrinkage on both predictions was evaluated.

Keywords: Coupled heat and mass transfer; Evaporation; Moving boundary; Multiphyics; Shrinkage.

1. Introduction

Roasting in a convection oven is a common way of frying whole meat in households, in professional kitchens and in the ready-meal industry. Mass and heat transfer play an important role in the roasting process. It is essential that their interaction and mechanisms are well understood to allow for better control and optimisation of the roasting process. The effect of shrinkage on meat roasting is often neglected due to the complexity of the process [1]-[4]. However, it is necessary to incorporate such effects into a heat and mass transfer model of meat roasting, because shrinkage is considerable (7-19 % on a area basis [5], and 11-20.3 % on diameter basis [6]) and plays a key role in the water transport during the roasting process) [7].

Several researchers have formulated different hypotheses to model mass transfer during roasting, mostly from the perspective of diffusion [1]-[3] while disagreements are often seen with regard to other types of water transport mechanisms [8]-[10]. Purely diffusion based models do not adequately describe the moisture transport phenomena during meat cooking because the effects of water binding capacity and shrinkage phenomena are not considered. These are, however, main driving mechanisms for the exudation of water during the cooking or roasting of meat, and some of the early studies on this topic agree with this fact [5],[8]-[10]. Roasting of meat causes the muscle protein to denature, resulting in a decrease in water holding capacity and leading to shrinkage of the protein network. Shrinkage of the network ultimately induces a pressure gradient inside meat muscle. The excess pressure induces a transport of water inside the meat [11], and in the end leads to water loss from the meat.

Most of the published work on the modelling of mass and heat transfer during meat roasting does not at all consider shrinkage, and thus the governing model equations were typically solved using a fixed boundary, where the evaporation interface and the material boundary remain the same for the entire roasting period [1]-[3],[11]. Usually, the reason for making such assumptions is that model equations become considerably simpler and thus easier to solve. However, the model based on such fixed boundary assumptions may not be valid for meat that is heated above the denaturation temperature, where the meat shrinks considerably, loses water and changes its dimensions. When temperatures exceed the denaturation temperature, shrinkage phenomena should therefore be taken into account in the heat and mass transfer model, in order to successfully describe heat and water transport inside the meat product. Therefore the objective of this work is to develop a model of heat and mass transfer by taking into account the shrinkage effect (moving boundary and pressure driven transport) and ultimately to describe and predict heat and mass transfer processes for meat roasting in a convection oven.

Nomenclature

C	Moisture content (wet basis) (kg /kg)	*Greek letters*	
C_{eq}	Water holding capacity at equilibrium (kg /kg)	β	Shrinkage coefficient
c_p	Specific heat (J/(kg.^{o}C))	ρ	Density (kg/m^3)
D	Diffusion coefficient (m^2/s)	μ_w	Viscosity (kg/(m.s))
E	Elastic modulus(N/m^2)	∇	Gradient(1/m)
f	Fraction of energy used for evaporation (J/kg)		
H	Latent heat of vaporization (J/kg)	*Subscripts*	
h	Heat transfer coefficient (W/(m^2 ^{o}C)	*av*	Average
K	Permeability (m^2)	*eq*	Equilibrium
k	Thermal conductivity (W/(m. ^{o}C))	*c*	Carbohydrate
m	Mass (kg)	*d*	Solid
P	Pressure (Pa)	*evp*	*evaporation*
q	Heat flux (W/m^2	*f*	Fat
T	Temperature (^{o}C)	*i*	Component
t	Time (s)	*m*	Meat
$T\sigma$	Sigmoidal temperature(^{o}C)	*p*	Protein
R	Radius (m)	*w*	Water
y_i	Mass fraction of component i *(kg/kg)*	*0*	Initial value
Z	Length (m)	*oven*	Oven
V	Volume (m^3)	*s*	Surface
v	Interface velocity (m/s)	*r*	Radial direction
u	velocity of water (m/s)	z	Length direction

