# Understanding *"Mutual Inductance"* using COMSOL Multiphysics

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**Abstract:** A teaching platform that could be used to help students understand concepts such as; flux linking and mutual inductance has been developed using the AC/DC module of COMSOL Multiphysics. This is achieved through the accurate determination of different magnetic flux density components within the proposed geometry. Furthermore, based on the structure configuration, students can use obtained magnetic flux density components to define the appropriate normal component relative to any selected closed surfaces. Consequently, by simple and direct boundary integration, students will immediately get access to accurate values of flux linking and mutual inductance, even for complex geometries, where mathematical calculation becomes prohibitive. The simplicity of the procedure permits students get experience and appreciation with different induction-related concepts, which are considered distant from students' imagination, especially at early levels.

A reconfigurable 2D geometry which consists of three different parallel concentric loops has been used for demonstration; where all required parameters are evaluated and related in a simple straight forward step-by-step flow to assist students assimilate induction-related issues.

**Keywords:** Mutual inductance, flux linking, Faraday's law, induction.

## 1. Introduction

One of the fundamental basic concepts in early undergraduate electromagnetic courses is the flux linking and associated parameters such as; mutual inductance and induced voltage. From the mathematical point of view, the calculation of these parameters represents a difficult task, especially if accurate values are required.

The problem becomes more complicated, if a complex geometry is to be investigated, where there are no approximate formulas to be compared with, as a final verification step.

From the teaching point of view, concepts such as flux linking, mutual inductance, Faraday's law and induced voltage, always represent real challenging tasks. The fact that all these issues are based on the accurate estimation of the behavior of the normal component of the magnetic flux density over a specified closed area necessitates the use of some visualization tool that can precisely evaluate and present the distribution of different magnetic flux density components over any specified part of the geometry.

This would be very helpful for junior students in first course of electromagnetism, where they are still having some difficulty with the imagination of the magnetic flux density distribution in a specified region. In addition, it would help students use their engineering judgment, based on the geometry configuration and flux visualization, to determine the appropriate normal component of the magnetic flux density over any selected surface area. Once this is achieved, a second step is also required to evaluate the integration of the appropriate normal component of the magnetic flux density over the surface of the closed area to get the amount of flux linking.

Moreover, to help students understand required concepts and get experience, the integration step permits students to define any closed surface and perform the required integration to evaluate, and even compare, the corresponding amount of flux linking and its dependence on different associated parameters. Finally, having evaluated the amount of flux linking, Faraday's law of induction could be directly verified, where the definition of the mutual inductance becomes easily understood.

The paper aims at the development of a teaching platform, based on the simulation environment of COMSOL Multiphysics, which could be used by students and instructors to get familiarity with induction-related parameters. The used geometry consists of three parallel concentric loops as shown in Fig. 1.

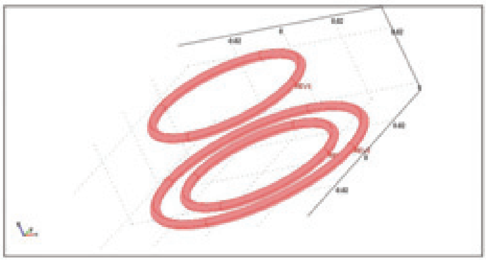

**Figure 1:** Configuration of three concentric loops (3D view)

Due to the structure rotational symmetry, the 2D axial symmetry mode is used to reduce the required amount of data storage and corresponding computational time. The 2D view of the geometry is shown in Fig. 2.

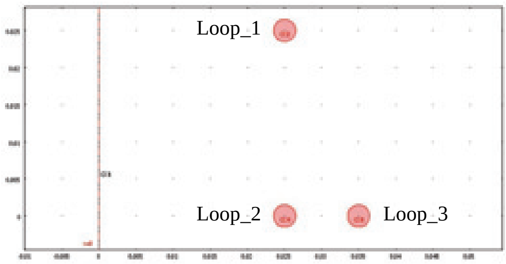


**Figure 2:** 2D Configuration of three concentric loops

Specifically, for the calculation of induced voltage and current in any of the available loops due to an excitation current in another loop, only one loop is excited, and the magnetic flux density is then calculated over the closed area formed by the second loop. The third loop is set as an air, which has no contribution to the problem formed by the two selected loops. By doing so, the used geometry could be used to study three possible cases;

Case (1); loop_1 is excited, flux is calculated over the closed surface formed by loop_2, giving rise to the induced voltage and current in loop_2.

Case (2); loop_1 is excited, flux is calculated over the closed surface formed by loop_3, giving rise to the induced voltage and current in loop_3.

Case (3); loop_3 is excited, flux is calculated over the closed surface formed by loop_2 giving rise to the induced voltage and current in loop_2.

Although it seems that there are still other possible cases, it is easy to verify that the mentioned three possibilities are the only different cases.

