Presented at the COMSOL Conference 2009 Milan

Simulating Superconductors in AC Environment: Two Complementary COMSOL Models

Roberto Brambilla[1] and Francesco Grilli[2]

[1] ENEA - Ricerca sul Sistema Elettrico S.p.A., Milano, Italy
[2] Karlsruhe Institute of Technology, Karlsruhe, Germany

francesco.grilli@kit.edu

October 16, 2009

Outline

- Motivation
- Finite-element models
  - 2-D & 1-D, two different approaches
  - Governing equations
  - Pros & Cons
  - Examples
- Conclusion

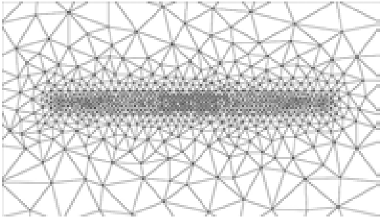
Motivation

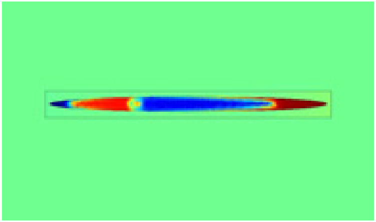
- High-temperature superconductors (HTS) very promising for power applications
- AC losses still too high
- Necessary to investigate loss dynamics
- What type of models?

Finite-element models: 2-D model

- Some simplifications can be made
- 2-D model enough in most cases
- Maxwell equations + non-linear resistivity
- $\rho(J) = \frac{E_c}{J_c} \left| \frac{J}{J_c} \right|^{n-1}$
- Implemented in COMSOL's PDE General Form
- Magnetic field components as state variables
- Edge elements guarantee continuity of tangential field component across adjacent elements

Example of mesh and results in 2-D



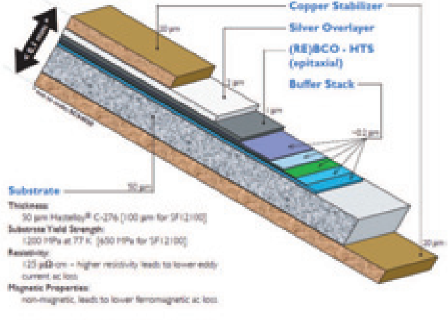


Limitations of 2-D model

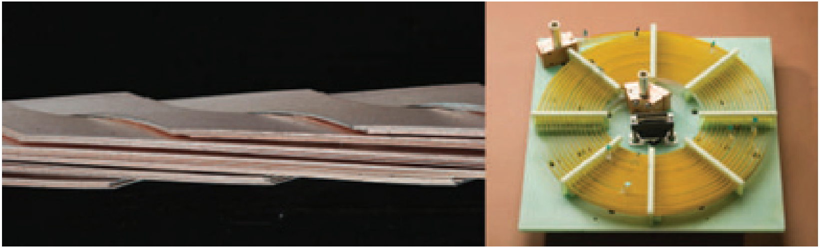
- Second-generation HTS tapes
- 4-10 mm wide, 1 μm thick
- Huge increase of DOFs
- Tapes modeled as 1-D objects
- Solve integral equation for J

$$\rho J(x,t) = \mu d \left[\frac{1}{2\pi} \int_{-a}^{a} \dot{J}(\xi,t) \log|x-\xi| \mathrm{d}\xi + \int_{-a}^{x} \dot{H}_{ey}(\xi,t) \mathrm{d}\xi \right] + C(t)$$

YBCO coated conductor tapes and devices



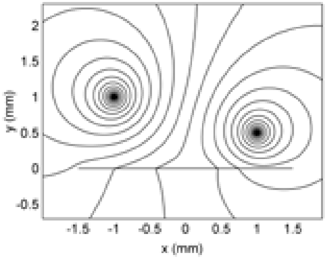
YBCO coated conductor tapes and devices

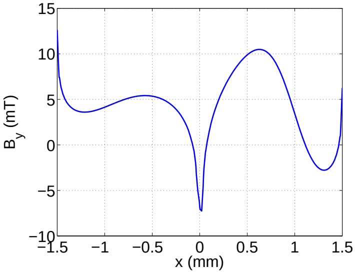


Use of COMSOL features in the 1-D model

- $\rho J(x,t) =$
  $\mu d \left[\frac{1}{2\pi} \int_{-a}^{a} \dot{J}(\xi,t) \log|x-\xi| \mathrm{d}\xi + \int_{-a}^{x} \dot{H}_{ey}(\xi,t) \mathrm{d}\xi \right] + C(t)$
- Use of integral constraints to impose the total current
  $\int_{-a}^{a} J(x,t)\mathrm{d}x = I(t)$
- Integrals transformed into *Boundary Integral Coupling Variables* (BICV) by using COMSOL's operator `dest()`
- Examples:
  - $f(x) = \int_{a}^{b} K(x,y)\mathrm{d}y \Rightarrow \mathtt{K}(\mathtt{dest}(\mathtt{x}),\mathtt{x})$
  - $f(x) = \int_{a}^{x} g(y)\mathrm{d}y \Rightarrow \mathtt{g}(\mathtt{x}) * (\mathtt{x} < \mathtt{dest}(\mathtt{x}))$