2. Mathematical Model of Heat and Mass Transfer

2.1 Process Descriptions and Problem Formulation

The product (meat) is heated in a convection oven by circulating hot air at 175^{o}C. Heat is supplied to the product surface by convective heat transfer. The heat is transferred from the surface the product to the center of the product mainly by conduction. Meanwhile, moisture is transported within the product via convection and diffusion processes, and moves from the inside of the product to its surface. With increase in temperature, muscle protein denatures, leading to a decrease in its water holding capacity and shrinkage of the protein network. The shrinkage of network ultimately induces a pressure gradient inside the meat muscle and excess water is expelled to the surface by convection phenomena. Simultaneously, liquid water is evaporated at the product surface and diffuses to the surrounding fluid (hot air). As the meat sample shrinks the interface or the surface at which water is evaporated changes with time. The most important mechanisms occurring during the convection oven roasting process are described in Fig.1, as shown below.

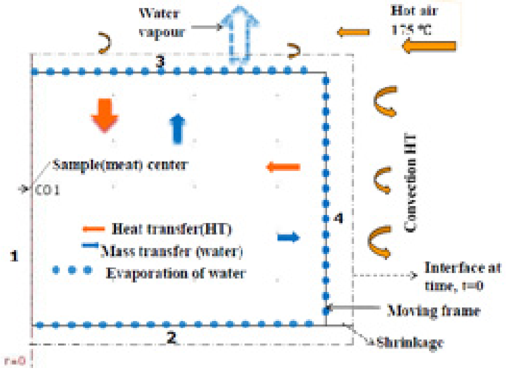

Figure 1: A schematic representation of coupled heat and mass transfer accompanied by shrinkage and evaporation processes

2.2 Assumptions:

In this study the following basic assumptions are made to formulate the governing coupled mass and heat transfer equations for a cylindrical body of meat:

a) Fat transport is negligible (lean meat is considered having less than 2% fat)

b) The crust is thin (this is observed when inspecting a cut through the cooked meat) and does not hinder transport of water to the surface. Evaporation therefore takes place at the surface (moving interface)

c) No internal heat generation and no chemical reaction.

d) Dissolved matter lost with water can be neglected in the material and energy balance [7].

e) The process can be represented two dimensions, due to symmetry of the cylindrical body that is modelled.

f) The initial distributions of water content and temperature are uniform.

2.3 Governing equations

Using conservation of energy, the heat transfer within meat is assumed to be given by (1)

$$\rho_m c_{p_m} \frac{\partial T}{\partial t} + \nabla(-k_m \nabla T) + \rho_w c_{p_w} u_w \nabla T = 0 \qquad (1)$$

From the conservation of mass, the governing equation for water transport within the product is given by (2)

$$\frac{\partial C}{\partial t} + \nabla(C u_w) = \nabla D \nabla C \qquad (2)$$

The relationship between the velocity and pressure gradient (that drives the moisture transport) inside the meat can be expressed using Darcy's law of porous media :

$$u_w = \frac{-K}{\mu_w} \nabla P \qquad (3)$$

The pressure (swelling pressure) is proportional to the excess moisture concentration within the meat [11]-[10] and the expression for swelling pressure P is given as

$$P = E\left(C - Ceq(T)\right) \qquad (4)$$

The expression for the water holding capacity is given by an empirical, sigmoid relation [11], [13]

$$C_{eq(T)} = 0.745 - \frac{0.345}{(1 + 30\exp(-0.25(T - T\sigma)))} \qquad (5)$$

The expression for velocity can be re-written using Eq. (3-5)

$$u_w = \frac{-KE}{\mu_w} \nabla\left(C - C_{eq}\right) \qquad (6)$$

2.4 Boundary Conditions

2.4.1 Heat Transfer Boundary Condition

For product subjected to convection roasting (boundary 2, 3 and 4, see Fig.1), the governing heat transfer equation (1) is solved using

$$-n.\left(k_m \nabla T + u_w c_{pw} \rho_w T\right) = h\left(T_{oven} - T_s\right) - q_{evp} \qquad (7)$$

For the sample center line, boundary 1 (see Fig.1), axial symmetry boundary is applied:

$$-n.\left(k_m \nabla T + u_w c_{pw} \rho_w T\right)\Big|_{r=0} = 0 \ ; \ t > 0 \qquad (8)$$