The geometry dimensions are; wire radius=1.5 mm, loop_1 radius=2.5 cm, loop_2 radius=2.5 cm, and loop_3 radius= 3.5 cm (loop radius is measured from the wire center to the symmetry axis). In addition, loop_2 and loop_3 are in the same plane; z = 0, while loop_1 is placed at z = 2.5 cm. Finally, loop material may be set as copper or as air, if the loop is to be considered as inactive.

## 2. Problem Formulation

Figure 3 shows two separated closed circuits *C1* and *C2*, of surface areas $S_1$ and $S_2$, respectively.

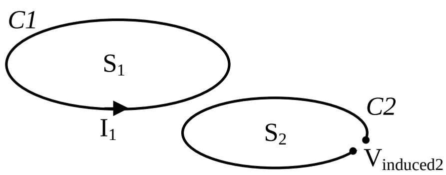


**Figure 3:** Two closed circuits C1 and C2

Consider *C1* excited with a time-harmonic current $I_1$ that produces magnetic field, then using Faraday's law of induction, the induced voltage in *C2* is given by;

$$V_{induced\,2} = -\frac{d\phi}{dt}[V]$$

With $\phi$ as the magnetic flux linking $S_2$ expressed by;

$$\phi = \oint_{S_2} B.ds\,[Wb]$$

Where; the integration is performed over the closed surface $S_2$ and B is the magnetic flux density.

It is clear that the value of flux linking $S_2$ depends on the following parameters; magnitude of the magnetic flux density B, the area $S_2$ and the angle that B makes with respect to the unit vector normal to the surface $S_2$.

Assuming time-dependence in the form $e^{j\omega t}$, the induced voltage in *C2* may be rewritten as;

$$V_{induced\,2} = -j\omega\phi = -j\omega\oint_{S_2} B.ds\,[V]$$

For a specified closed surface $S_2$, the above equation involves both the magnitude of normal component of the magnetic flux density relative to $S_2$ and the area of the surface.

## 3. Use of COMSOL Multiphysics

To familiarize students with the effect of different factors, in the previous equation, on the induced voltage, the proposed structure shown in Fig. 2 is simulated using the ***Azimuthal Induction Current Vector Potential*** application mode.

The following parameters have been set; loop_3 is set to be copper wire with an excitation by a loop potential of one volt and frequency of 50 Hz. The selected loop, where induced voltage and current are to be evaluated, is set as copper, while the third one is set as air. The purpose of this first simulation is to help students get experience and feeling with the dependence of the induced voltage and current, in loop_1 and loop_2, on their relative position with respect to the excitation loop_3. In other words, this reflects the dependence of the induced voltage on the magnitude of the magnetic flux density, as both loop_1 and loop_2 are of same area.

The magnitude of the azimuthal component of the electric field ($E_{phi}$) is shown in Fig. 4.

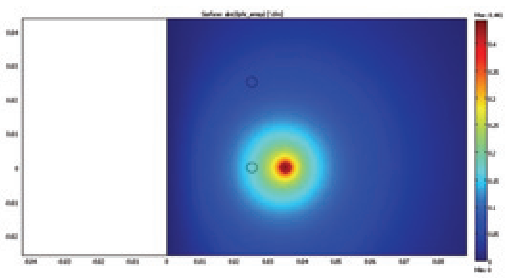

**Figure 4:** Azimuthal component ($E_{phi}$) of electric field

The induced voltages in loop_1 and loop_2 are evaluated as follows;

I. From the data display of subdomain parameters, the value of $E_{phi}$ at the wire center corresponding to loop_1 and loop_2 are obtained;

$E_{phi\,1}$ (loop_1)= 0.058∠-100.3 V/m
$E_{phi\,2}$ (loop_2)= 0.168∠-100.3 V/m

II. Using the definition of the induced voltage across a closed path in terms of the electric field, multiply each obtained value of $E_{phi}$ by the perimeter length of the corresponding loop, the induced voltage in each loop is calculated as;

$V_{induced\,1}$ (loop_1) = $9.1\times10^{-3}$∠-100.3 V
$V_{induced\,2}$ (loop_2) = $26.4\times10^{-3}$∠-100.3 V

As both loops are of the same area, these values demonstrate only the dependence of the induced voltage on the magnitude of the magnetic flux density B.

Once the induced voltage in each loop is evaluated, it is also possible to visualize and evaluate the induced current in each of these loops. The distribution of the magnitude of the azimuthal ($J_{iphi}$) component of the induced current density over the wire cross section for loop_1 and loop_2 is presented in Fig. 5.

