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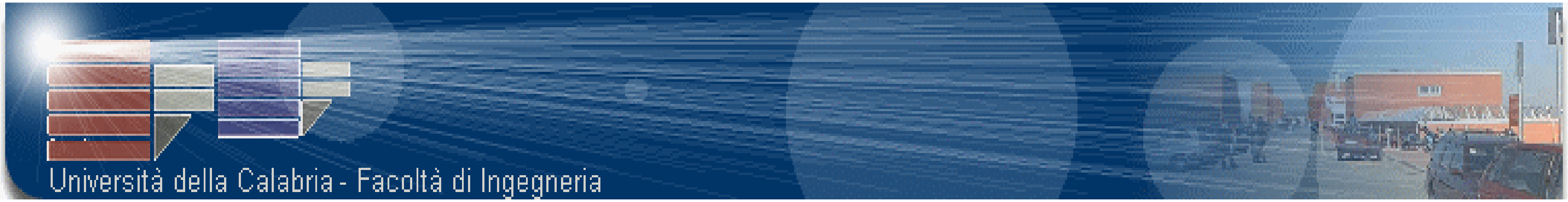
Dynamic Crack Propagation in Fiber Reinforced Composites

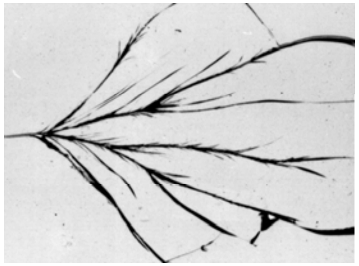
P. Lonetti, C. Caruso, A. Manna

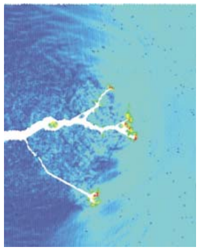


DYNAMIC FRACTURE MECHANICS

MONOLITIC MATERIALS

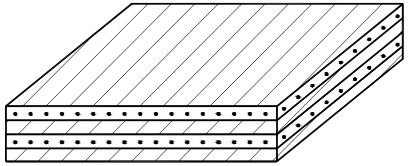
- Crack Branching phenomena
- Crack speeds are limited
- Unknown path of the crack

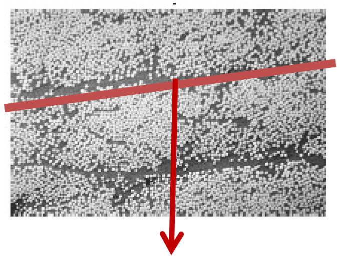


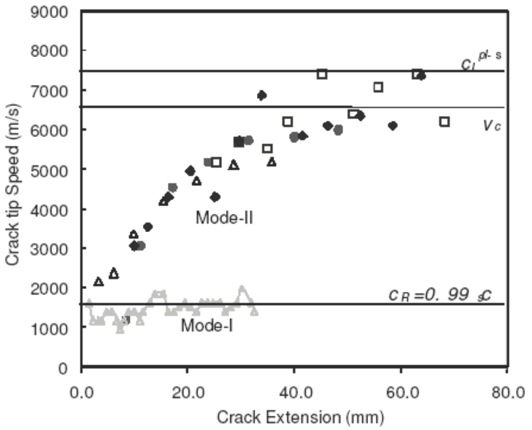


Ravi-Chandar and Knauss, *Int J Fract*, 1984

COMPOSITE STRUCTURES







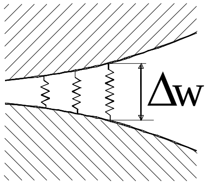
(Rosakis, A.J., “Intersonic shear cracks and fault ruptures propagation”, Advances in Physics, 2002)

- High crack speed
- Crack constrained along the interfaces

DYNAMIC CRACK GROWTH MODELING

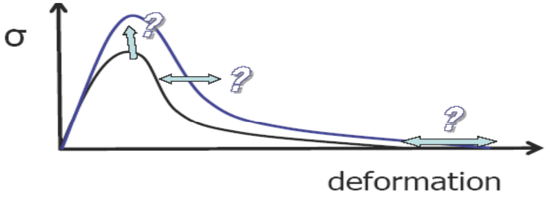
Cohesive modeling

- Interface elements are introduced at the crack region
- Damaged constitutive relationship is required



Fracture Mechanics approaches

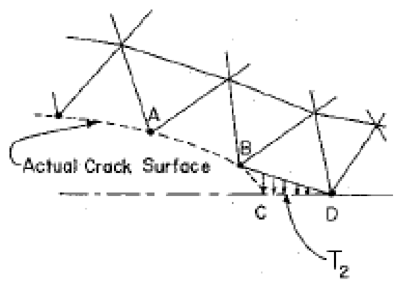
- **Static analyses:**
  (the time dependence is neglected "a priori")
- **Steady state crack growth approaches:**
  (Moving reference system with the tip, crack tip speed is constant)
- **Unsteady models :**
  Full Time dependence , inertial forces, loading rate,….

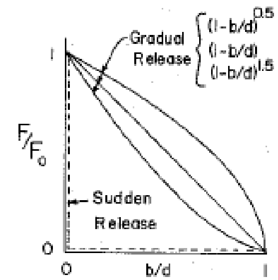


DYNAMIC CRACK GROWTH MODELING

Node release technique

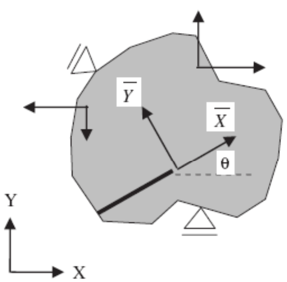
- Gradual release of the nodal forces behind the crack tip

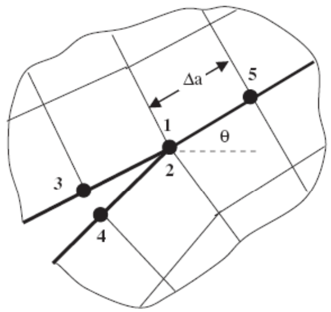




Virtual crack closure methods

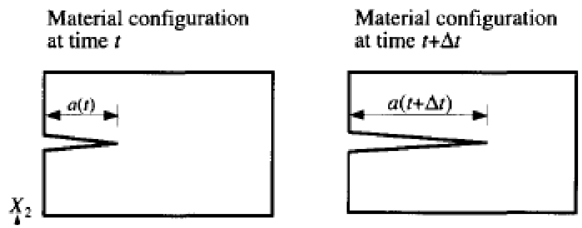
- The ERR is evaluated by the mutual work at the crack tip and behind the crack tip





Moving mesh methodology

- The nodes are moved to predict changes of the geometry produced by the crack motion



MOTIVATION OF THE WORK AND SUMMARY

AIM OF THE WORK

- **Propose a generalized modeling based on Fracture mechanics and moving mesh methodology to predict the dynamic behavior of composite laminated structures**

SUMMARY

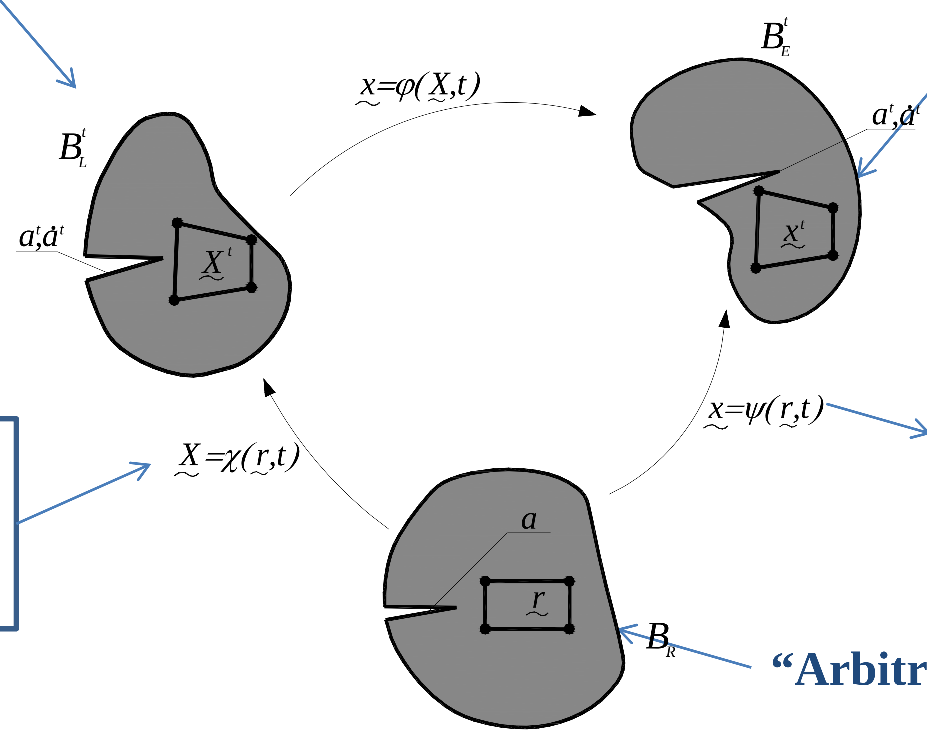
- **Review the main equations of the ALE formulation in view of the Dynamic Fracture Mechanics approach**
- **Evaluate the specialized expressions of the ERR by the use of the decomposition methodology of the J-integral and propose a proper mixed mode crack toughness criterion**
- **Develop the finite element implementation. Propose validation by means of comparisons with experimental data and a parametric study to analyze dynamic crack behavior (i.e. crack arrest phenomena, allowable tip speeds and rate dependence of the interfacial crack growth)**

BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION

INDEX:

“Lagrangian Approach”

“Eulerian Approach”



$$\chi : B_R \to B_L$$
$$X = \chi(r, t)$$

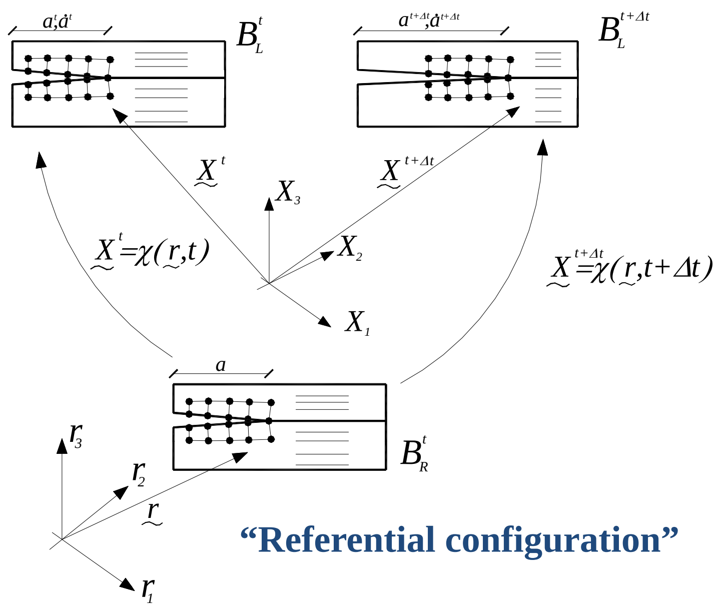
$$\psi : B_L \to B_E$$
$$x = \psi(r, t)$$

“Arbitrary Lagrangian Eulerian”

The Reference configuration is fixed and independent of any placement of the material body

BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION

time "t" **time "t+dt"**



"Referential configuration"

Physical quantities:

$$\underset{\sim}{v} = \frac{d}{dt}\varphi\left(\underset{\sim}{X},t\right)\Big|_{\underset{\sim}{X}} , \qquad \underset{\sim}{X}' = \frac{d}{dt}\chi\left(\underset{\sim}{r},t\right)\Big|_{\underset{\sim}{r}}$$

"Material" "Referential"

$$\dot{f} = f' - \underset{\sim}{X}'\frac{d}{d\underset{\sim}{X}} f\left(\underset{\sim}{X},t\right)$$

Time derivative rule

Physical fields in ALE formulation

$$\ddot{\underset{\sim}{u}} = \underset{\sim}{u}'' - 2\nabla_x \underset{\sim}{u}'\cdot\underset{\sim}{X}' - \nabla_x \underset{\sim}{u}\underset{\sim}{X}'' + \nabla_x\left(\nabla_x \underset{\sim}{u}\right)\underset{\sim}{X}'\,\underset{\sim}{X}' + \nabla_x \underset{\sim}{u}\nabla_x \underset{\sim}{X}'\,\underset{\sim}{X}'$$

"Material accel."

$$\nabla_{\underset{\sim}{x}}\underset{\sim}{u} = \nabla_{\underset{\sim}{r}}\underset{\sim}{u}\ \underset{\sim}{J}^{-1}$$

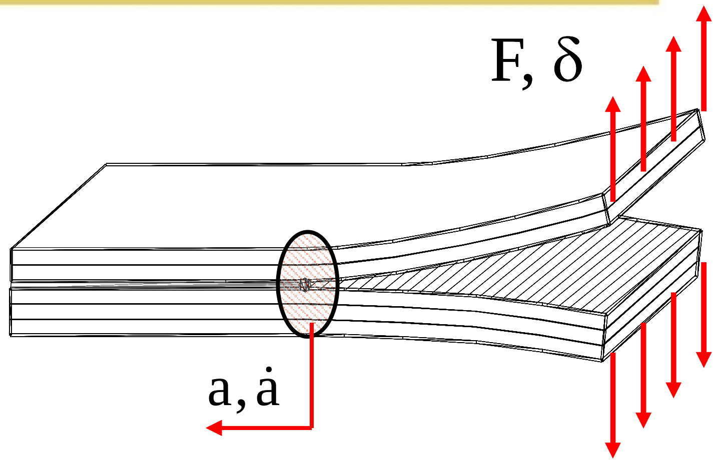
"Grad. transform."

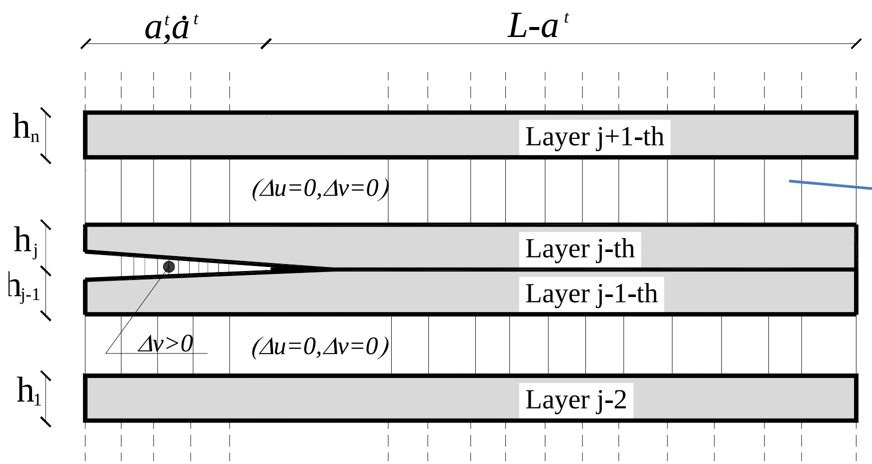
$$\det \underset{\sim}{J} \neq 0$$

"one-to-one relationship"

DESCRIPTION OF THE DELAMINATION MODEL

- **Multi-layer Modeling**
- **2D Kinematic formulation**
- **The laminate is divided into *n* mathematical layer representing the staking sequence**





- **Compatibility equations LMM:**

$$\Delta u_i = u_{i+1} - u_i = 0, \quad \Delta v_i = v_{i+1} - v_i = 0,$$

"undelaminated interfaces"

$$\Delta v_i = v_{i+1} - v_i \geq 0,$$

"delaminated interfaces"

DESCRIPTION OF THE DELAMINATION MODEL IN THE REFERENTIAL CONFIGURATION

Governing Equations: “Principle of d’Alembert”

$$\sum_{i=1}^{n}\int_{V_i} \tilde{\sigma}\,\delta\tilde{\nabla}\tilde{u}\,dV + \sum_{i=1}^{n}\int_{V_i} \rho\ddot{\tilde{u}}\,\delta\tilde{u}\,dV = \sum_{i=1}^{n}\int_{\Omega_i} \tilde{t}\,\delta\tilde{u}\,dA + \sum_{i=1}^{n}\int_{V_i} \tilde{f}\,\delta\tilde{u}\,dV$$

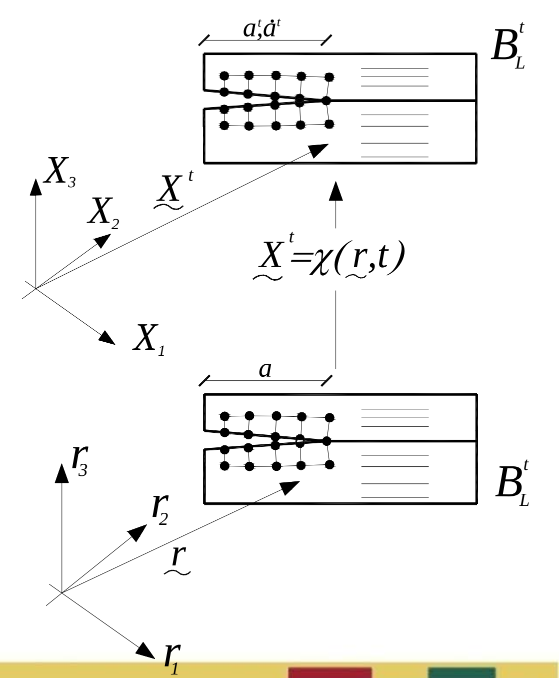
Internal work **External work**

$$\sum_{i=1}^{n}\int_{V_i} \tilde{\sigma}\,\delta\nabla\tilde{u}\,dV = \sum_{i=1}^{n}\int_{V_{ri}} \tilde{C}\left(\tilde{\nabla}_{\tilde{r}}\tilde{u}\tilde{J}^{-1}\right)\delta\left(\tilde{\nabla}_{\tilde{r}}\tilde{u}\tilde{J}^{-1}\right)\det\left(J\right)dV_r$$

