Dynamic Crack Propagation in Fiber Reinforced Composites

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Abstract: A generalized model to predict dynamic crack propagation in fiber composite structures is proposed. The proposed approach is based on a generalized formulation based on the Fracture Mechanics approach and Moving mesh methodology. Consistently to the Fracture Mechanics, the crack propagation depends from the energy release rate and its mode components, which are calculated by means of the decomposition methodology of the J-integral expression. The geometry variation, produced by the crack advance, is taken into account by means of a moving mesh strategy based on the Arbitrary Lagrangian-Eulerian formulation. The governing equations are solved numerically by using Comsol Multiphysics. Comparisons with experimental results are reported to validate the proposed modeling.

Keywords: composite materials, delamination phenomena, ALE formulation, energy release rate

1. Introduction

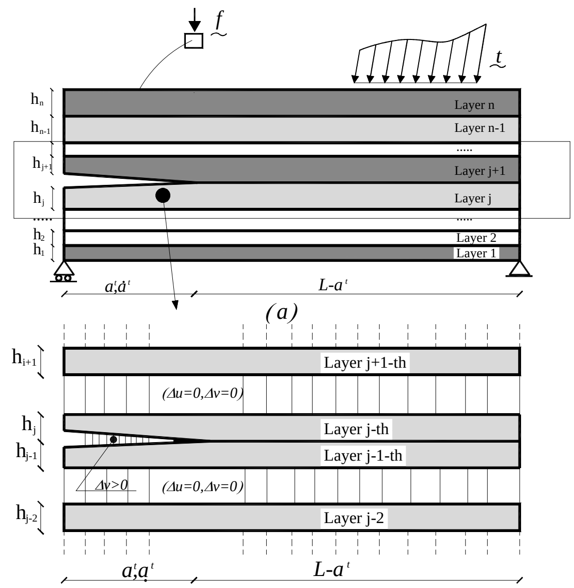
The simulation of propagating cracks in materials is essentially complex in nature, since many phenomena affect the crack growth, such as high speed crack propagation and multiple cracks with branching mechanisms. During the last decades many efforts have been made to analyze dynamic fracture behavior, giving rise to several studies devoted to predicting crack growth phenomena [1]. In the framework of composite structures, most research efforts were confined to static or low velocity crack propagation whereas dynamic delamination phenomena are not completely investigated. Indeed, a brief review of the literature denotes that many papers are concerned to analyze the main features of crack growth, neglecting a priori the inertial contributions arising from fast propagating phenomena. In this paper, the structural modeling is developed by means of a finite element formulation based on a plane stress behavior, whereas the crack growth is predicted by a Fracture Mechanics approach. In order to simulate the dynamic crack growth, the proposed modeling utilizes a fracture toughness criterion based on the energy release rate (ERR) and the corresponding mode components. The ERR is evaluated by means of the decomposition methodology of the J-integral expression, which is proposed in the framework of the dynamic crack propagation [2]. The change of the geometry, produced by a crack advance, is taken into account by means of a moving mesh strategy based on an Arbitrary Lagrangia-Eulerian (ALE) formulation, based on Winslow regularization technique.

2. Formulation of the Damaged Model

The general formulation of the structural model is consistent to a 2D plane stress approach, in which the behavior of each lamina is homogeneous, linear and elastic (Fig.1).

In order to reproduce multilayered laminate, the general model is based on the assemblage of 2D layer connected by the interfaces (Fig.1a) In particular, at the interface plane in which delamination occurs the crack faces are subjected to contact spring model to avoid compenetration effects during the crack motion. Moreover, at the interfaces not affected by interfacial cracks, constrain equations are introduced to reproduce continuity condition in the displacement fields (Fig.1b). It is worth noting that the interfacial defects are assumed to propagate along the interfaces between the laminas, which are basically weak planes, in which the delamination defects are able to growth, producing high stiffness and strength reduction [3]. This assumption can be motivated from physical point of view, since many experimental observations have shown that the evolution of such interfacial defects (known as delaminations) proceeds along a prescribed path almost fixed in the interface zones, leading to measured crack speeds ranging also in the framework of intersonic crack propagation.

