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Hyperbolic Heat Transfer Equation for Radiofrequency Heating: Comparison between Analytical and COMSOL solutions

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Radiofrequency heating (RFH) has been employed in a multitude of minimally intrusive operational techniques in the modern surgery.

Changing the corneal curvature to correct refractive errors.

Waring 4th, G.O. and Durrie, D.S.,
*J. Refract. Surg.,***24***, 419-423, (2008).*

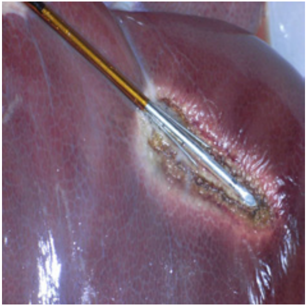
Elimination of cardiac arrhythmias.

Geha, A.S. and Abdelhady, K.,
World J. Surg., **32***, 46-49, (2008).*

Thermal destruction of tumors.

Zhu, J.C. and Yan, T.D. and Morris, D.L.,
Ann. Surg. Oncol., **15***, 1765-1774, (2008).*

RFH is based on the point heating of target zones of human tissue in which a great amount of heat is transferred on a very small time scale.



The equation for the thermal current conservation with a given internal heat source $S(\mathbf{x}, t)$,

$$\nabla \cdot \mathbf{q}(\mathbf{x}, t) + \frac{k}{\alpha}\frac{\partial T}{\partial t}(\mathbf{x}, t) = S(\mathbf{x}, t). \tag{1}$$

$\mathbf{q}(\mathbf{x}, t)$: thermal flux

$T(\mathbf{x}, t)$: temperature at point $\mathbf{x} \in D$ in the biological tissue domain $D \subset \mathbb{R}^3$ at time $t \in \mathbb{R}_+$

k: Thermal conductivity

$\alpha = \frac{k}{\eta c}$: Diffusivity, where η is the density and c is the specific heat of the material. ρc is the volumetric heat capacity.

Parabolic Heat Transfer Equation (PHTE)

$\mathbf{q}(\mathbf{x}, t) = -k\nabla T(\mathbf{x}, t).$

$$\nabla \cdot (-k\nabla T(\mathbf{x}, t)) + \frac{k}{\alpha}\frac{\partial T}{\partial t}(\mathbf{x}, t) = S(\mathbf{x}, t). \tag{2}$$

Bioheat Equation: $S(\mathbf{x}, t) = S_s(\mathbf{x}, t) + S_p(\mathbf{x}, t) + S_m(\mathbf{x}, t)$

Hyperbolic Heat Transfer Equation (HHTE)

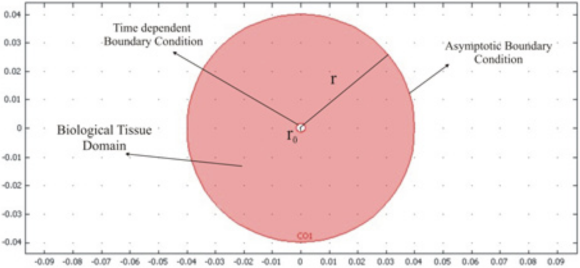
$$-\Delta T(\mathbf{x}, t) + \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}(\mathbf{x}, t) + \tau\frac{\partial^2 T}{\partial t^2}(\mathbf{x}, t)\right) = \frac{1}{k}\left(S(\mathbf{x}, t) + \tau\frac{\partial S}{\partial t}(\mathbf{x}, t)\right)$$

τ: thermal relaxation time.

We have considered an annulus D, in cartesian coordinates

$$D := \{(x, y) \in \mathbb{R}^2 \ : \ r_0^2 \leq x^2 + y^2 \leq R_0^2\}$$

$r_0 = 0.0015$ m and $R_0 = 0.4$ m.