$\tilde{J}$: Jacobian

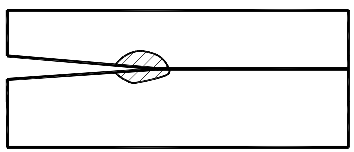
$$\sum_{i=1}^{n}\int_{V_i} \rho\ddot{\tilde{u}}\,\delta\tilde{u}\,dV = \sum_{i=1}^{n}\int_{V_{ri}} \rho\Big[\tilde{u}'' - 2\nabla_{\tilde{r}}\tilde{u}'\,\tilde{J}^{-1}\cdot\tilde{X}' - \left(\nabla_{\tilde{r}}\tilde{u}\,\tilde{J}^{-1}\right)\cdot\tilde{X}'' +$$
$$+\nabla_{\tilde{r}}\left(\nabla_{\tilde{r}}\tilde{u}\,\tilde{J}^{-1}\right)\tilde{J}^{-1}\tilde{X}'\,\tilde{X}' + \nabla_{\tilde{r}}\tilde{u}\,\tilde{J}^{-1}\cdot\left(\nabla_{\tilde{r}}\tilde{X}'\tilde{J}^{-1}\right)\tilde{X}'\Big]\delta\tilde{u}\det\left(J\right)dV_r$$

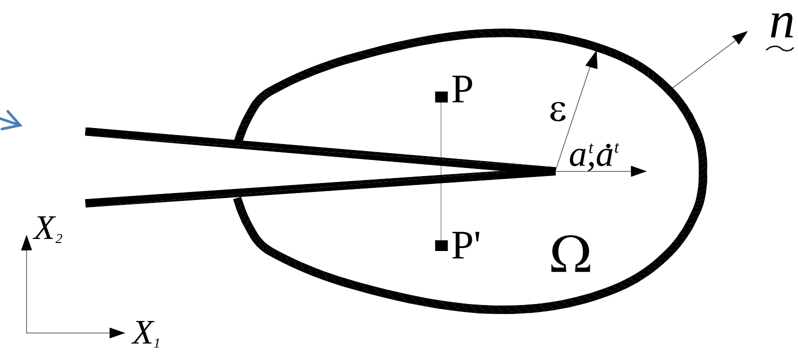
$$\sum_{i=1}^{n}\int_{\Omega_i} \tilde{t}\,\delta\tilde{u}\,dA + \sum_{i=1}^{n}\int_{V_i} \tilde{f}\,\delta\tilde{u}\,dV = \sum_{i=1}^{n}\int_{\Omega_{ri}} \tilde{t}\,\delta\tilde{u}\det(\bar{J})\,d\Omega_r + \sum_{i=1}^{n}\int_{V_{ri}} \tilde{f}\,\delta\tilde{u}\det(J)\,dV_r$$



ERR RATE EVALUATION : J-INTEGRAL APPROACH

Revision of the J-integral Dec. procedure (Rigby & Aliabady, 1998, Greco & Lonetti, 2009)





Expressions of the ERR

$$J = \lim_{\varepsilon \to 0} \oint_{\Omega} \left[(W + K) n_1 - \tilde{t} \frac{\partial \tilde{u}}{\partial \tilde{X}} \right] ds$$

$$J = \oint_{\partial\Omega} \left[(W + K) n_1 - \tilde{t} \frac{\partial \tilde{u}}{\partial \tilde{X}} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}} - \tilde{f} \right) \nabla \tilde{u} - \rho \dot{\tilde{u}} \nabla \dot{\tilde{u}} \right] dA$$

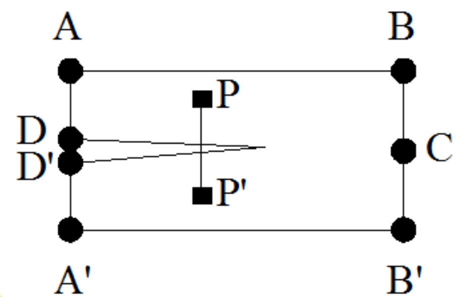
“Path independent”

(Nishioka,T, 2001)\

Decomposition of the ERR into symmetric and antisymmetric fields

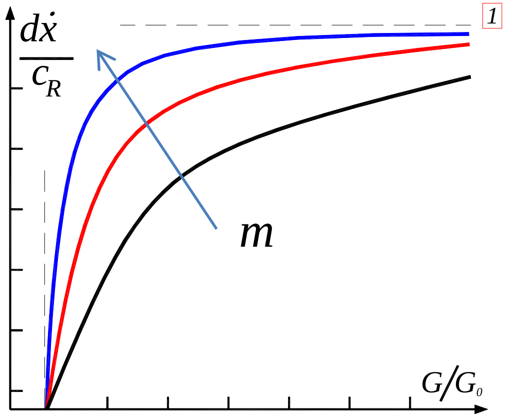
$$J_I = G_I = \oint_{\partial\Omega} \left[\left(W^S + K^S \right) n_1 - \sigma_{ij}^S n_j \frac{\partial u^S}{\partial x} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}}^S - \tilde{f}^S \right) \nabla \tilde{u}^S - \rho \dot{\tilde{u}}^S \nabla \dot{\tilde{u}}^S \right] dA,$$

$$J_{II} = G_{II} = \oint_{\partial\Omega} \left[\left(W^{AS} + K^{AS} \right) n_1 - \sigma_{ij}^{AS} n_j \frac{\partial u^{AS}}{\partial x} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}}^{AS} - \tilde{f}^{AS} \right) \nabla \tilde{u}^{AS} - \rho \dot{\tilde{u}}^{AS} \nabla \dot{\tilde{u}}^{AS} \right] dA,$$



DYNAMIC CRACK PROPAGATION ANALYSIS: GROWTH CRITERION

Crack growth criterion



$$G_D = \frac{G_0}{1-\left(\frac{c_t}{V_R}\right)^m}$$

Material parameter (m)

"Critical value of the ERR"

(Freund, 1990; Ravi-Chandar, 2004)

$c \to V_R \qquad G_D(c_t) = \infty$ — **"Rayleigh wave speed"**

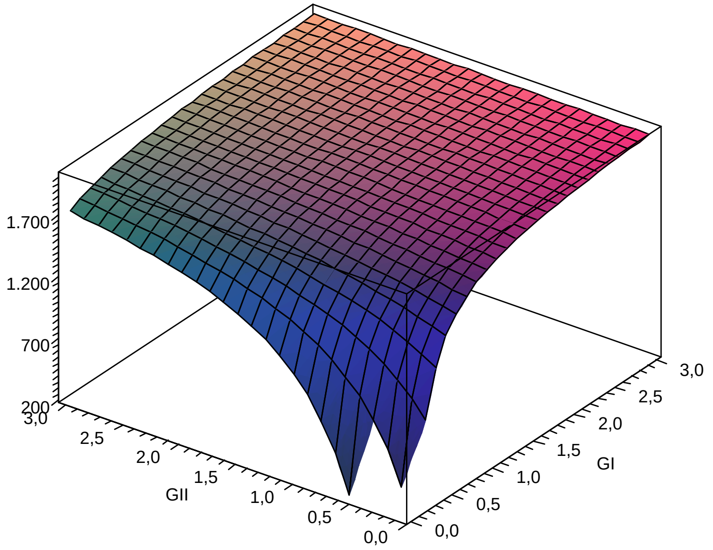
$c \to 0 \qquad G_D(c_t) = G_0(0)$ — **"initiation value"**

1) Mixed mode crack growth criterion

$$g_f = \frac{G_I}{G_{ID}(c_t)} + \frac{G_{II}}{G_{IID}(c_t)} - 1 \le 0$$

Material parameter (m)

$$G_{ID}(c_t) = \frac{G_{0I}}{1-\left(\frac{c_t}{V_R}\right)^m}, \qquad G_{IID}(c_t) = \frac{G_{0II}}{1-\left(\frac{c_t}{V_R}\right)^m}$$