In order to reproduce the geometry changes in the continuum, plane stress and ALE formulation are defined in the same reference mesh. Moreover, the motion of the crack is expressed as a function of an explicit criterion written in terms of crack tip speed and ERRs mode components.



(b)

Figure 1. a) Schematic illustration of the delaminated model subjected to general loading. b) Compatibility conditions at the interfaces.

The dynamic energy release rate (ERR) is calculated by using the J-integral concept [4], evaluated on a contour surrounding the crack tip. In the case of a mixed mode loading condition, the ERRs components are expressed by means the decomposition technique as follows[2]:

$$(J_I, J_{II}) = (G_I, G_{II}) =$$
$$= \oint_{\partial\Omega}\left[\left(W^{S,AS}+K^{S,AS}\right)n_1-\sigma_{ij}^{S,AS}n_j\frac{\partial u^{S,AS}}{\partial x}\right]ds+ \quad (1)$$
$$+\int_{\Omega}\left[\rho\left(\ddot{\underset{\sim}{u}}^{S,AS}-\underset{\sim}{f}^{S,AS}\right)\nabla\underset{\sim}{u}^{S,AS}-\rho\dot{\underset{\sim}{u}}^{S,AS}\nabla\dot{\underset{\sim}{u}}^{S,AS}\right]dA$$

where, $\underset{\sim}{u}$ is the displacement, $\underset{\sim}{t}$ the traction vector, $\underset{\sim}{f}$ is the body force, $\underset{\sim}{\rho}$ is the volume density, W is the strain energy density and K is the kinetic energy and the superscripts (S) and (AS) in Eq.(1) represent the symmetric and antisymmetric components with respect to the crack tip plane.

In order to evaluate the crack growth phenomenon, a fracture criterion based on the crack tip variables should be introduced. From experimental point of view, several observations have shown that the crack growth is strictly dependent from the instantaneous crack tip speed [2, 6]. In the proposed modeling, the crack criterion is based on a logarithmic three parameters evolution law. In particular, it depends for low range of the crack tip speed on an initial value of the ERR, which is, typically, close to the initial crack toughness of the material. Moreover, as far as the ERR grows the crack tip speed reaches asymptotically the Rayleigh wave speed of the material, namely V_R. Therefore, fracture toughness is assumed to be governed by a mixed mode crack tip criterion, solving the following equation for the crack tip speed as far as it is equal to zero:

$$\frac{G_I}{G_{ID}(c_t)}+\frac{G_{II}}{G_{IID}(c_t)}-1\le 0 \quad (2)$$

with

$$G_{ID}(c_{tip})=\frac{G_{0I}}{1-\left(\dfrac{c_t}{V_R}\right)^m},\; G_{IID}(c_{tip})=\frac{G_{0II}}{1-\left(\dfrac{c_t}{V_R}\right)^m}$$

where m is a material parameter predicting the evolution on the speed range and c_t is the speed of the crack tip, (G_{0I}, G_{0II}) correspond to the initial mode components toughness.

3. Governing Equations for Laminated Structures and ALE formulation

The governing equations in the material configuration can be written by means of the principle of virtual works of inertial, external and internal forces:

$$\int_{V^i}\underset{\sim}{\sigma}^i\delta\nabla\underset{\sim}{u}^i dV+\int_{V^i}\rho_i\ddot{\underset{\sim}{u}}^i\delta\underset{\sim}{u}^i dV==\int_{\Omega^i}\underset{\sim}{t}^i\delta\underset{\sim}{u}^i\underset{\sim}{n}dA+ \quad (3)$$
$$+\int_{V^i}\underset{\sim}{f}^i\delta\underset{\sim}{u}^i dV++\int_{\Omega^i_{UD}}\underset{\sim}{\lambda}^i\delta\Delta\underset{\sim}{u}^i\underset{\sim}{n}dA+\int_{\Omega^i_D}p^i\delta\Delta\underset{\sim}{u}^i\underset{\sim}{n}dA$$

