Second order drift forces on "Offshore" Wave Energy Converters

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Abstract: Objective of this contribution is to present a procedure for evaluating second order drift forces on floating bodies, often the most important loading component for mooring design, in case of high waves propagating in relatively shallow water depths.
The non linearity associated to this condition, which is typical of installations involving wave energy converters, makes this problem particularly interesting. In particular, the note focuses on the a second order force term that depends on the second order potential.
Peculiarity of the method is a simple way to compute the product of two phasor variables.
It is shown in great details how to carry out the computation by means of Comsol Multiphysics.

Keywords: Wave Energy Converters, Floating Breakwaters, Drift forces, Moorings, Floating bodies, Newman approximation.

1. Introduction

This study deals with low frequency second order wave forces acting on stationary floating bodies such as Wave Energy Converters (WECs).

Second order average and low frequency wave forces have an important role in the dynamic of floating bodies and in the design of their moorings [1]. The horizontal components are also known as wave drift forces since, under their influence, a floating unrestrained vessel will "drift away", i.e. carry out a steady slow drift motion along the wave direction.

The low frequency components in the drift forces are associated with the frequencies of groups of waves present in an irregular wave train. Usually the force spectrum has some power at the moored vessel eigenfrequencies, and since the damping of low horizontal motions of moored structures is generally very low, a resonant behavior occurs with large amplitude motions.

It is current procedure to evaluate only the average second order force. This approximation was proposed by Newman [2]. It is shown for instance in [3] that the Newman approximation underestimates the low-frequency loads in water of finite depth.

WECs are designed to withstand extreme waves, e.g. with height H=25 m, and are placed at a depth equal to half of the average wavelength, e.g. in depths d=60 m. This means that the non-linear parameter H/d is far larger than required by the Newmann hypothesis (H/d<0.1) and the computed drift force is underestimated by one order of magnitude!

In short, the current design of moorings often leads to strong errors when WECs are considered.

The evaluation of all contributions of the second order forces under non-linear conditions is therefore a very hot and practical topic, recently studied by many authors [4;5;6].

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2 Hydrodynamic theory

This section briefly introduces the hydrodynamic theory which forms the basis for computations of the mean and low frequency second order wave drift forces on floating objects.

The fluid around the body is assumed to be ideal, the flow irrotational.

An eulerian cartesian coordinate system is defined and forms the basis for the independent variables x and y, with the origin at the mean free surface, x positive toward right and y positive upward.

2.1 The boundary value problem

The formulation to second order of the hydrodynamic boundary problem can be found for instance in [7] and will not be given here for obvious reasons of brevity.

The velocity field of the fluid can be described as the gradient of a potential Φ which is the sum of an incident field Φ_W, a scattered one Φ_S (which originates due to the presence of any obstacle) and a radiated one (given as the sum of velocities times the radiated potentials per unit displacement rate):

$$\Phi = \Phi_W + \Phi_S + \sum Vi\Phi_i \qquad (1)$$

The total potential is required to satisfy the partial differential equation (PDE):

$$\nabla^2\Phi=0 \qquad (2)$$

The advantage of the decomposition is that the diffraction hydrodynamic problem does not involve the FB dynamics and can be solved first. The radiation hydrodynamic problem, describing the effect of a forced motion of unit amplitude, is solved separately. The actual body periodic motion is obtained at last, deriving the hydrodynamic forces by the diffraction problem and the added mass and damping by the radiated potential.

The motions of the body and the potential Φ have to be determined taking into account certain boundary conditions. On the free surface, the condition is non-linear.