MOVING MESH METHOD: FOUNDAMENTAL EQUATIONS

ALE formulation to describe mesh motion

$$\nabla^2_{\underset{\sim}{X}} \Delta X_1 = 0, \quad \nabla^2_{\underset{\sim}{X}} \Delta X_2 = 0.$$

$$\Delta X_1 = X_1 - r_1 \quad \Delta X_2 = X_2 - r_2$$

"Mesh displacements of nodes Should be regular"

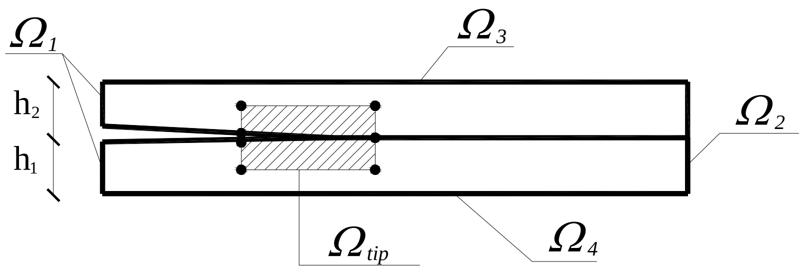
Mesh regularization technique
"Winslow Smoothing method"

Minimize the mesh warping

Boundary conditions

$$\left(\Delta X_1 = 0,\ \Delta X_2 = 0\right) \quad \text{on } \Omega_1 \cup \Omega_2,$$
$$\Delta X_2 = 0 \quad \text{on } \Omega_3 \cup \Omega_4$$
$$\Delta X_1' = 0 \quad \Leftrightarrow if \;\; g_f < 0 \quad \text{on } \Omega,$$
$$\Delta X_1' = c_t \quad \Leftrightarrow if \;\; g_f \geq 0 \text{ on } \Omega,$$
$$\Delta X_2' = 0 \quad \text{on } \Omega$$
$$\Delta X_1(0) = 0,\ \Delta X_2(0) = 0,\ \Delta \dot{X}_1(0) = 0,\ \Delta \dot{X}_2(0) = 0$$

Example: DCB scheme



VARIATIONAL FORMULATION AND FE IMPLEMENTATION

INDEX:

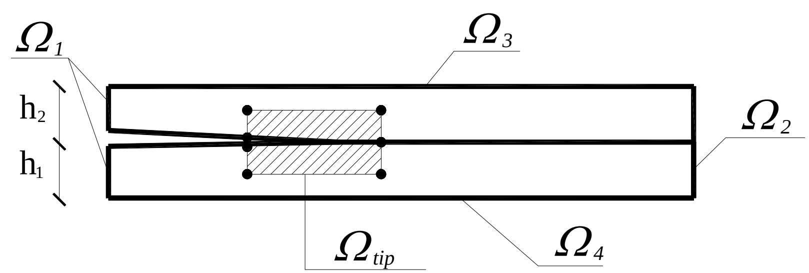
- **Weak forms: coupled equations for the ALE and PS formulations:**

$$\sum_{i=1}^{n}\int_{V_{ri}} C\left(\nabla_r u J^{-1}\right)\delta\left(\nabla_r u J^{-1}\right)\det\left(J\right)dV_r + \sum_{i=1}^{n}\int_{V_{ri}}\rho[u'' - 2\nabla_r u' J^{-1}\cdot X' - \left(\nabla_r u J^{-1}\right)\cdot X'' +$$
$$+\nabla_r\left(\nabla_r u J^{-1}\right)J^{-1}X' X' + \nabla_r u J^{-1}\cdot\left(\nabla_r X' J^{-1}\right)X']\delta u \det\left(J\right)dV_r$$
$$= \sum_{i=1}^{n}\int_{\Omega_{ri}} t\,\delta u \det(\bar{J})\,d\Omega_r + \sum_{i=1}^{n}\int_{V_{ri}} f\,\delta u \det(J)\,dV_r$$

PS

$$\int_{V_r}\left(\nabla_r \Delta X J^{-1}\right)\cdot\left(\nabla_r w J^{-1}\right)\det\left(J\right)dV_r + \oint_{\Omega_r}\left[\delta\lambda\left(X' - c_t\right)i + \lambda\delta\dot{X} i\right]\left(\bar{\bar{J}}\right)ds = 0,$$

ALE



Explicit equations for PS+ALE

Implicit equation the crack growth

→ **Crack growth criterion**

VARIATIONAL FORMULATION AND FE IMPLEMENTATION

INDEX:

FE approximation by "Comsol Multiphysics":

- **Quadratic Lagrangian interpolation functions for displacements, velocity and acceleration fields**
- **Quadratic Lagrangian interpolation functions for mesh points displacements**

Non Linear Equations System

FE equations

$$\sum_{i=1}^{n} \underset{\sim}{M}_i \underset{\sim}{U}''_i + \sum_{i=1}^{n} \underset{\sim}{C}_i \underset{\sim}{U}'_i + \sum_{i=1}^{n} \left(\underset{\sim}{K}_i + \underset{\sim}{K}_{0i} + \underset{\sim}{K}_{1i} + \underset{\frown}{K}_{2i} \right) \underset{\sim}{U}_i + \sum_{i=1}^{n} \underset{\sim}{T}_i + \sum_{i=1}^{n} \underset{\sim}{P}_i = 0$$

$$\underset{\sim}{W} \cdot \Delta \underset{\sim}{X} + \underset{\sim}{Q} \cdot \Delta \underset{\sim}{X}' + \underset{\sim}{L} = 0,$$

CHECK MESH ELEMENTS QUALITY

Solution Procedure

- **Implicit time integration scheme based on variable-step-size backward differentiation formula**

Iterative-incremental Solving procedure

RESULTS: VALIDATION OF THE STRUCTURAL MODEL

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ALE MODEL

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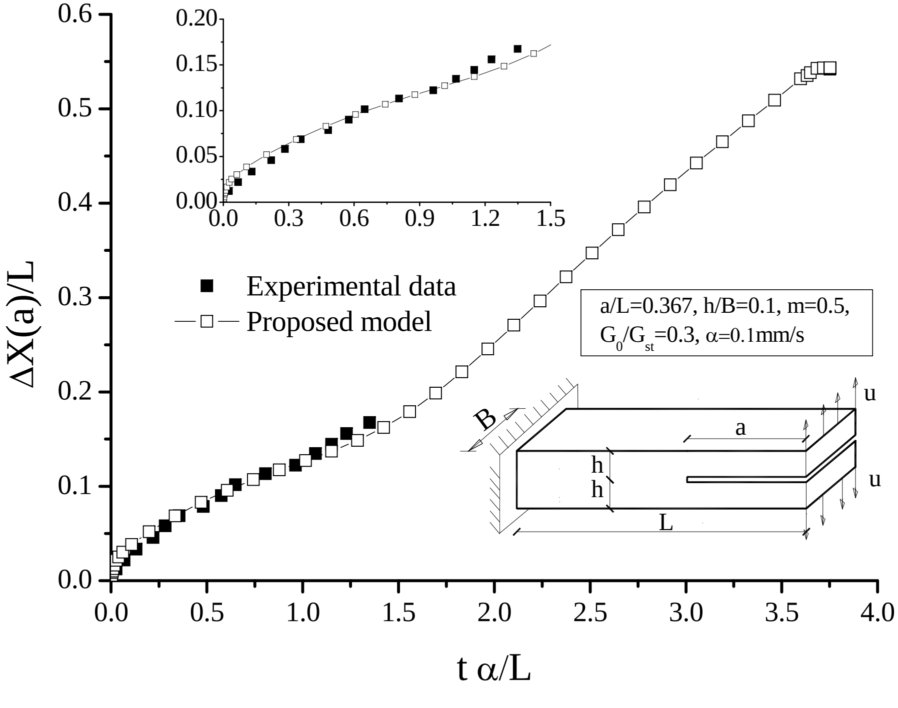
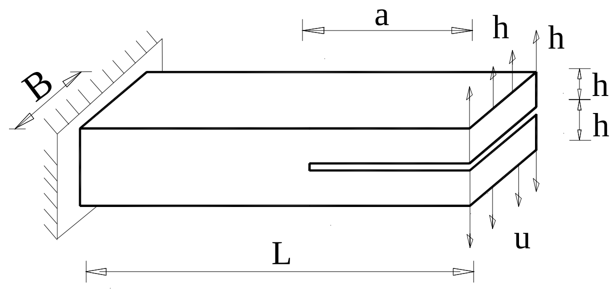
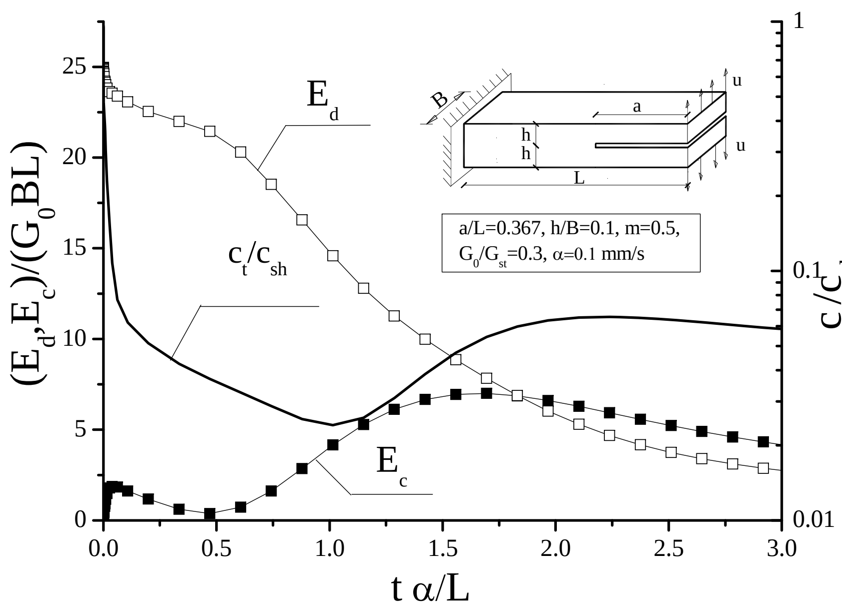
FE MODEL