where i=1,n_l represent the number of layers of the laminates, $\underset{\sim}{n}$ is the unit normal vector, $\underset{\sim}{f}^i$ is the volume forces vector, $\underset{\sim}{\lambda}^i$ is the Lagrange multipliers traction forces vector of the i-th interface Cauchy stress tensor, dV and dA are the volume and the loaded area in the material configuration, (V^i, Ω^i) are the volume, the area of the laminate and $(\Omega^i_{UD}, \Omega^i_D)$ are the undelaminated and the delaminated areas. The governing equations are completed by a weak statement of displacement continuity at the interfaces in which perfect adhesion occurs:

$$\int_{\Omega_i} \delta \underset{\sim}{\lambda}^i \Delta \underset{\sim}{u}^i \underset{\sim}{n}^i \, dA = 0 \tag{4}$$

with $\underset{\sim}{\lambda}^{iT} = \left[\lambda_1^i \quad \lambda_2^i \right]$ is the Lagrange multiplier vector, $\Delta \underset{\sim}{u}^{iT} = \left[u_1^i - u_1^{i+1} \quad u_2^i - u_2^{i+1} \right]$ are the relative displacements prescribed to be equal by continuity requirements and $\underset{\sim}{n}^i$ is the normal vector of the undelaminate area

Consistently to ALE formulation, the motion of the body is described in the referential configuration and thus Eq.(3), should be reformulated to take into account for the transformation rule between Lagrangian and referential coordinate systems. According to ALE formulation, the time derivatives of a generic physical field, in referential and material configurations can be related by the following relationship [5]:

$$\dot{\mathrm{f}} = \mathrm{f}' - \underset{\sim}{\mathrm{X}}' \frac{\mathrm{d}}{\mathrm{d}\underset{\sim}{\mathrm{X}}} \mathrm{f}\left(\underset{\sim}{\mathrm{X}}, \mathrm{t}\right) \quad \text{with} \quad \underset{\sim}{\mathrm{X}}' = \frac{\mathrm{d}}{\mathrm{dt}} \chi\left(\underset{\sim}{\mathrm{r}}, \mathrm{t}\right)\Big|_{\underset{\sim}{\mathrm{r}}} \tag{5}$$

where $\underset{\sim}{\mathrm{X}}'$ represents the relative velocity of the grid points in the material reference system. Moreover, in those cases in which gradient operators are involved, the transformation rules from the material and referential configurations require the evaluation of the Jacobian (J) by means of the following relationship:

$$\underset{\sim}{\nabla}_{\underset{\sim}{\mathrm{X}}} \underset{\sim}{\mathrm{f}} = \underset{\sim}{\nabla}_{\underset{\sim}{\mathrm{r}}} \underset{\sim}{\mathrm{f}} \; \underset{\sim}{\mathrm{J}}^{-1} \quad \text{with } \underset{\sim}{\mathrm{J}} = \frac{\mathrm{d}\underset{\sim}{\mathrm{X}}}{\mathrm{d}\underset{\sim}{\mathrm{r}}} \tag{6}$$

Substituting Eq.(5)-(6) into Eq.s(3)-(4) the governing equations in the referential configuration are determined.

