An Acoustical Finite Element Model of Perforated Elements

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Abstract: The present work deals with a numerical investigation of resonating systems used for noise control applications. In literature one can find analytical models based on fluid-dynamics concepts for evaluating losses occurring across the holes of the perforates.
In the paper an acoustical formulation based on the equivalent dissipative fluid approach will be analyzed. It will be firstly applied to simple acoustical resonators and normal incidence sound absorption coefficient will be calculated in a virtual plane wave tube. Successively the same formulation will be used in order to model perforated sheets utilized in mufflers and silencers for increasing sound transmission loss at low frequencies. Results for both analyses will be compared with experimental measurements and well extablished analytical models.

Keywords: resonator, muffler, perforates.

1. Introduction

Resonating systems are widely used in several noise control applications (i.e low frequency absorbers, mufflers and silencers, etc…). They are made of two main components: (i) a sheet perforated with small holes that can be regularly or irregularly spaced and (ii) a series of cavities.

In literature it is possible to find analytical formulas based on fluid-dynamics concepts for evaluating losses occurring across the holes of these panels [1,2,3]. Those formulations make use of correction functions in order to account the losses due to a sudden variation of surface when the sound wave is impinging the perforated sheet [4]. However if the geometry of the system is complicated a numerical approach (i.e. the finite element analysis) is required.

In the present paper an acoustical formulation for modeling losses in the openings will be presented and validated against experimetal tests. This paper is organized as follows. The description of the theoretical background of the proposed formulation will be presented in the next section. In the third and fourth sections the formulation will be applied to acoustical resonators and reactive mufflers respectively and results will be compared and discussed. Finally, concluding remarks will be given in the last section.

2. Theorethical backgroud

In modeling sound propagation through perforated sheets, losses are accounted by introducing corrections related to the distorsion of flow and increment of the mass participating to the total motion of air, as depicted in Figure 1.

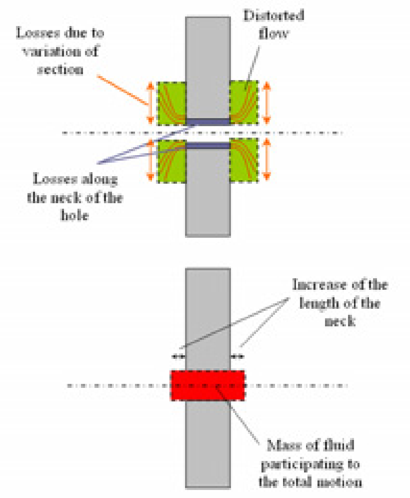


Figure 1. Physical phenomena involved in the propagation through holes.

The FEM modeling of the acoustical domains corresponding to the holes is a crucial point. In fact, from previous studies it has been demonstrated that using real valued density and sound speed for the air, the FEM model does not allow to account correctly losses described in Figure 1.

The formulation here presented is based on the equivalent dissipative fluid approach. According to this theory the hole can be considered as a fluid completely described by means of "equivalent" sound speed c and density ρ, complex valued, that are able to justify internal energy losses and phase shifts between pressure and particle velocity. In the widely used equivalent fluid model of Johnson-Champoux-Allard [5] (mainly utilized for porous materials) expressions for the $\boldsymbol{\rho}$ and $\boldsymbol{c}$ are proposed as:

$$(1) \qquad \rho = \frac{\alpha_\infty \rho_0}{\phi} + \frac{\sigma}{i\omega}\sqrt{1+\frac{4i\alpha_\infty{}^2\eta\rho_0\omega}{\sigma^2\Lambda^2\phi^2}}$$

$$c = \sqrt{\frac{\kappa \cdot P_0 / \phi\rho}{\kappa-(\kappa-1)\left[1+\dfrac{8\eta}{i\rho_0\omega N_p\Lambda'^2}\sqrt{1+\dfrac{i\rho_0\omega N_p\Lambda'^2}{16\eta}}\right]^{-1}}}$$

with ρ_0 and η the density and the viscosity of air, N_p the Prandtl number, κ the specific heat ratio and P_0 the static pressure.

From eq. (1) these effective quantities depend on five macroscopic parameters:

- the **airflow resistivity** $\boldsymbol{\sigma}$: it is a measure of the resistance that air flow meets passing through a structure;
- the **open porosity** $\boldsymbol{\phi}$: it is a measure of the fractional amount of air volume in the interconnected pores within the tested material;
- the **tortuosity** $\boldsymbol{\alpha_\infty}$: it is an adimensional quantity which takes into account the sinuous fluid paths through the porous material;
- the **viscous** $\boldsymbol{\Lambda}$ and **thermal** $\boldsymbol{\Lambda'}$ **characteristic lengths** : they have been introduced to describe the viscous forces and the thermal exchanges between a porous frame and its saturating fluid at high frequencies.