RESULTS

CONCLUSIONS







- **DCB mode I loading scheme**
- **Comparisons with experimental data**
- **AS 3501-6 Graphite/Epoxy**

RESULTS: EFFECT OF THE LOADING RATE

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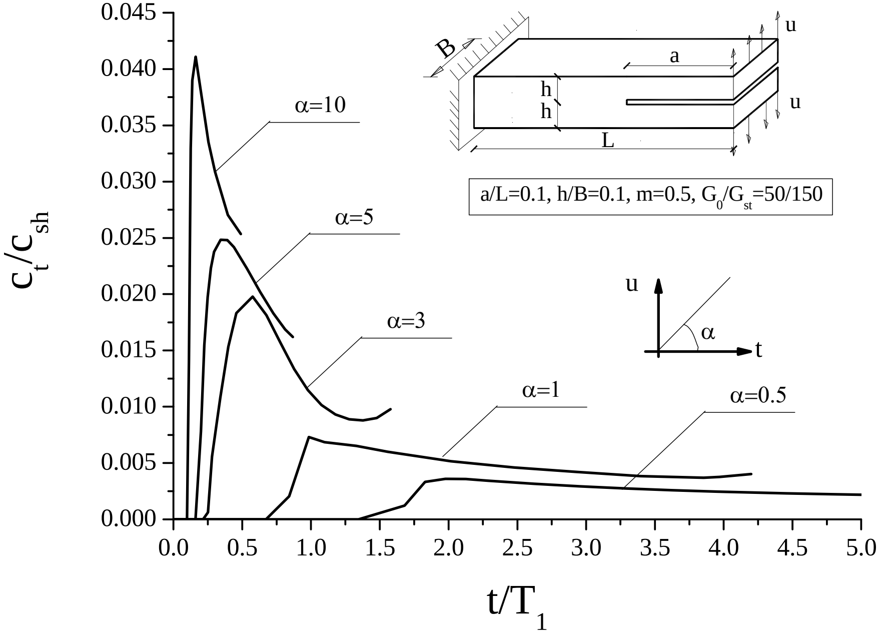
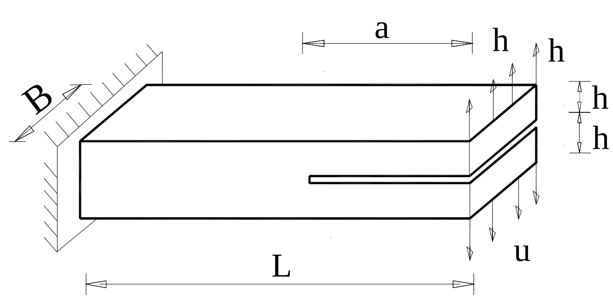
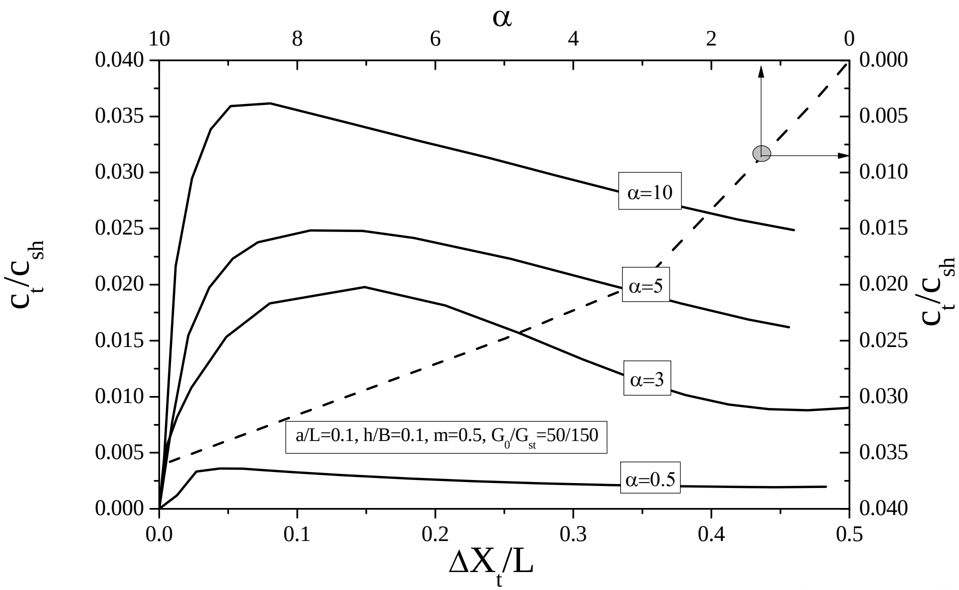


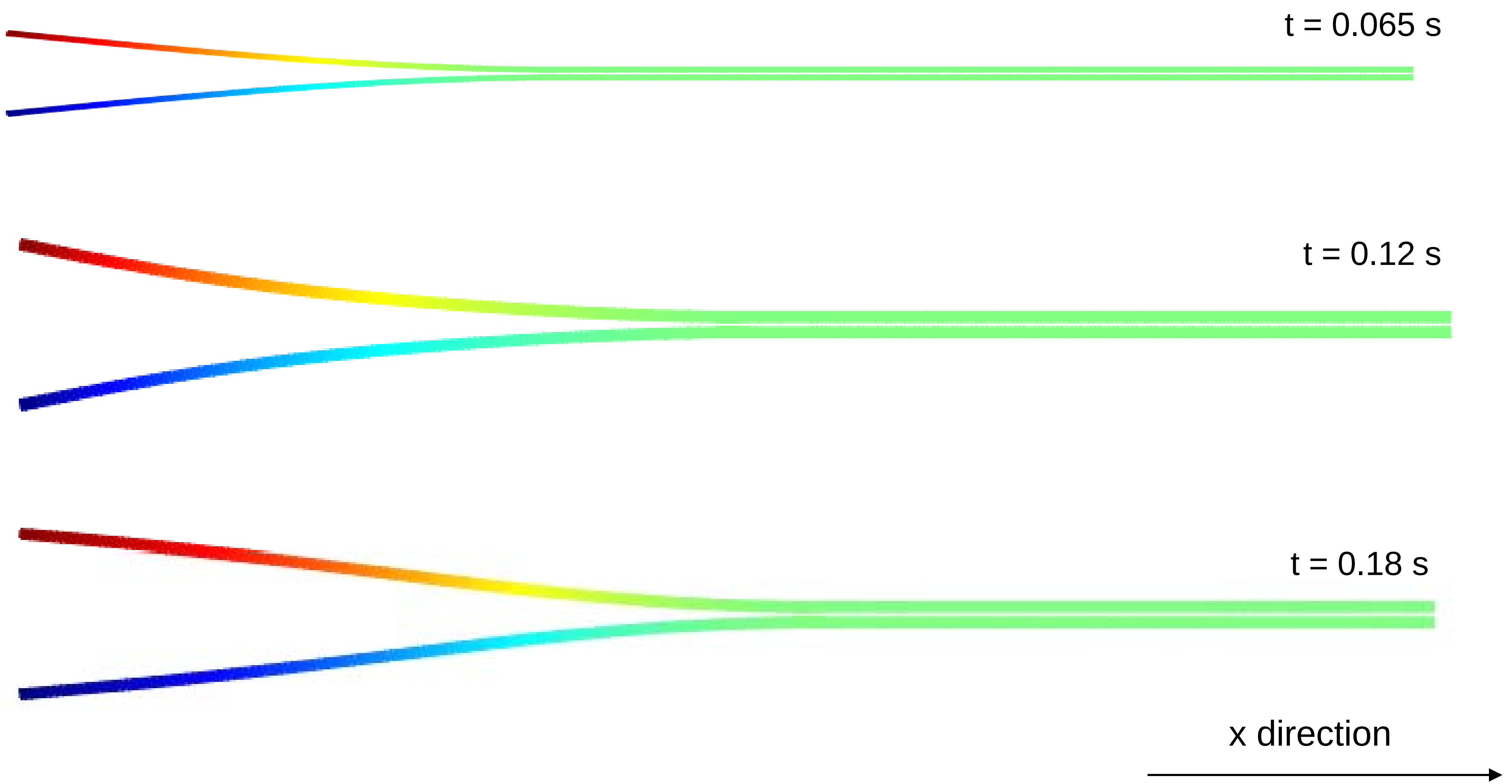




- **DCB mode I loading scheme**
- **Influence of the loading rate**
- **Evolution of the crack tip speed**