Where the term on the left-hand side of equation (7) refers to heat transferred by conduction and convection from the outer surface to the inside of the meat sample, the first term on the right-hand side is heat penetrating from the oven (hot air) to the product by means of convection, and the second term on the right-hand side denotes heat dissipation for evaporation of the water at the interface. The initial condition has the following form (9):

$$T(r, z) = T_0 = \text{const} \quad \text{at} \quad t = 0 \qquad (9)$$

2.4.2 Mass Transfer Boundary Condition

For product subjected to convection roasting (boundary 2, 3 and 4), the governing mass transfer equation (2) is solved using (10)

$$n..\left(-D\nabla C + u_w C\right) = \frac{q_{evp}}{H_{evp}\rho}\left(C - C_{eq}\right) \qquad (10)$$

For boundary 1(at r = 0), the axial symmetry boundary condition applies:

$$n.\left(-D\nabla C + u_w C\right)\Big|_{r=0} = 0 \ ; \ t > 0 \qquad (11)$$

The initial condition has the following form (12):

$$C(r, z) = C_0 = \text{const} \quad \text{at} \quad t = 0 \qquad (12)$$

2.4 Shrinkage

The methods used to consider material shrinkage differ greatly throughout the literature [14]. It is often considered that the change of dimensions (shrinkage) is proportional to the volume of liquid water removed [14]. For meat cooking, Sun and Du found a good correlation between the shrinkage (volume based dimensions change) and cooking loss, (a higher shrinkage leads to more cooking loss, and vice versa) [15]. The action of roasting causes denaturation of meat proteins, which allows for dehydration and shrinkage of the meat, and the simultaneous formation of air filled pores [6]. By assuming that the relationship between volume of water removed and shrinkage holds for roasting of meat, with an additional consideration for the effect of pore formation, the following theoretical expressions are formulated.

The volume of a cylindrical meat sample at any given time is expressed in terms of the initial volume (V_o) and volume of water lost ($V_{w,l}$) as

$$V = V_0 - \beta V_{w,l} \tag{13}$$

The coefficient β is used to describe the effect of pore formation during roasting process. For shrinkage, the value of β varies between 0 and 1. If β is 1, there is no pore formation (i.e. the volume of water removed is equal to the volume deformation) and if β = 0, then there is no shrinkage (i.e the volume water lost is entirely replaced by air and no deformation occurs). The fraction (1-β) is the fraction of the volume of water removed from the meat during roasting that is replaced by pore space (filled with air). For minced meat, this value is roughly estimated (for a mass loss of 15%, the corresponding pore formation is 3%) to be around 0.2, and in that case β = 0.8 [6].

For isotropic shrinkage [16], Eq. (13) can be re-written as:

$$\begin{aligned} V &= V_0\left(1-\frac{\beta V_{w,l}}{V_0}\right) \\ &= \pi R_0^2\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{2/3} Z_0\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{1/3} = \pi R^2.Z \end{aligned} \tag{14}$$

From (14) the expressions for Z and R are given as:

$$Z = Z_0\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{1/3} \tag{15}$$

$$R = R_0\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{1/3} \tag{16}$$

Differentiating (15) and (16) with respect to time, the interface velocity components can be obtained as:

$$v_z = \frac{dZ}{dt} = -\frac{Z_0\beta}{3V_0}\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{-2/3}\frac{d}{dt}\left(V_{w,l}\right) \tag{17}$$

$$v_r = \frac{dR}{dt} = -\frac{R_0\beta}{3V_0}\left(1-\frac{\beta V_{w,l}}{V_0}\right)^{-2/3}\frac{d}{dt}\left(V_{w,l}\right) \tag{18}$$