In accordance with classical hydrodynamic theory it is hereafter assumed that the motions of the body and the velocity potential of the flow, as well as all other derivable quantities (fluid velocity, wave height, pressure, hydrodynamic forces) may be expanded in convergent power series with respect to a small parameter ε.

$$\Phi=\varepsilon\,\Phi^{(1)}+ \varepsilon^2\,\Phi^{(2)}+O(\varepsilon^3) \qquad (3)$$

where $^{(i)}$ is i-th order variation. All orders are subject to the same domain equation (Eq. 2).

$$\nabla^2\Phi^{(1)}=0 \qquad (4a)$$

$$\nabla^2\Phi^{(2)}=0 \qquad (4a)$$

Note that some quantities, such as wave elevation η or the body displacement X may also have a non-zero static value, denoted with the superscript$^{(0)}$:

$$h = h^{(0)} + e\, h^{(1)} + e^2\,\Phi^{(2)}+O(e^3) \qquad (5)$$

$$\overline{X} = \overline{X}^{(0)} + e\;\overline{X}^{(1)} + e^2\,\overline{X}^{(1)} + O(e^3) \qquad (6)$$

where the line above the *X* denotes its vectorial nature, that in 2D Comsol Multiphysics notation becomes [x,0,y].

Of course also the normal to the body does change as a consequence of the body rotation, and it is expressed by:

$$\overline{N} = \overline{n} + e\;\overline{N}^{(1)} + O(e^2) \qquad (7)$$

where:

$$\overline{N}^{(1)} = a^{(1)} \times \overline{n} \qquad (8)$$

$a^{(1)}$ being the first order angular motion vector, in 2D equal to [0,θ,0].

In Comsol, this means to define the following expressions on the body boundaries:

$$N1x=n_x\,\theta;\; N1y=-n_y\,\theta \qquad (9)$$

The second order potential is again obtained as the sum of contributions:

$$\Phi^{(2)} = \Phi_w^{(2)} + \Phi_s^{(2)} + \sum V_i^{(2)}\Phi_i^{(2)} \qquad (10)$$

where the subscript (w) stands for wave induced, (s) stands for scattered and there is a sum of radiated potentials per unit velocities relative to roll, heave and surge.

$\Phi_w^{(2)}$ accounts for the 2nd order boundary condition at the free surface:

$$-\Gamma n = \Phi_{w,z}^{(2)} = -1/g[-w^2\Phi_w^{(2)} + 2\nabla\Phi^{(1)}\nabla\Phi^{(1)} + iw\Phi^{(1)}\left(\Phi_{,zz}^{(1)} - \Phi_{,z}^{(1)}w^2/g\right)] \qquad (11)$$

where the last two addends in the RHS are second order terms in the Bernoulli Equation due to velocity and pressure, which are obtained by the first order solution.

At the bed and at the floating body the linear condition (requiring that fluid and boundary have the same velocity) can be applied.

The second order potential for scattered waves $\Phi_s^{(2)}$ accounts for the mismatch due to first order solution ate the boundaries with the bottom and the body (moving with first order oscillations). The second order potential of radiated waves $\Phi_r^{(2)}$ accounts for the effect of second order velocities.

More precisely, and the boundary condition at the bed and at the floating body for the scattered and radiated potential are:

$$\nabla\Phi_s^{(2)} n = (X^{(1)}\cdot\nabla)\nabla\Phi^{(1)} n + -(V^{(1)} - \nabla\Phi^{(1)})N^{(1)} \qquad (12)$$

Where $X^{(1)}$ can be obviously derived by the first order velocity vector divided by -iω.

$$\left|V_i^{(2)}\right|\nabla\Phi_i^{(2)} n = V_i^{(2)} n \qquad (13)$$

The free surface boundary condition for $\Phi_s^{(2)}$ and $\Phi_s^{(2)}$ is merely the linear one.