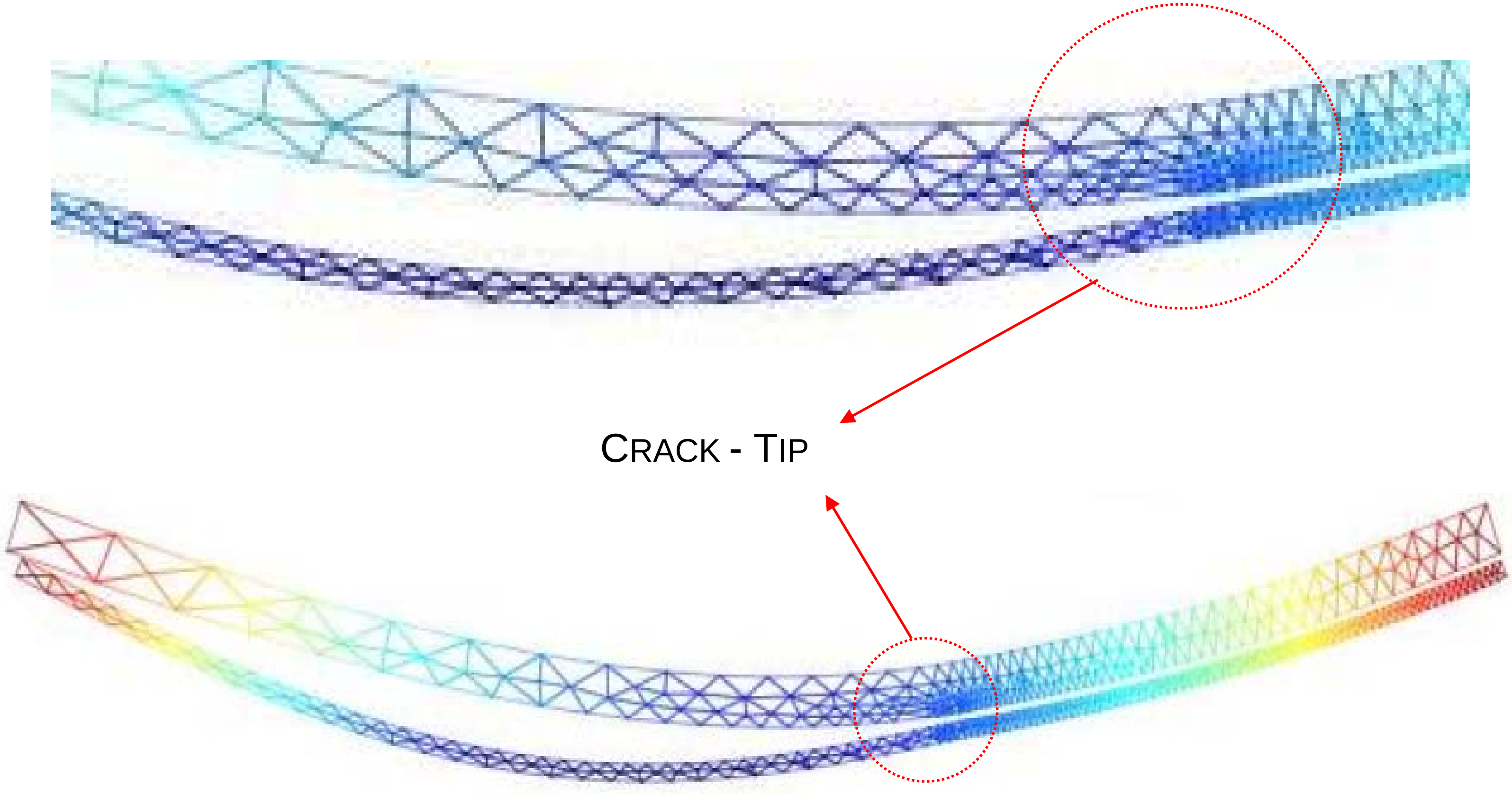
DEFORMED SHAPE OF THE BEAM UNDER *MODE I* LOADING CONDITIONS

Horizontal displacement of the crack-tip front

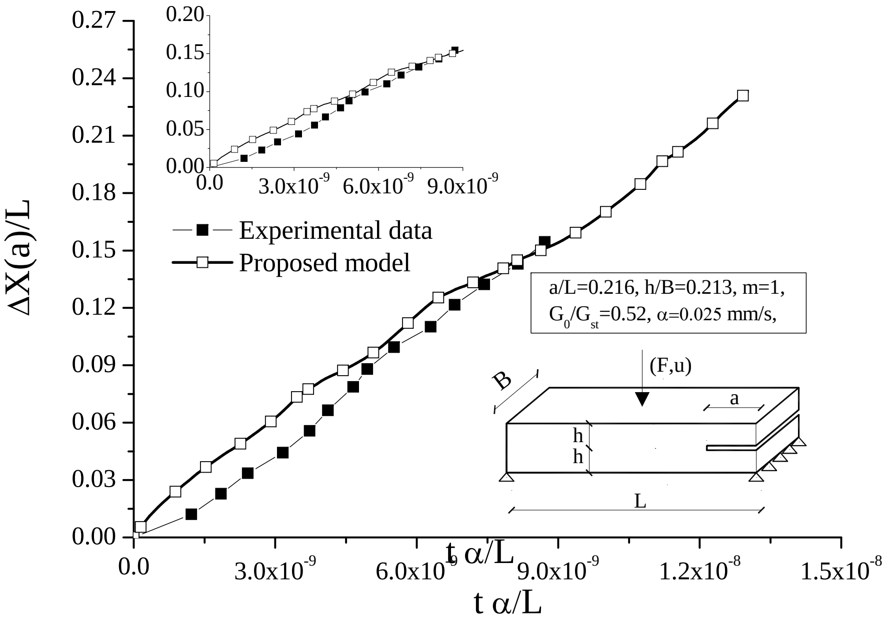


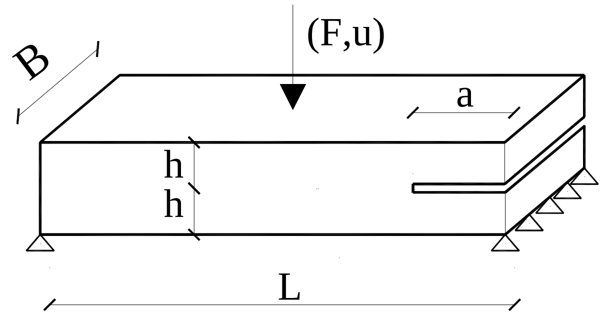
DEFORMED SHAPES OF THE BEAM UNDER *MIXED MODE* LOADING CONDITIONS

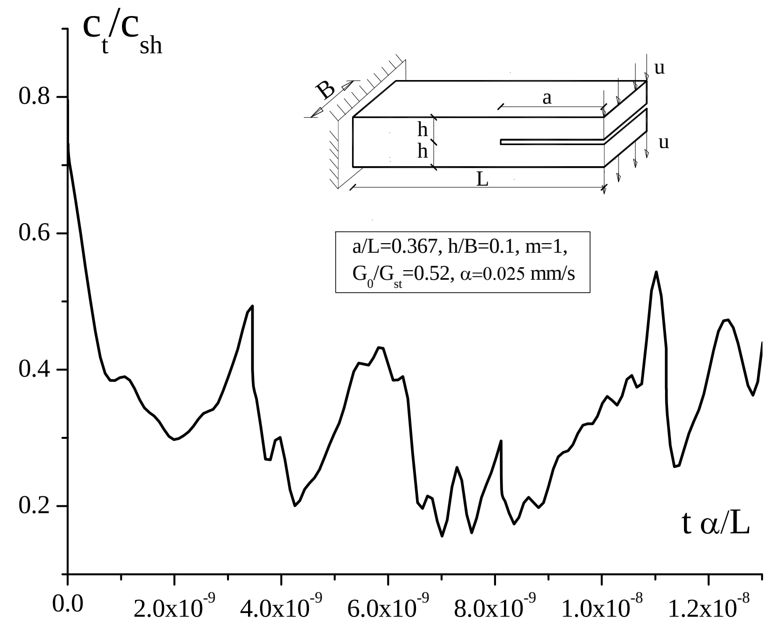
TRIANGULAR MESH ELEMENTS