$V_{w,l}$ can be expressed as function of water content as in Eq. (19) :

$$V_{w,l} = \frac{m_d\left(X_0 - X\right)}{\rho_w} = \frac{\rho_0 V_0\left(1-C_0\right)}{\rho_w}\left(\frac{C_0}{1-C_0} - \frac{C_{av}}{1-C_{av}}\right) \tag{19}$$

and the rate change of $V_{w,l}$ is given by (20) :

$$\frac{dV_{w,l}}{dt} = -\frac{\rho_0 V_0\left(1-C_0\right)}{\rho_w}\left(\frac{1}{1-C_{av}}\right)^2\frac{dC_{av}}{dt} \tag{20}$$

at the sample center (boundary r = 0)

$$v_r = 0 \tag{21}$$

3. Numerical Method

The above model equations (system of partial differential equations) describing coupled heat and mass transfer in convection roasting of meat were solved using the finite element software, COMSOL Multiphyics®version3.5. A 2D cylindrical geometry of dimensions (radius of 20 mm and length of 54 mm) was built in COMSOL for numerical simulations. The coupled partial differential equations for heat and mass transfer along with the boundary condition were solved using the *Chemical Engineering module* (transient heat transfer and transient mass transfer) and the *moving mesh module (ALE)*. The incorporation of ALE gives the ability to track the position of the product-air interface. The input parameter values and the algebraic expressions in the model are given in table 1.

4. Result and Discussion

4.1 Temperature and water content distributions

In the meat roasting process, temperature and water content distributions are important factors which determine the quality of the product. The water content distribution is influenced by the temperature distribution. Fig 2a and 2b show simulated spatial temperature and moisture distribution, respectively, for 2D cylindrical meat sample at different times of roasting process (*t* = 0, 500, 1000, 1500, 2000, 2500, 3000, and 3500 s). Generally, inside the meat sample, the temperature increases with increase in time, whereas water content and dimensions are decrease with increase in time. From that the figure, a change of dimensions - a moving boundary - can be noticed.

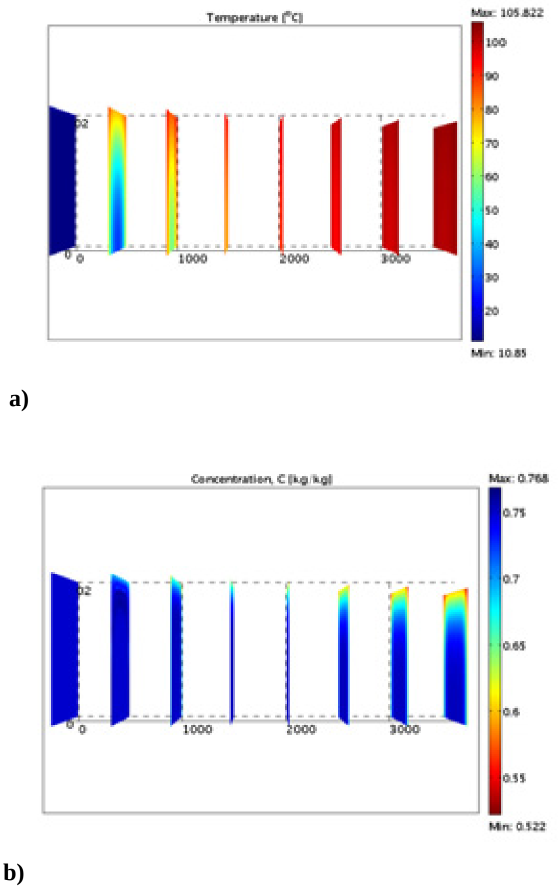

Figure 2. a) Temperature distribution, and b) water content distribution at (*t* = 0, 500, 1000, 1500, 2500, 2000, 3000, and 3500 s)

Fig. 2a, illustrates the progress of the temperature distribution during meat roasting in a convection oven. Initially, there is a sharp increase in surface temperature because of the large temperature difference between hot air (175^{o}C) and the meat (13^{o}C). At *t* = 500 s, the surface of the meat is at a much higher temperature than the inside part of the meat sample, and a large temperature gradient is developed in the region close to the surface,(see Fig. 2a and Fig. 3). When the roasting process proceeds, this large temperature gradient shifts gradually from near the surface to inside of the product. Moreover, its magnitude decreases as a function of time, as the heat energy is slowly penetrating into the centre of the product, thereby raising its temperature (Fig. 3). In the final period of this roasting experiment, at time *t* = 3000 s, the temperature of the meat is almost uniform.