Internal Sources for the HHTE

$$S(\mathbf{x}, t) = S_s(\mathbf{x}, t) + S_p(\mathbf{x}, t) + S_m(\mathbf{x}, t)$$

S_s **Surgical source: Radiofrequency Heating**

$$S_s(\mathbf{r}, t) = \frac{P r_0}{4\pi r^4} H(t)$$

S_p **Metabolic Source: Blood perfussion**

$$S_p(\mathbf{r}, t) = -\eta_b c_b \omega_b (T - T_0)$$

S_m **Metabolic Source**

We do not consider internal heat sources related to metabolic activity.

$$S_m(\mathbf{r}, t) = 0$$

Initial and Boundary conditions

$$T(r,0) = T_0, \quad \forall\, r > r_0 \tag{3}$$

$$\frac{\partial T}{\partial t}(r,0) = 0, \quad \forall\, r > r_0 \tag{4}$$

$$\lim_{r\longrightarrow\infty} T(r,t) = T_0 \quad \forall\, t > 0. \tag{5}$$

Electrode Boundary $r = r_0$

$k_{electrode} >> k_{tissue}$

$$\frac{\tau\ \eta_0\ c_0\ r_0}{3\ k}\left(\frac{1}{\tau}\ \frac{\partial T}{\partial t}(r_0,t) + \frac{\partial^2 T}{\partial t^2}(r_0,t)\right) = \frac{\partial T}{\partial r}(r_0,t). \tag{6}$$

Governing Equations

In spherical coordinates the resulting governing equation is

$$- \alpha \left(\frac{\partial^2 T}{\partial r^2}(r,t) + \frac{2}{r}\ \frac{\partial T}{\partial r}(r,t) \right) + \zeta \frac{\partial T}{\partial t}(r,t) + \tau \frac{\partial^2 T}{\partial t^2} = \frac{P\ \alpha\ r_0}{4\ \pi\ k\ r^4} \left(H(t) + \tau \delta(t) \right) - B(T - T_0) \tag{7}$$

where $H(t)$ is the Heaviside function, $\delta(t)$ is Dirac's function, $B = \frac{\alpha \eta_b c_b w_b}{k}$ and $\zeta = 1 + \tau B$.

Geometry $\Rightarrow$ spherical coordinates
Radial symmetry $\Rightarrow$ 1D approach, r being the dimensional variable.

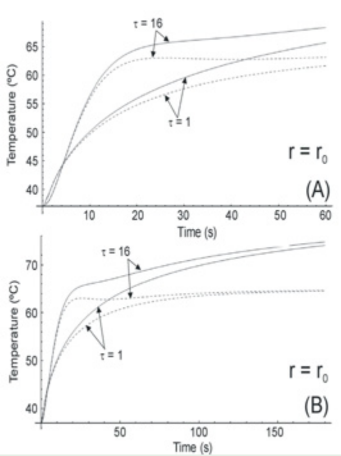
Analytical Solution

$$T(r,t) := V\left(\frac{r}{r_0}, \frac{\alpha t}{r_0^2}, \frac{\alpha \tau}{r_0^2}, m\right)\frac{P}{4\pi k r_0} + T_0. \tag{8}$$

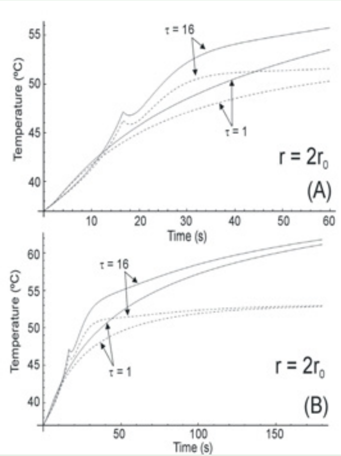
where $V\left(\frac{r}{r_0}, \frac{\alpha t}{r_0^2}, \frac{\alpha \tau}{r_0^2}, m\right)$ is a combination of several inverse Laplace transforms.

M.J. Rivera *et al.*, *Modeling the radiofrequency ablation of the biological tissue using the Hyperbolic Heat Equation: Analytical and numerical solution*
Submitted to *Physics in Medicine and Biology*

$r = r_0$


τ = 16
65
60
55
50
45
40
Temperature (ºC)
τ = 1
r = r0
(A)
10
20
30
40
50
60
Time (s)
τ = 16
70
60
50
40
Temperature (ºC)
τ = 1
r = r0
(B)
50
100
150
Time (s)

$r = 2r_0$


τ = 16
τ = 1
r = 2r0
(A)
Temperature (ºC)
Time (s)
55
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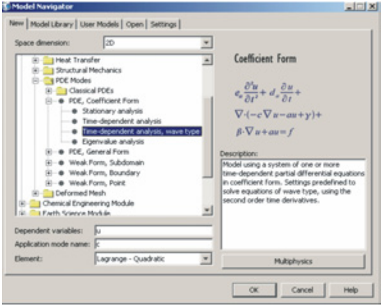
50
60
τ = 16
τ = 1
r = 2r0
(B)
Temperature (ºC)
Time (s)
60
55
50
45
40
50
100
150

COMSOL Multiphysics software version 3.2b.
Two-dimensional problem using the **PDE coefficient Form option** for a **Time-dependent analysis, wave type**.