RESULTS : MODE II ENF SCHEME



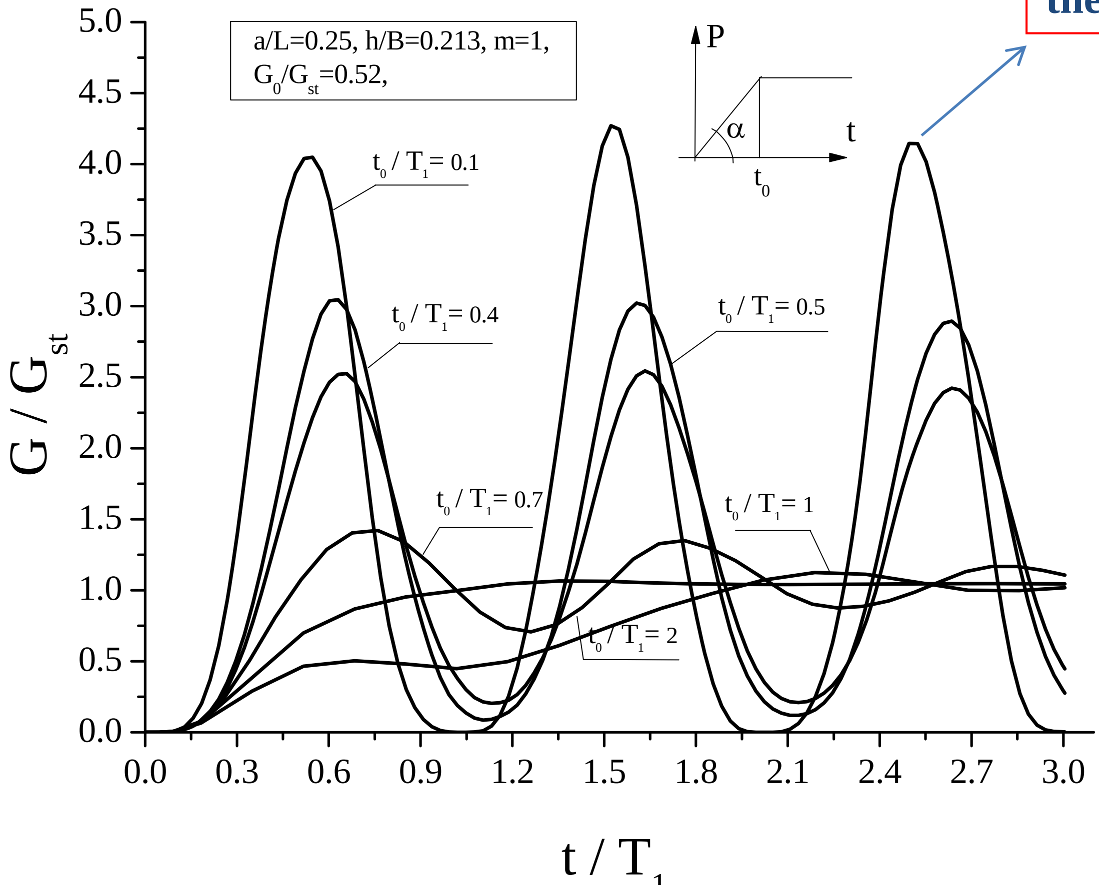


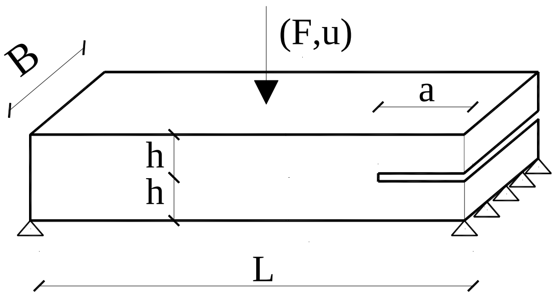


- **ENF mode II loading scheme**
- **Comparisons with experimental data**
- **S2/8553 Glass/Epoxy**

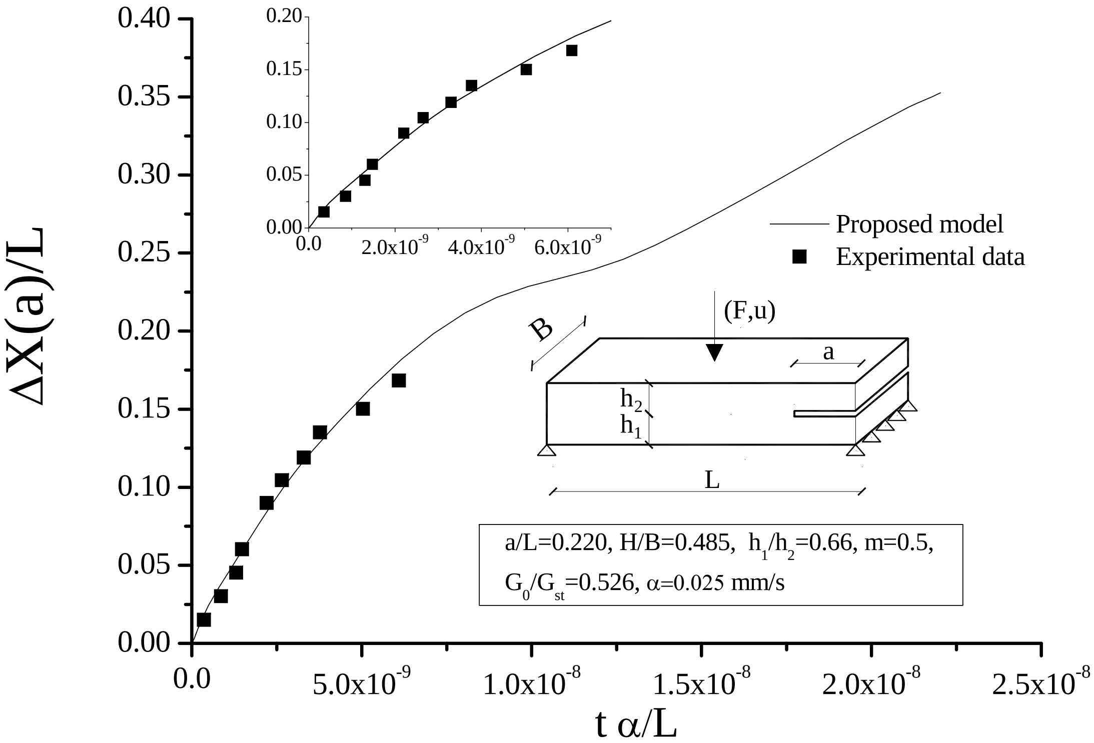
RESULTS : MODE II ENF SCHEME

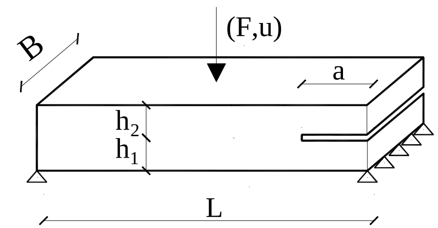
High amplifications in the ERR prediction