Fig. 2b, illustrates the progress of the water content distribution within the meat product during the roasting process. The water content distribution changes from being uniform (= initial condition) to a non-uniform profile. The increase in temperature (to the denaturation temperature zone) causes the meat to reduce its water holding capacity and induces shrinkage. The reduction of the water holding capacity and the shrinkage of the meat protein network cause the meat to exudate water to the surface, which is lost by evaporation at the surface. As a result, the water content gradient is developed within the meat, as shown by iso-concentration lines at t = 500 s (Fig. 4). A large water concentration gradient is observed near the surface and the gradient gradually shifts towards the interior of the product (Fig. 2b). The water transport depends upon the material properties (permeability and elastic modulus), the diffusivity coefficient and the pressure gradient.

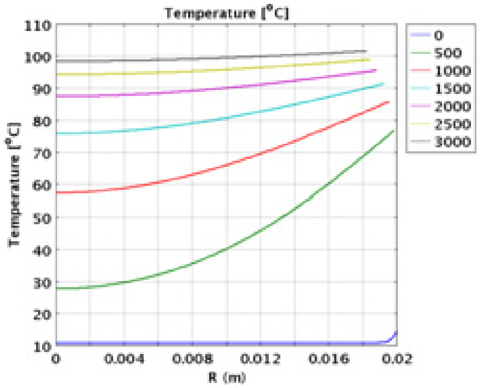

Figure 3 Temperature profile across cylindrical sample (Z = 0)

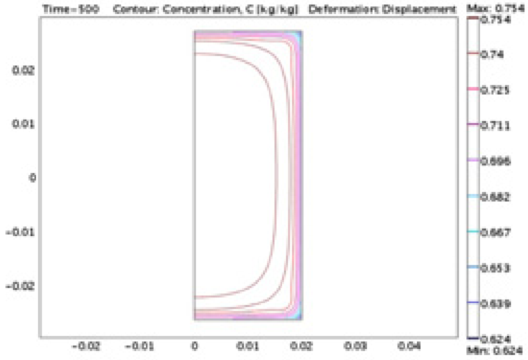

Figure 4 .Iso-concentration, C (kg/kg) at t = 500 s

4.2 **Effect of Moving Boundary**

The temperature profiles with moving boundary (MB) and fixed boundary (FB) are compared in Fig. 5a and 5b. From Fig. 5a, the center and the surface temperature values predicted by both methods coincide well at the beginning of the process (*t* = 0 to *t* = 1000 s). But later one, (*t* > 1000 s), the two predictions start deviating from each other. The FB predicts lower center temperature than MB. However, the FB predicts higher water content than MB (Fig 5b).

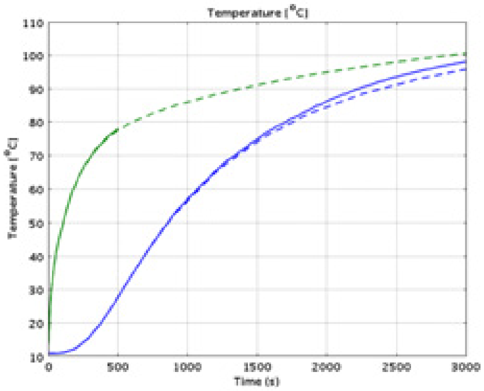

Figure 5a Temperature profile –MB -- FB (blue) center (0, 0) (green) surface (R = 0.02, Z = 0)

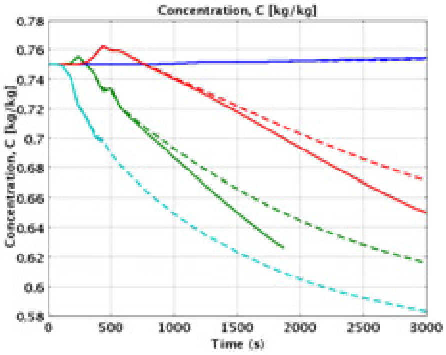

Figure 5b Water content profile – (MB), -- (FB), blue is center (0, 0), red is at (0.017, 0), green is at (0.019, 0), and is surface (0.02, 0).