In the framework of ALE formulation the computational mesh may moves arbitrary with respect to the material body. In particular, the mesh movements should be addressed to reduce distortions of the mesh elements and to handle for changes of the geometry produced by the crack growth. In the proposed formulation, a smoothing variational method based on Winslow approach is utilized, in which the horizontal and vertical mesh displacements of the generic i-th lamina are evaluated by solving the following expressions:

$$\nabla^2_{\underset{\sim}{X}} \Delta X_1^i = 0, \quad \nabla^2_{\underset{\sim}{X}} \Delta X_2^i = 0. \tag{7}$$

Internal and external boundary conditions need to be introduced to reproduce the crack growth. Weak forms of smoothing differential equations are derived, by multiplying Eq.s **Error! Reference source not found.** by a weight functions $w_1^i\left(X_1, X_2\right)$, $w_2^i\left(X_1, X_2\right)$ and then integrating by part. Moreover, the boundary conditions regarding the prescribed crack tip speed, i.e. Eq.**Error! Reference source not found.**.3, is taken into account for by means of non-ideal weak constraint based on the Lagrangian multiplier method. Therefore, the resulting equations regarding the ALE formulation are:

$$\begin{aligned} &\int_{V_r^i} \left(\nabla_{\underset{\sim}{r}} \Delta \underset{\sim}{X} \underset{\sim}{J}^{-1} \right)^i \cdot \left(\nabla_{\underset{\sim}{r}} \underset{\sim}{w} \underset{\sim}{J}^{-1} \right)^{Ti} \underset{\sim}{\mathrm{I}} \det\left(J^i \right) dV_r^i + \\ &+ \oint_{\Omega_{tip}^i} \left[\delta \underset{\sim}{\lambda} \left(\underset{\sim}{X}' - \underset{\sim}{c}_t \right) \underset{\sim}{k} + \underset{\sim}{\lambda} \delta \dot{\underset{\sim}{X}} \underset{\sim}{k} \right] \det\left(\overline{\overline{J}} \right) ds_r^i = 0, \end{aligned} \tag{8}$$

where $\underset{\sim}{w}^{iT} = \left[\underset{\sim}{w}_1, \underset{\sim}{w}_2 \right]$ is the weight function vector, $\underset{\sim}{k}^T = [1, 0]$ is the propagation direction vector of the interfacial crack, $\underset{\sim}{c}_t^{iT} = \left[c_t, 0 \right]$ is the crack tip speed vector, $\det\left(\overline{\overline{J}} \right)$ is the determinant of a scalar metric representing the ratio of differential length, $\underset{\sim}{\mathrm{I}}$ is the identity 2x2 matrix.

4. Finite Element Approximation

Governing equations given by Eq.(8) and Eq.(12) with boundary conditions described by Eqs.(10) introduce a non linear set of equations, which have been solved numerically, using a user customized finite element program COMSOL Multiphysics TM version 3.5. It is worth noting that since second order time derivative of the variables involved in the main equations are not provided in the ALE application mode, customized version of these relationships are implemented.

Finite element expressions are written for 2D plane stress modeling, utilizing Lagrangian interpolation shape functions. The governing equations regarding the plane stress and the ALE formulations are solved by using a finite element isoparametric approach. The algebraic equations are solved by using an implicit time integration scheme based on variable-order variable-step-size backward differentiation formulas (BDF). During the time integration, due to the fast speeds involved in the crack advance, a small time step size is utilized. In order to compute accurately the ERR with the aid of J-integral formulas, a fine discretization meh and a standard numerical integration method (quadrature) is adopted on the contour line and on the area surrounding the crack tip. The time integration procedure at each iteration step checks the crack advance criterion . In the case it is satisfied a prescribed crack speed, evaluated by solving the crack criterion, is applied to the crack tip area and thus moving boundary conditions are applied at the crack front. In order to avoid mesh distortions in the crack tip region and lost of accuracy in the evaluation of the crack tip solution, a remeshing algorithm is need. The remeshing procedure is performed by using COMSOL finite element program, by checking that the minimum value of the mesh quality parameter regarding the geometry of the element in the undistorted configuration should be greater than a fixed tolerance. Once this condition is not satisfied, a remeshing procedure is performed on the basis of the deformed geometry in the actual reference system and a restart with an updating procedure from the previous converged time step is developed (Fig.2).