2.2 The second order force for 1 frequency

Second order drift forces can be derived based on potential flow assumptions, simply as integral of the pressures around the hull. If we separate the hydrostatic forces and include the hydrodynamic reaction forces in the added mass and damping coefficient, the following well known equations for the forces and the moments are obtained:

$$\overline{F}^{(2)} = -\oint_{wl} \frac{1}{2} r g \left(h_R^{(1)}\right)^2 \overline{n}\, dl +$$

$$\oint_{So} \frac{1}{2} r \left|\nabla\Phi^{(1)}\right|^{(2)} + r\left(\overline{X}^{(1)}\overline{\nabla}\Phi_t^{(1)}\right)\overline{n}\, dS$$
$$+\overline{a}^{(1)} \times \left(M\ddot{\overline{X}}^{(1)}\right) + \oint_{So} \left(r\Phi_{w,t}^{(2)} + r\Phi_{d,t}^{(2)}\right)\overline{n}\, dS \qquad (14)$$

and

$$\overline{M}^{(2)} = -\oint_{wl} \frac{1}{2} r g \left(h_R^{(1)}\right)^2 \left(\overline{x}\cdot\overline{n}\right) dl +$$

$$+\oint_{So} \frac{1}{2} r \left|\nabla\Phi^{(1)}\right|^{(2)} + r\left(\overline{X}^{(1)}\overline{\nabla}\Phi_t^{(1)}\right)\left(\overline{x}\cdot\overline{n}\right) dS$$
$$+\overline{a}^{(1)} \times \left(I\ddot{\overline{a}}^{(1)}\right) + \oint_{So} \left(r\Phi_{w,t}^{(2)} + r\Phi_{d,t}^{(2)}\right)\left(\overline{x}\cdot\overline{n}\right) dS \qquad (15)$$

where η_R is the relative wave elevation, i.e. the elevation minus the vertical displacement of the floating body.

Obviously, that the mean value of these oscillating contributions is not zero in general.

It should be noted that 4 of the 5 terms in Eqs. (14-15) are caused by 1st order potentials only. The 5th term, involving $\Phi_w^{(2)}$, is the focus of the following investigation. But first, in next Sub-Section, we need to recall why the frequency of the second order contribution in irregular waves may have some power at the period of the wave group (where the wave spectrum may be completely flat).

3 Effect of wave superposition

It is a common practice in offshore engineering to use linear summation of Fourier wave components to simulate random sea surface. When quadratic terms appear, the nonlinear wave-wave interactions induce new terms which oscillate at different frequencies.

For instance the product of two sinusoids having angular frequencies w_1 and w_2 is equal to the sum of two sinusoids with half amplitude and oscillating with frequencies $\omega_S=(w_1 + w_2)$ and $\omega_D=(w_1 - w_2)$.

It can therefore be easily understood that second order quantities contain both a low and a high frequency component.

The consequences of such high component on the mooring system is not relevant, but the low frequency terms is. It is obvious to see it by considering the extreme case with $\omega_1 = \omega_2$, when a steady component arises.

Eqs. 14-15 are easily modified to account for the nonlinear force arising by the combination of two waves with unit amplitude and frequencies ω_1 and ω_2: it is simply necessary to consider each term as the sum of two waves.

For the first 4 terms depending only on first order quantities, in order to evaluate the responce to any wave super there are several well known procedures based on the quadratic transfer function.

In the following, we will suggest a procedure to obtain all terms in case of two superimposed waves.

3.1 The peculiarity of the computation

Any sinusoidal variable $A(x,t)$ can be described by the complex variable $f(x)$:

$$\begin{aligned} A(x,t) &= a(x)\cos(wt+\varepsilon(x)) = \\ &= \mathrm{Re}[a(x)\, e^{(i\varepsilon(x))}\, e^{(iwt)}] = \mathrm{Re}[f(x)\, e^{(iwt)}] \end{aligned} \qquad (16)$$

where $f(x)=a(x)\, e^{(i\varepsilon(x))}$.

That is convenient in linear theory since the following relation holds:

$$A(x_1,t)+A(x_2,t) = \mathrm{Re}[(f(x_1)+ f(x_1))\, e^{(iwt)}] \qquad (17)$$

and it is possible and convenient to study the linear relations involving A(x,t) by solving only f(x).

Special attention is needed when products are involved, for which case we argued that:

$$\begin{aligned} A(x_1,t)*A(x_2,t) = &+ \mathrm{Re}[0.5 f(x_1) f(x_2) e^{(i2wt)}] + \\ &+ \mathrm{Re}[0.5 f(x_1) f(x_2)^C] \end{aligned} \qquad (18)$$

where the superscript ^{C} stands for complex conjugate. Eq. (18) merely states that the product of two functions with same frequency is a constant plus a contribution that oscillates in time with twice such frequency.