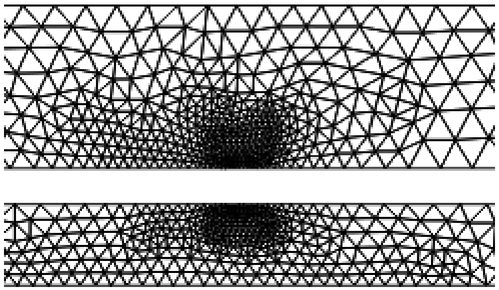


RESULTS : MIXED MODE ANALYSIS



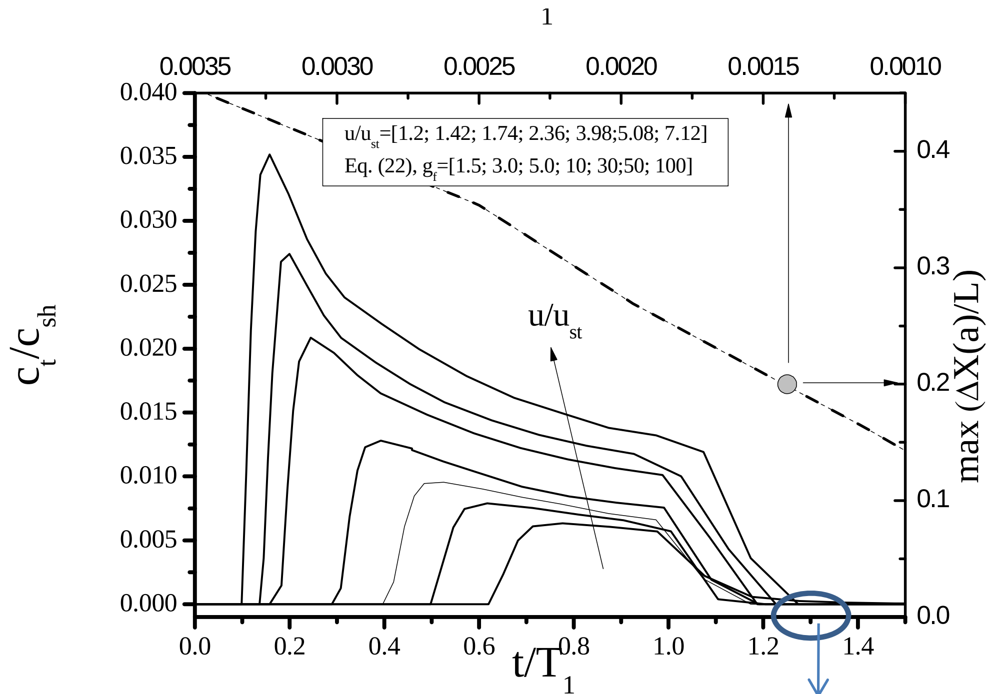


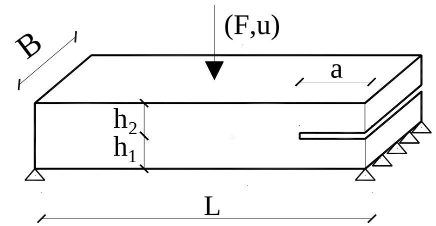
Mesh tip discretization

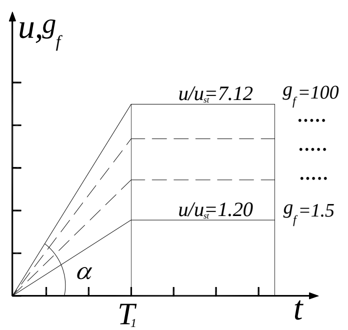


- **AS 3501-6 Graphite/Epoxy**

RESULTS : MIXED MODE ANALYSIS

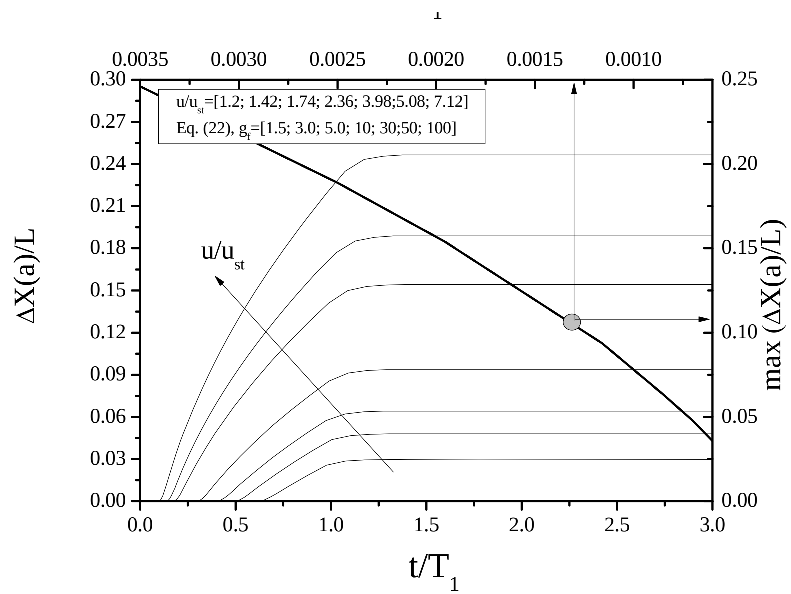




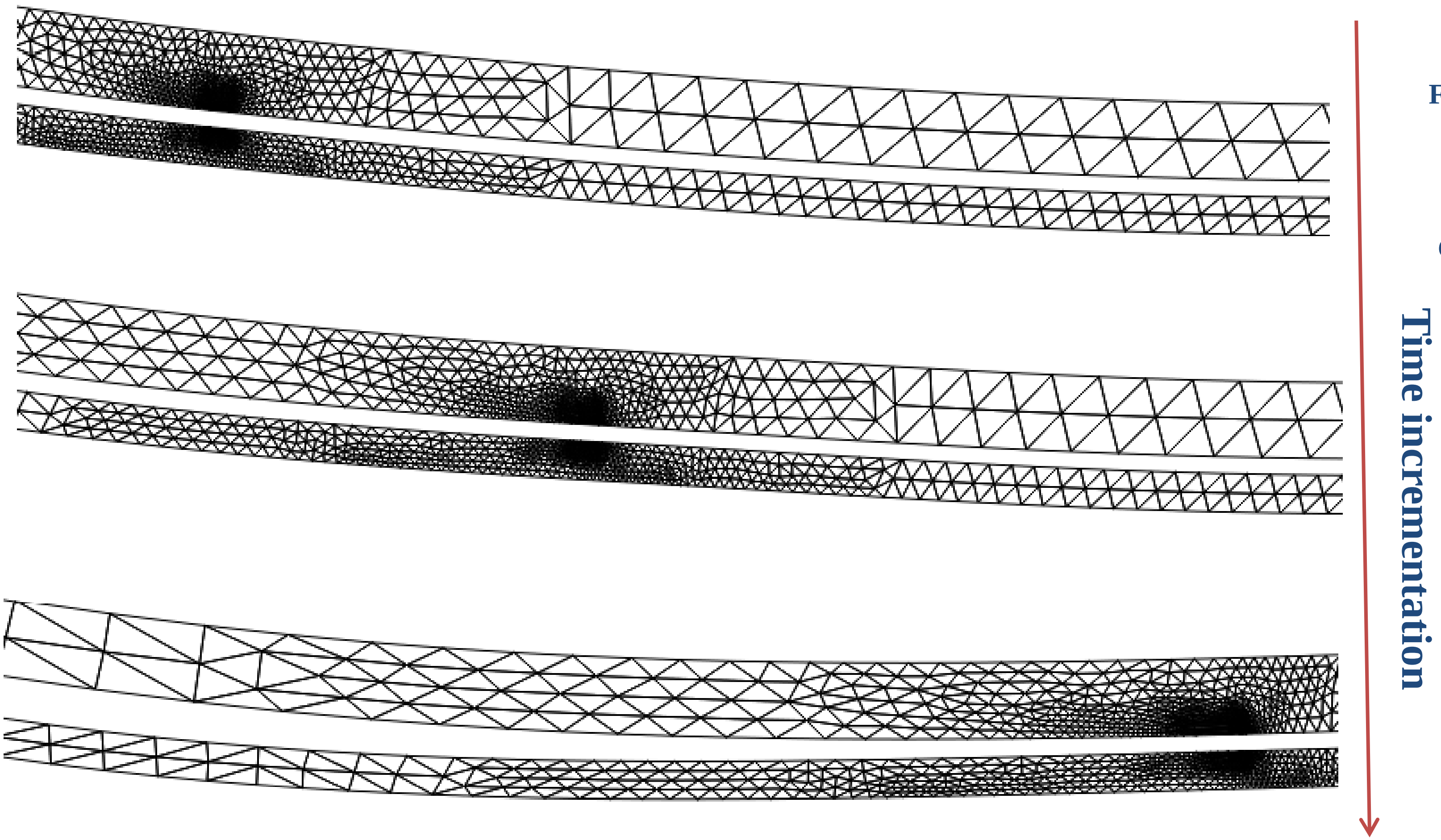


Crack arrest phenomenon

Loading curves



RESULTS : MIXED MODE ANALYSIS



CONCLUDING REMARKS

- **A delamination model for general loading conditions based on moving mesh methodology and fracture mechanics is proposed.**
- **New expressions of the ERR mode components based on the J-integral decomposition procedure.**
- **Comparisons with experimental data are proposed to validate the delamination modelling**
- **The analyzed parametric study shows that delamination phenomena are quite influenced by the loading rate, inertial effects leading to high amplifications in the ERR prediction and the crack growth.**