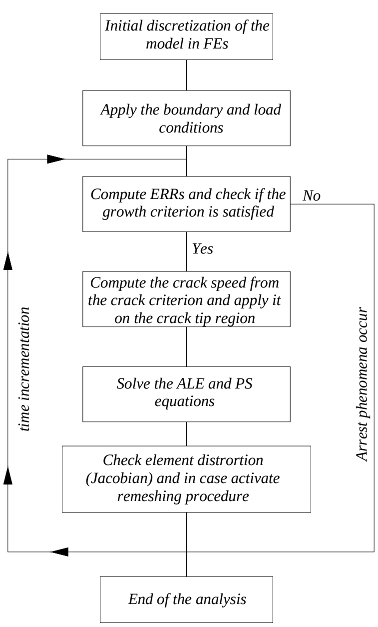


Figure 2. Flow chart of the FEM integration algorithm.

5. Results

Comparisons with experimental results are performed to validate the proposed modeling for mode I and mixed mode loading conditions. A double cantilever beam (DCB) specimen, under mode I loading condition is analyzed [6]. The material tested is AS 3501-6 Graphite/Epoxy, the laminate is formed by unidirectional laminas and the specimen is 260mm long, 20mm wide and 3.7mm thick. The material properties and parameters regarding the crack criterion are reported in Tab.n.1.The loading rate at the left end points is 0.1 mm/s, whereas the crack propagation is produced introducing a strip of adhesive film at the crack tip, which enforces the crack to growth at high speeds.

E_1 [Mpa]	E_2=E_3 [Mpa]	G_{12}= G_{23} [Mpa]	ρ [Kg/m^3]	ν_{12}= ν_{13}	m	G_0
142E3	10.3E3	7.2E3	1580	0.27	0.5	300

Tab.1. Mechanical properties of unidirectional fiber-reinforced AS4 graphite epoxy.

In order to reproduce correctly the experimental results, at first the laminate ends are displaced statically to an initial value of ERR, leaving the crack tip fixed in the initial position. It is worth noting that since the relationship between crack tip speed and ERR toughness is not provided in the experimental results, the material parameters, involved in the definition of the crack criterion, are taken as adjustable variables.

The FE model involve in the computation 50000 variables and On a Dual Core processor at 2500Mz the CPU time required for performing the time history for each case was approximately 10 min. In Fig.3 the mesh model and with a particular of the integration path surrounding the crack tip is reported.

In Fig. 4 comparisons with experimental results, in terms of time history of the crack tip displacements are reported. The agreement between proposed and experimental results is noted. In Fig. 5, the evolution of the strains and kinetic energies and the energy expended in the crack growth are also reported. Moreover, the crack tip speed reaches his maximum value especially during the initiation phase with value comparable to the ones of the material wave characteristic.

In order to analyze the crack tip motion in Fig.6, results concerning the crack tip motion for different time steps are reported. It is worth noting that only the small region surrounding the crack tip require an accurate discretization of the mesh size, since the mesh motion is enforced to evolve during the crack propagation rigidly, avoiding as a result distortions of the crack tip region.