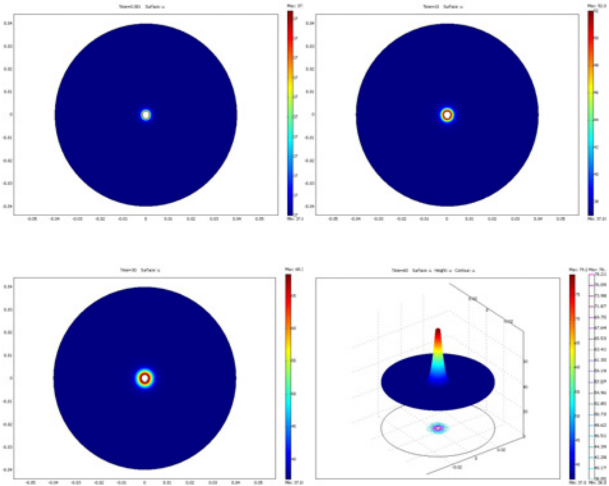
Initial Conditions

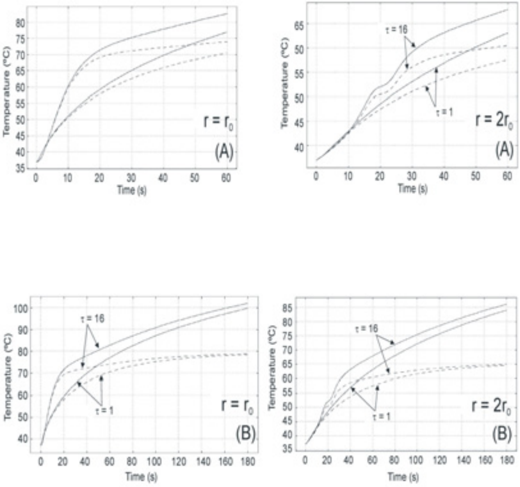
$$\begin{aligned} q = r &= 0 \\ h &= 1 \\ g &= T_0 \end{aligned} \tag{9}$$

Boundary Conditions

$$\begin{aligned} q = h = r &= 0 \\ g = \frac{-\rho_0 ce_0 r_0}{(3\rho ce)}(\tau\ utt &+ ut) \end{aligned} \tag{10}$$

$$\omega_b = 0 \ s^{-1}, \ \tau = 16 \text{ s}$$




Temperature (ºC)
Time (s)
r = r0
(A)
τ = 16
τ = 1
r = 2r0
(A)
r = r0
(B)
r = 2r0
(B)

$$\left(\frac{T_{COMSOL} - T_{analytitcal}}{T_{analytitcal}} \times 100\right)$$

Differences $\tau = 1$

		$t = 30$ s	$t = 60$ s	$t = 120$ s	$t = 180$ s
		$\tau = 1$ s			
no perfusion	$r = r_0$	9.2%	16.9%	29.3%	35.1%
	$r = 2r_0$	9.2%	17.8%	27.3%	37.7%
perfusion	$r = r_0$	12.5%	16.5%	18%	21.9%
	$r = 2r_0$	7.4%	14%	19%	21.7%

Differences $\tau = 16$

		$t = 30$ s	$t = 60$ s	$t = 120$ s	$t = 180$ s
		$\tau = 16$ s			
no perfusion	$r = r_0$	13.6%	20.8%	30.6%	36%
	$r = 2r_0$	12.3%	21.4%	32.2%	37.9%
perfusion	$r = r_0$	12.7%	21.3%	21%	21.9%
	$r = 2r_0$	12%	14.2%	19.8%	22.6%

Temperature peaks

COMSOL Multiphysics reproduces the temperature peaks (cuspidal-type singularities) which travel through the medium at finite speed. These peaks cannot be observed in the PHTE model.

Effect of the blood perfusion

The blood perfusion involves lower temperatures than those predicted by the solution without perfusion. In this respect, the HHTE behaves like in the case of PHTE.

Differences between analytical and numerical results

There are temperature differences between the analytical and numerical solutions which in general increase with time but are lower when the blood perfusion term is considered.

Thank you for your attention!