In the proposed formulation the most relaible expressions for afore-mentioned five parameters have been demonstrated to be:

$$(2) \quad \sigma = \left(\frac{2d}{R}+4\right)\frac{R_s}{\phi}\frac{1}{d}, \; \phi=1, \; \alpha_\infty=1 \text{ and } \Lambda=\Lambda'=R$$

where d [m] is the length of the hole, R [m] is the radius of the hole and :

$$(3) \qquad R_s = \frac{1}{2}\sqrt{2\eta\omega\rho_0}$$

Details about the theory of the proposed formulation can be found in [6,7].

3. The acoustical resonators

In order to validate the analytical formulation (1) it has been firstly applied to simple acoustical resonators. Simulated sound absorption coefficient as well as complex reflection coefficient have been compared with experimental data obtained in an impedance tube by means of the transfer function method and two microphones technique [8] in the frequency range between 50 and 1600 Hz.

Five different resonators have been tested in 42 different configurations (in terms of perforation ratio, radius of the holes, thickness of the plate and air gaps). In Figure 2 a picture of the resonators is shown.

Figure 2. Acoustical resonators.

The FEM model has been developed in Comsol Multiphysics 3.5a. An example of the impedance tube and resonator is depicted in Figure 3.

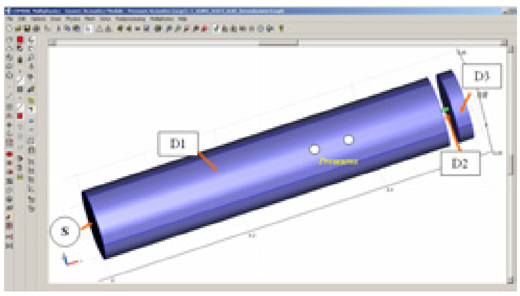


Figure 3. FEM model of a resonator.

According to labels in Figure 3 a brief description of the domains and boundary conditions is given as follows:

- ***domains:***
  - D1 (tube) : ρ=1.21 kg/m^3 and c=343 m/s
  - D2 (hole) : ρ and c from eq. (1)
  - D3 (air gap) : ρ=1.21 kg/m^3 and c=343 m/s
- ***boundaries:***
  - S (source) : radiation condition (p=1 Pa)
  - remaining : hard wall (particle velocity is equal to zero on those boundaries).

Sound pressure is determined at two positions and the surface acoustical properties are calculated by implementing the transfer function technique [8]. Mesh has been created according to the rule of 10 finite elements per wavelength. As an example in Figure 4 the sound absorption coefficient is shown for two resonators:

(a) 7 holes of radius 3 mm, thickness 10 mm and air gap of 10 mm;

(b) 1 hole of radius 5 mm thickness 15 mm and air gap of 20 mm;

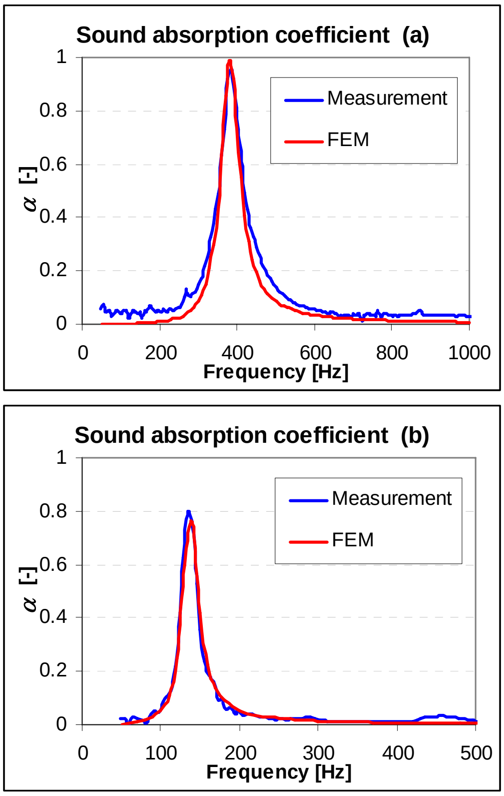


Figure 4. Normal incidence sound absorption coefficient of two resonators.

Figure 5 shows the comparison between complex reflection coefficients (in amplitude and phase) for resonator (a).