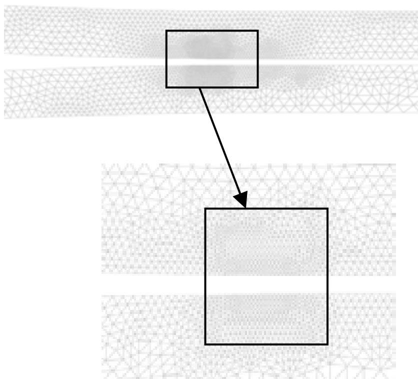


Figure 3. Mesh model and discretization of the crack tip region

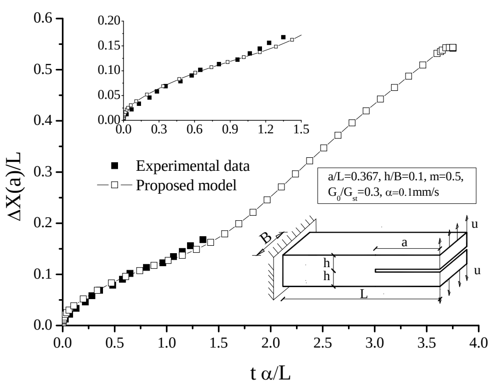


Figure 4. Mode I dynamic crack growth in a DCB scheme. Comparison between experimental data and proposed results: time history of the crack tip displacement ΔX.

7. Conclusions

A general model based on Fracture Mechanics and ALE formulation is proposed. The analysis is developed in an unsteady dynamic crack propagation, in which the influence of time dependence and the inertial forces are taken into account. The moving mesh strategy combined with a Fracture Mechanics approach is able to predict properly the time dependent behavior of delamination phenomena. The proposed modeling is based on a generalized mixed mode dynamic fracture toughness criterion, which depends on a limited number of adjustable variables. Comparisons with experimental results for loading conditions involving mode mix at high speeds of the crack tip are proposed**.**

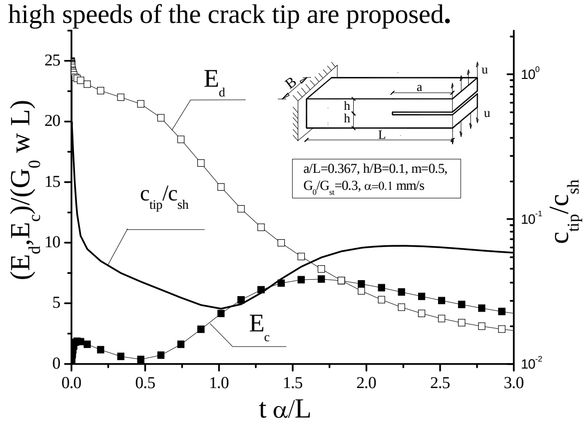


Figure 5. Mode I dynamic crack growth in a DCB scheme. Time history of the strain energy, kinetic energy and crack tip speed.

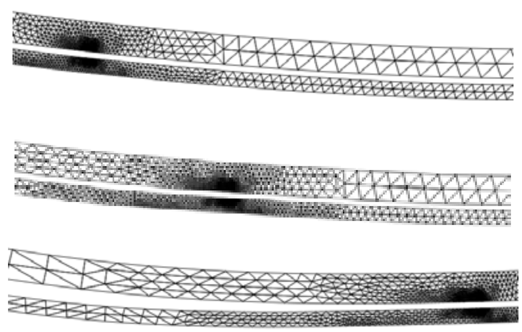


Figure 2. Crack-tip propagation.

8. References

1. Freund LB., Dynamic Fracture Mechanics, *Cambridge University Press*, Leyden (1990)
2. Greco F., Lonetti P., *Mixed mode dynamic delamination in fiber reinforced composites*, Vol. 40, Issue 5, Pages 379-392, Composites Part B:Engineering (2009)
3. Barbero EJ, Finite Element Analysis of Composite Materials. CRC: Boca Raton, FL, 2007.
4. Nishioka,T Tokudome H., Kinoshita M., Dynamic fracture-path prediction in impact fracture phenomena using moving finite element method based on Delaunay automatic mesh generation, International Journal of Solids and Structures, Vol. 38, pp. 5273-5301, 2001.
5. J. Donea et al., Arbitrary Lagrangian-Eulerian methods, *Encyclopedia of Computational Mechanics*
6. C. Guo and C.T. Sun, Dynamic mode-I crack propagation in a carbon/epoxy composite, *Composites Science and Technology*, Vol.58, Pages 1405-1410, (1998)