Similarly, if we deal with the product of two functions A and B with different frequencies,

$$A(x,t)=a(x)\cos(w_1 t+\varepsilon_1(x)) = + \mathrm{Re}[f(x_1)e^{(iw1t)}]$$
$$B(x,t)=b(x)\cos(w_2 t+\varepsilon_2(x)) = + \mathrm{Re}[y(x_1)e^{(iw2t)}]$$

the product becomes:

$$A*B = \mathrm{Re}[0.5 f y\ e^{(iw_S t)} + 0.5 f y^C\ e^{(iw_D t)}]) \qquad (19)$$

Taking advantage of Eq. 18-19, we have a method to solve the products of phasor variables, and hence the second order potential problem. Note that our attention is mainly devoted to the lower frequency part of the products.

3.2 The governing equations for two frequencies

In this SubSection we present the governing equation of the second order potential problem when the incident wave is a superposition of two linear waves with frequencies ω_i and ω_j.

The two first order potential is obtained with the model described in [8]. The total first order potential $\Phi_{ij}^{(1)}$ is simply the sum of two incident potentials, two scattered and six (in 2D) radiated ones.

The second order potential is defined as:

$$\Phi^{(2)} = \Phi_W^{(2)} + \Phi_S^{(2)} + \sum V_i^{(2)} \Phi_i^{(2)} \tag{19}$$

As shown above, second order term $\Phi_w^{(2)}$ should compensate for the second order "error" at the surface non-linear boundary condition produced by global 1st order solution.

We divide the 'error' in two contributions, that we know oscillating at the frequency of the sum and difference. The second order potentials required to 'absorb' such 'error' are then also divided in two parts.

For instance, the second order potential of the incident waves $\Phi_w^{(2)}$ -dealing with the free surface boundary condition- is:

$$\Phi_w{}^{(2)} = \Phi_{wHF}^{(2)} + \Phi_{wLF}^{(2)} \tag{20}$$

where HF and LF stand for High and Low frequency.

The two contributions are searched separately, applying the boundary condition, that for the low frequency term with frequency ω_D (remember that $\omega_D=\omega_i-\omega_j$), reads:

$$\begin{aligned}\Phi_{wLF,z}^{(2)} &= -1/g[-w_D{}^2 \Phi_{wLF}^{(2)} + \nabla\Phi_{ij}^{(1)} \nabla\Phi_{ij}^{(1)} + \\ &+ \frac{1}{2} i w_D \Phi_{ij}^{(1)} \left(\Phi_{ij,zz}^{(1)} - \Phi_{ij,ztt}^{(1)} / g\right)]\end{aligned} \tag{21}$$

where the product of phasor variables are carried out according to Eqs. (18-19).

All the derivatives of $\Phi_{ij}^{(1)}$ must be supplied as global variables, considering that the two waves that give origin to it have different time derivatives.

For the scattered and radiated second order potentials, the body boundary condition does not change if the high or the low frequencies component is considered.

For the scattered one, the boundary condition is (cfr. Eq. 13):

$$\begin{aligned}\nabla\Phi_{sLF}^{(2)} n &= \frac{1}{2}(X_1{}^{(1)} \cdot \nabla + X_2{}^{(1)} \cdot \nabla)\nabla\Phi^{(1)} n + \\ &- (V_1{}^{(1)} + V_2{}^{(1)} - \nabla\Phi^{(1)}) N^{(1)}\end{aligned} \tag{22}$$

where $N^{(1)}$ also accounts for the first order displacement due to both waves.