4.3 Relative Change of Dimension

Fig. 6 shows the plot of the relative dimension change, R/R_o, in the r-direction. In the first part of the roasting process (until *t* = 300 s), there is no shrinkage. The product (meat) starts shrinking slowly from *t* = 300 s to 500 s. In the second period (between *t* = 500 s to *t* = 2000 s), the relative change of dimension is large (steep profile). In this zone, a major part of the meat is in the denaturation zone (where a reduction of water holding capacity and shrinkage of protein network take place). In the third period, (after *t* = 2000 s), the relative change of deformations (shrinkage rate) is reduced. After t = 3500 s, the rate of change of the relative dimension has clearly diminished. The probable reasons for such situation are; 1) the mechanical properties of the meat have changed (e.g. elastic modulus increase) and 2) reduction of the water content near the surface, which make the product more rigid and less susceptible to deformations.

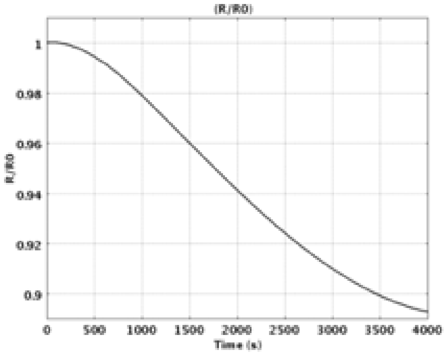

Figure 6 Relative length of cylinder as function of time (R/R0)

5. Conclusions

A first-principles-based model of heat and mass transfer with moving boundary is developed for a convection meat roasting process. The model equations were solved using COMSOL Multiphyics$^{®}$version3.5. Temperature and water content distributions as function of position and time were predicted. Using the model better insight of the process mechanisms is obtained, which would otherwise not be possible. The novelty of the developed model is its capability to incorporate the effect of the shrinkage and water holding capacity. Such model can be helpful in understanding the physics of meat roasting, and can be used to improve prediction of temperature and moisture loss.

6. References

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7. Acknowledgement

The Author would like to thank DTU for a Ph.D. grant under the aegis of Food-DTU.

8. Appendix

Table 1: **Parameters values, thermophysical properties and other expression.**

	Value or expression	*Reference*
y_p	0.2 kg/kg	*[5]*
y_c	0.02 kg/kg	*Initial mass fraction*
y_f	0.03 kg/kg	
y_w	0.75 kg/kg	
ρ_f	920 kg/m^3	*[17]*
ρ_p	1320 kg/m^3	*[17]*

ρ_c	1600 kg/m^3	*[17]*
ρ_w	1000 kg/m^3	*[17]*
k_m	0.47 W/(m. ^{o}C)	*[17]*
$c_{p,w}$	4170 J/(kg$^\cdot$ ^{o}C	*[17]*
H_{evap}	2.3 10^6 J/kg	
h	33.4 (W/(m$^{2\cdot}$ ^{o}C)	*Measured*
K	10^{-17}-10^{-19} (raw meat) 10^{-17}m^2	*[18]*
T_{0ven}	175 °C	*Set*
T_0	13 °C	*Set*
C_0	0.75 kg /kg	*[5][7]*
β	0.8	[6]
$D = 2.23e\text{-}5\exp(-3382.212/T)$		*[16]*
$-\log \mu_w = 0.0072\,T + 2.8658$		*Using data [19]*
$\rho_m = \dfrac{1}{\sum \dfrac{y_i}{\rho_i}}$		*[17]*
$c_{pm} = (1.6y_c + 2y_p + 2y_f + 4.2y_w).10^3$		*[17]*
$E_{(T)} = E_o + \dfrac{E_{mx}}{(1+\exp(-E_n(T-E_D)))}$		*Using data [5]*
For whole meat, E_0=*12 kpa*, E_{mx}=83 kpa at T=80 ^{o}C; E_n=0.3, and E_D=60		
$q_{evp} = fh(T_{oven} - T_s)$		