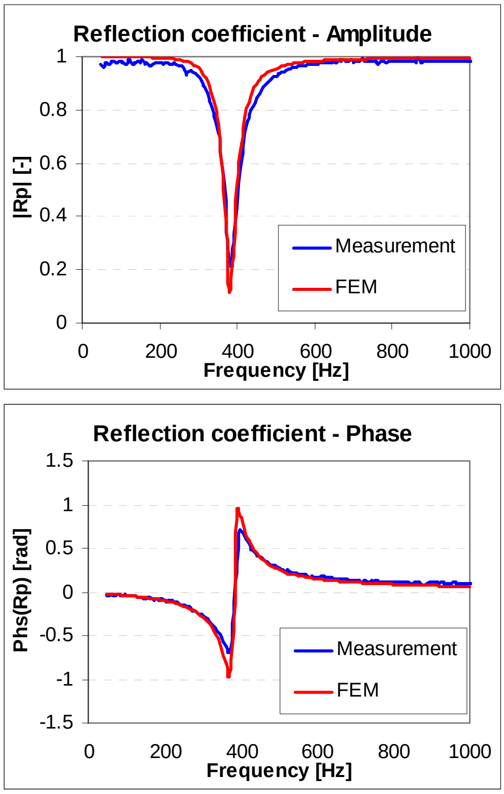


Figure 5. Complex reflection coefficient resonator (a)

Results from the complete set of FEM simulations have been compared with experimental data in terms of resonance frequency (f_{res}), amplitude of the absorption curve at the resonance frequency ($\alpha_{\max}$) and half power width of the same curve (δ). The comparison between the mean values of the deviations between the three simulated and measured parameters is reported in Table 1.

Table 1: Comparison between experimental and numerical results

	Δf_{ris} [Hz]	α_{max} [-]	δ [Hz]
Mean value	**5.7**	**0.08**	**11.9**
Standard deviation	**6.7**	**0.06**	**10.6**

The analysis of Table 1 shows that the proposed formulation is able to predict the acoustical surface properties of the resonators with good accuracy.

4. Perforated elements in mufflers

Once the acoustical formulation has been validated for modelling losses in plane perforated panels, it has been applied for predicting the sound transmission loss of reactive mufflers. It has to be underlined that perforates in mufflers could be implemented in a finite element code (Comsol Multiphysics as an example) by substituting the 3D perforated shells with a 2D layer having a given impedance “jump” [9]. The last procedure (in this paper named *impedance layer*) permits to reduce considerably the computational time (reductions up to 75% has been found in the present work); however previous researches [10,11] have highlighted that this simulation is not reliable when the back cavity is filled with fibrous materials.

Mufflers and perforates were made of PVC. The perforated cylinders had three different perforation ratios that were 3.86% (hole radius 3mm), 6.86% (hole radius 4mm) and 7.71% (hole radius 3mm). The muffler has been tested also with the back cavity filled with low density polyester fiber (10 kg/m^3). A picture of the muffler and perforates is depicted in Figure 6.

The normal incidence sound transmission loss has been determined in the frequency range between 50 and 4000 Hz in a plane wave tube by means of a transfer matrix approach [12,13]. By measuring the sound pressure at four microphone positions it is possible to measure the complex reflection and transmission coefficient, the sound transmission loss and the fraction of energy absorbed within the system. A picture of the measurement set-up is shown in Figure 7 [13].

Figure 6. Muffler and perforates.

Figure 7. Measurement tube

The model of the plane wave tube and the reactive muffler has been developed in Comsol Multiphysics 3.5a and shown in Figure 8.

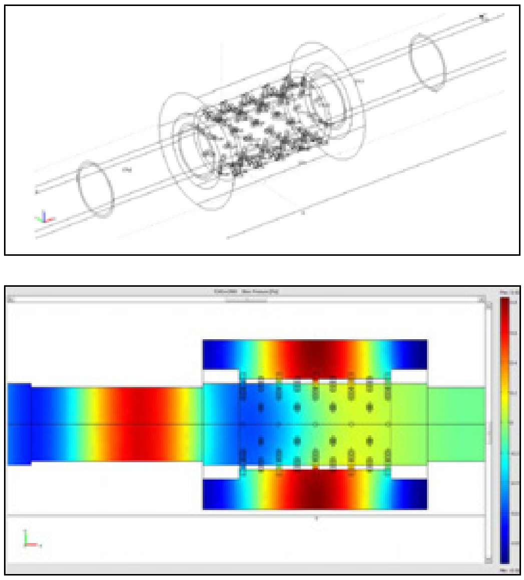


Figure 8. FEM model of the muffler and internal sound pressure distribution.

A sound source (with a given radiation condition of 1 Pa) has been positioned at one end of the tube and a perfectly anechoic termination (with a give impedance condition of 415rayls) at the other end. External lateral boundaries have been modelled as “hard”. It has to be emphasized that this condition is not true for the muffler since in real experiments part of the sound energy is outgoing through the solid structure of the muffler.