The boundary condition correspondent to Eq.(14) for the radiated problem is trivial:

$$\nabla\Phi_{rLF}^{(2)} n = \frac{1}{2}\left(V_1{}^{(2)} + V_2{}^{(2)}\right) n \tag{23}$$

Whereas the surface boundary conditions are all similar. The scattered low frequency potential is required to satisfy the:

$$\Phi_{sLF,z}^{(2)} = -1/g[-w_D{}^2 \Phi_{sLF}^{(2)}] \tag{24}$$

4 Model description

The domain consists of the water around a floating body, moored by 2 chains.

The body indefinitely long with square cross section, 0.3 m x 0.3 m, mass of 88 kg/m. It is constrained move only in the 2DV plane, having only 3 degrees of freedom.

Chains are loose, 80 g/m, two every 50 cm, initial angle 35°. Water depth is 0.8 m.

The scale is suited for wave flume testing but does not resemble any real object.

In order to describe the model, we propose its main structure.

Dependent variables:

- 2 set of 4 first order complex potentials (1 scattered and 3 radiated each), one for the two angular frequencies ω_1 and ω_2.
- 2 set of 2 second order complex potentials (wave and scattered), for the frequencies ω_S and ω_D.

Domain equation:

- Eq. (2) for all dependent variables.

Boundary Conditions:

- See section 2.

Global expressions:

- the incident first order potentials (for frequency 1 and 2), and its spatial derivatives;

- the first order velocities in x and z and the rotations, calling an external script file, that can account for chain non-linearities;
- the global first order potential, as the sum of incident, scattered and radiated potential for the two problems, including the spatial and temporal derivatives;

Boundary expressions:

- the second order normals defined in Eq.(7).

Boundary integration-coupling-variables:

- first order forces and moments for the two first order problem and the 5 terms of the second order forces;
- added masses and damping coefficients (all the 3x3 matrix for both problem);

Point integration-coupling-variables:

- relative wave elevation at the body, used in the first term of Eq. (14). The integration in 2D of this term is merely a difference of the two points placed at the intersection between body and waterlevel. The normal direction is undefined here and we used the sign of the distance between point and gravity center to define the force direction.

All of the above are complex 'phasor' variables.

The wave group in the incident wave field can be appreciated in Fig 1, that shows the global incident wave potential.

5 Model Results

The first results concerns the limit case $\omega_1=\omega_2$.

Fig 2 shows the case with infinitesimal wave amplitude. The first three steady terms of the second order forces are non-dimensionalized by $\rho g(\zeta^{(1)})^2$, where $\zeta^{(1)}$ is the first order incident wave amplitude, plotted Vs kL_b, where k is the wavenumber and L_b is the length of the floating body.

The same three terms are presented in Fig. 3, a well known example of second order forces [7], computed for a sphere of radius a. Plots cannot be directly compared, since they describe different structures. Nevertheless it is possible to see a similar trend in the main components. Term 4 is zero since there is no moment in the sphere, whereas it is non-zero for our example. Such term is not plotted in Fig. 2 to avoid confusion.

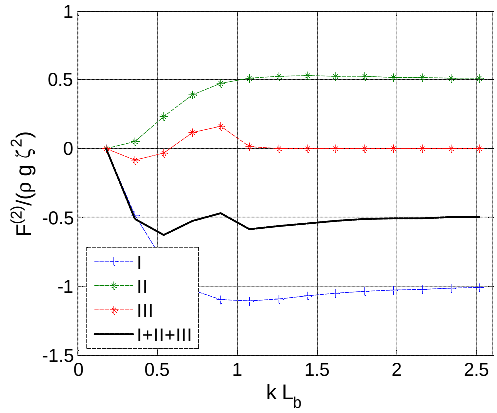

Fig. 2 First three terms of the drift force computed for a floating box, L_b=side length.

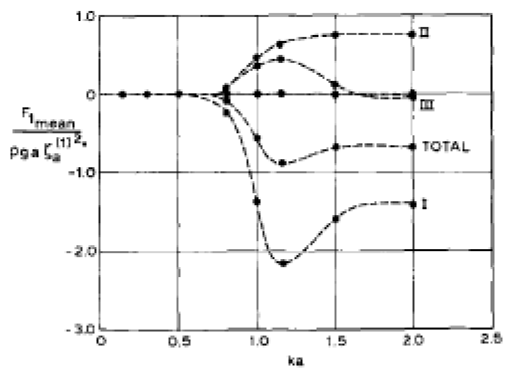

Fig. 3. First three terms of the drift force computed for a floating sphere, a=radius. [7].