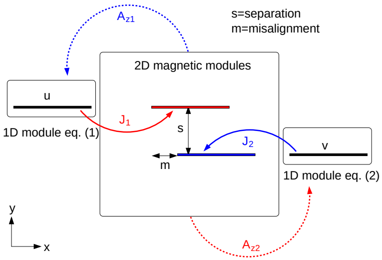
Examples of use of the 1-D model





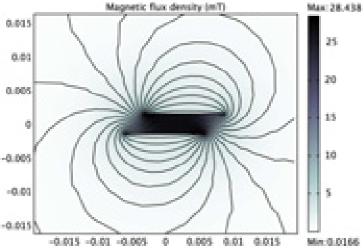
1-D model for interacting tapes

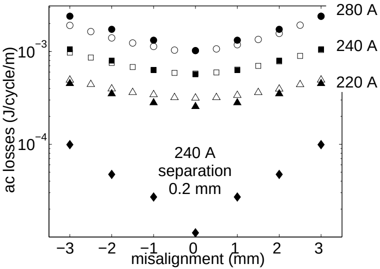
- Useful for tape assemblies, such as cables, coils
- Magnetic field produced by one tape becomes ‘source’ for the others
- Electromagnetic interaction mediated by 2-D magnetostatic models



Test against experiments

Two anti-parallel tapes with misalignment





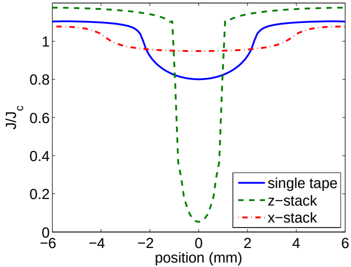
1-D model for interacting tapes

- Main drawback: one magnetostatic model for each interaction
- Include interaction directly in the integral equations
- Feasible only for certain geometries

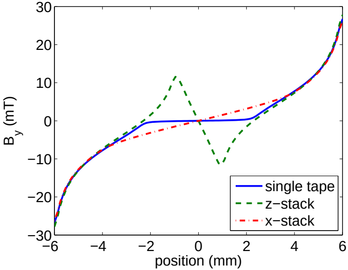
Integral equations for interacting tapes

- z-stack
  $\rho J(x,t) = \frac{\mu d}{2\pi} \int_{-a}^{a} \dot{J}(\xi,t) \log \sinh \frac{\pi|x-\xi|}{S} \mathrm{d}\xi + C(t)$
- bifilar z-stack
  $\rho J(x,t) = \frac{\mu d}{2\pi} \int_{-a}^{a} \dot{J}(\xi,t) \log \tanh \frac{\pi|x-\xi|}{2S} \mathrm{d}\xi + C(t)$
- x-array $\rho J(x,t) = \frac{\mu d}{2\pi} \int_{-a}^{a} \dot{J}(\xi,t) \log \sin \frac{\pi|x-\xi|}{L} \mathrm{d}\xi + C(t) = \mu d K_X(x,t) + C(t)$
- 2-layer x-array
  $\rho J(x,t) = \mu d \left[K_X(x,t) \pm K_{2P}(x,t)\right] + C(t)$
  $K_{2P}(x) = \frac{1}{4\pi} \int_{-a}^{a} \dot{J}(\xi) \log \left[\cosh(2\pi S/L) - \cos(2\pi(x-\xi)/L)\right] d\xi$

Examples



J/J_c
position (mm)
single tape
z−stack
x−stack

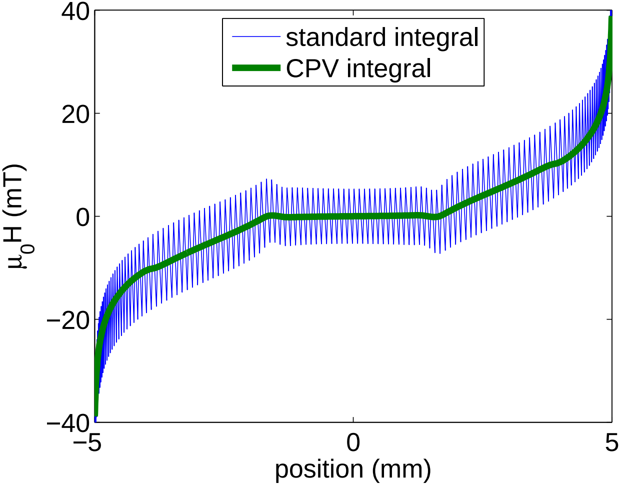


B_y (mT)
position (mm)
single tape
z−stack
x−stack

Computing the magnetic field in the 1-D model

- Very simple in 2-D, B available from state variable H
- Not immediate in 1-D, B needs to be calculated from J
- $B_y(x) = \frac{\mu_0}{2\pi} \int_{-a}^{a} \frac{J(\xi)}{\xi - x} \mathrm{d}\xi$
- Must be computed as Cauchy Principal Value (CPV), due to singularity in $\xi = x$
- Lacking feature, look for alternative method
- $B_y(x) = \Re \left[\frac{\mu_0}{2\pi} \int_{-a}^{a} \frac{J(\xi)}{\xi - x + i\epsilon} \mathrm{d}\xi \right]$
- Not mathematically rigorous, but ok for our purpose
- Useful for models with $J_c(B)$

Computing the magnetic field in the 1-D model



Conclusion

- Two complementary models for calculation of field/current distributions and ac losses
- Utilized for superconductors but useful for conductors of arbitrary resistivity
- 2-D model very flexible, but not practical for thin conductors
- 1-D model very fast, ideal for design optimization
- Both exploit COMSOL features:
  - Edge elements in 2-D
  - Integral equations in 1-D
- Both utilized for simulating configurations of practical interest, see bibliography in the conference paper