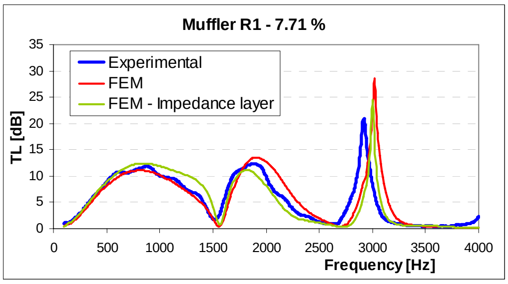
Domain conditions can be summarized as follows:

- holes: density and sound velocity of the fluid are modelled by using expression (1).
- Air: ρ=1.21 kg/m^3 and c=343 m/s.

- Fibrous materials: equivalent dissipative fluid approach: density and sound velocity have been determined by using a semi-empirical model developed for polyester fiber materials [14].

Mesh has been created according to the rule of 10 finite elements per wavelength.

Figure 9 shows the comparison between measured and simulated sound transmission loss of the muffler with a perforated cylinder (perforation ratio equal to 7.71 %). In the same figure the transmission loss has been reported for the case of FEM model with the layer impedance approach.



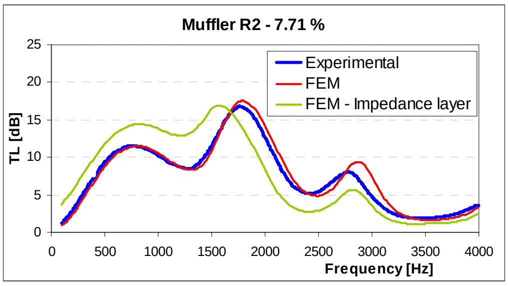


Figure 8. Sound transmission loss of muffler with perforation ratio of 7.71 % (a)R1: empty back cavity. (b)R2: back cavity filled with polyester fiber 10kg/m^3.

From previous figures it is possible to observe that when the cylinder is not in contact with an absorbing porous material both 3D full FEM and impedance layer FEM models provide reliable results when compared to measured data up to 2500 Hz. Differences are lower than 2-3 dB. Discrepancies can be observed at the cavity resonance occurring at 2900 Hz and they could be mainly due to small geometrical differences between real muffler and numerical model; moreover it has to be underlined that the expressions of the domain parameters in eq. (1) have been validated at normal incidence while within the muffler the flow is propagating mainly at grazing incidence.

On the other hand when a fibrous material is put in the back cavity the impedance layer model leads to wrong results in terms of amplitude and high resonance frequencies. On the contrary the proposed formulation permits to predict the transmission loss with good accuracy (differences lower than 2-3 dB in the entire frequency range). Once again discrepancies can be observed around 2900 Hz.

In Figure 10 the comparison between experimental and numerical internal absorption coefficient is reported for the muffler with perforation ratio of 7.71 % and back cavity filled with polyester fiber 10kg/m^3.

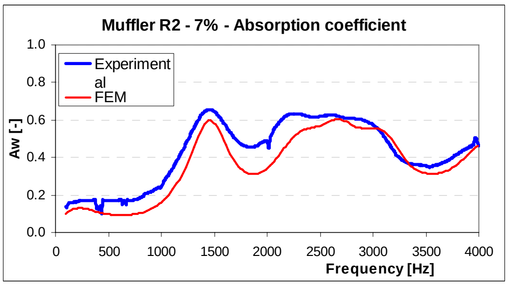


Figure 10. Internal absorption coefficient of the muffler with perforation ratio of 7.71 % and back cavity filled with polyester fiber 10kg/m^3.

The analysis of the figure shows that the FEM model underestimates the fraction of absorbed energy. As previously mentioned the boundaries of the FEM model are completely reflective and energy can not outgoing the solid structure of the muffler; a coupled structural-acoustical FEM model of the muffler should be implemented to account this effect.

5. Concluding remarks

In the present paper it has been presented an acoustical formulation for the FEM modelling in Comsol Multiphysics 3.5a of the losses occurring when a sound wave is passing through perforated sheets. The proposed approach is based on the concept of equivalent dissipative fluid; the perforated panel is modelled as a porous material with adequate values of physical properties (airflow resistivity, porosity, tortuosity and characteristic lengths) depending on its geometry.

Initially the formulation has been applied to plane acoustical resonators and sound absorption coefficient has been calculated with satisfying accuracy at normal incidence. Successively the same formulation has been utilized to model perforates within a muffler and to predict its normal incidence sound transmission loss. The FEM model has permitted a reliable prediction of the sound transmission loss also in the case of back cavity filled with fibrous material. It is interesting to underline that when a impedance layer approach was used to model the perforated sheet in contact with a rigid framed fibrous material results were completely wrong.

Future effort will be devoted to include mean flow and temperature effects through the holes of the perforates as well as the absorbing materials.

6. References

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