It can be seen that the drift force is zero at low frequencies, where the body floats with the waves and transmission is complete. Similarly, at high frequency, all the wave energy is reflected and the body is subject to a well predictable total nondimensional force, that is (for our simple example) equal to 1/2.

The second example computes the second order potential $\Phi_w^{(2)}$ with two different frequencies relative to a period of 1.0 s and 0.91 s and heights so that $(H_1+H_2)/d$ =1/4.

Time for 1 run is approximately 1 minute. The procedure is conveniently run by a script file, and a full matrix of 60 by 60 frequencies would take therefore 5 days to run.

Actually, not all combinations are important, but only those that produce a frequency difference which is of interest for the mooring system: the matrix needs to be filled only for a band of 3-5 frequencies, for a total in our example of 180-300 elements, that can be computed in 3-5 h. This is still a reasonable amount of time, since one simulation can run overnight.

Fig 4 shows for the case of two equal amplitude waves (H/d=0.2) with the two wave frequencies ω_1 and ω_2. Convergence is very rapid, and the linear solver takes 5 steps only, although a 5th order interpolating function with a fine grid is necessary to obtain better results.

The Low frequency component is shown in color scale, the high frequency component is given by deforming the surface elevation. It can be seen that wave distortion is significant.

The second order force in this case is again given by Eq. 14, but each term is formed by 4 low frequency contributions: two constants, due to potentials oscillating with ω_1 and ω_2, and two (equal) potentials oscillating with the frequency of the difference.

6. Conclusions

The note shows how to approach the problem of finding second order forces on a floating body which is placed in relatively shallow waters in comparison to waterdepth.

All five terms forming the second order drift force are described first with reference to a single regular wave and then with reference to a sum of waves.

One application is presented, showing reasonable results, in quantitative agreement with asymptotic behaviors. A finer grid should have been used in the computations to check/increase accuracy.

The importance of the 5th term of the drift force is also briefly investigated, although this part of the research is still at its earlier stage.

8. References

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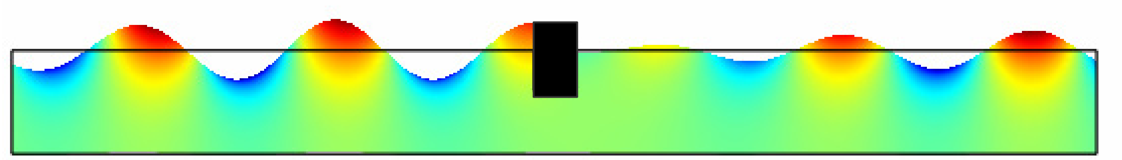

Fig. 1. 1st order waves of period 0.9 and 1 s (incident+scattered). Vertical deformation = 1. Note that there is a slight beat between the 0.9 and the 1.0 s incident waves (=wave height is not constant).

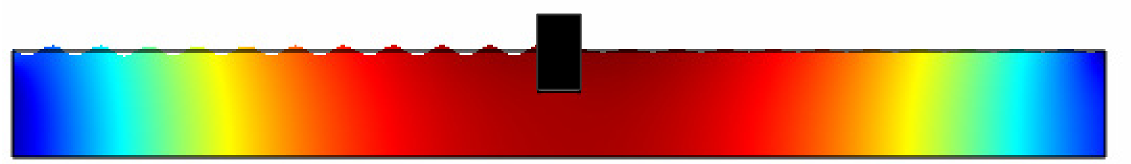

Fig 4. The 2nd order low frequency potential $\Phi_{wLF}^{(2)}$ in color scale, and elevation due to high & low frequencies components (2nd order wave correction and scattered), solving Fig 1. Vertical deformation = 1.