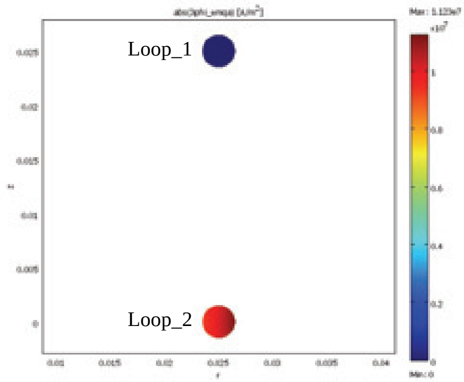


**Figure 5:** Current density in loop_1 and loop_2

The total current in each loop is then evaluated using subdomain integration for the induced azimuthal current density variable ($J_{iphi}$)

over the cross section area of loop_1 and loop_2, where obtained values are;

$J_{iphi}$ (loop_1) = 24.7 $\angle$-100 A
$J_{iphi}$ (loop_2) = 71.3 $\angle$-100 A

The second main issue in induction concept is the dependence of the induced voltage on loop enclosed surface area, which reflects the amount of flux linking. Using the proposed geometry, students can perform the following exercise which clarifies the concept of flux linking.

To compute and compare between the amount of flux linking loop_1 and loop_2, the normal component of the magnetic flux density should be integrated over the enclosed surface area by each loop. Based on the geometry configuration, the enclosed surface by each loop will be in the form of a circular disc in the x-y plane, and consequently the z-component of the magnetic flux density $B_z$ will be the required normal component. Now, to perform the surface integration and calculate the flux, a virtual surface should be created as a boundary over which the required magnetic flux component could be integrated. This is achieved by inserting lines connecting each wire center to the symmetry axis as shown in Fig. 6.

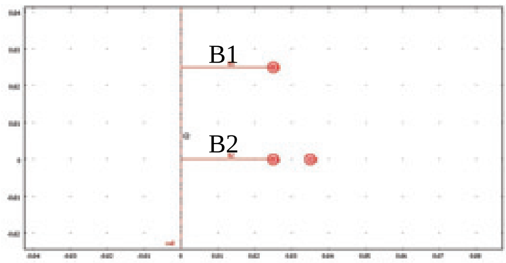


**Figure 6:** Inserted lines forming virtual surfaces

Furthermore, to visualize different structure elements if all wires and lines are resolved around the symmetry axis, Fig. 7 shows the obtained 3D version; where inserted lines now form surfaces that exactly match the corresponding loop enclosed surface area.

Based on the previous simulation, students can perform boundary integration on the variable $B_z$ component of the magnetic flux density over each of the inserted lines B1 and B2, representing the surface area enclosed by loop_1 and loop_2, respectively.

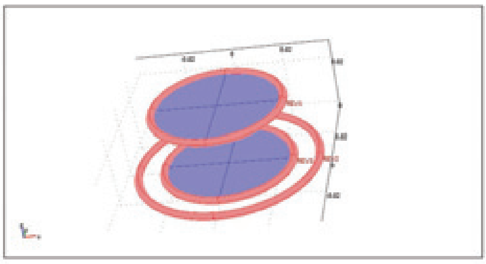

**Figure 7:** 3D configuration showing virtual closed surfaces for integration purpose

The following values for the total amount of flux linking loop_1 and loop_2 are obtained;

$\phi_1$ (loop_1) = 2.9x10$^{-5}\angle$-10.3 Wb
$\phi_2$ (loop_2) = 8.4x10$^{-5}\angle$-10.3 Wb

Using these values together with the corresponding values for the induced voltages, students can easily verify Faraday's law of induction.

At this point, it should be noted that in the used COMSOL model, loops form closed circuits where the induced voltage in each loop results in an induced current as previously presented. Consequently, the induced current in each loop will also generate a flux that will be added to the flux originally linking this loop due to the excited one. In this case, Faraday's law is verified based on its fundamental definition which relates the induced emf around a closed path to the rate of change of the magnetic flux with respect to time passing through the area enclosed by the path.

Now, as students are already familiarized with the concept of induction, the definition of the mutual inductance becomes meaningful.

Back to Faraday's law of induction, considering now the case where the induced open circuit voltage is required;

$$V_{induced_{ij}} = -j\omega\phi_{ij}\,[V]$$

Where; $V_{inducedij}$ and $\phi_{ij}$ are the induced voltage and flux linking circuit (i) due to current in circuit (j), respectively. In addition, due to the fact that circuit (i) does not involve any induced current (i.e., open circuit), $\phi_{ij}$ is a linear function of only $I_j$.

Consequently, it becomes possible to express $\phi_{ij}$ as;

$$\phi_{ij} = M_{ij} I_j \ [\mathrm{Wb}]$$

Where a new parameter $M_{ij}$ is defined and called the 'mutual inductance' is expressed as;

$$M_{ij} = \frac{\phi_{ij}}{I_j} [\mathrm{Wb/A = H}]$$

Using the definition of $M_{ij}$, Faraday's law may be rewritten as;

$$V_{induced_{ij}} = -j\omega M_{ij} I_j \ [\mathrm{V}]$$

With an alternative definition for $M_{ij}$ as;

$$M_{ij} = \frac{j}{\omega} \frac{V_{induced_{ij}}}{I_j} [\mathrm{H}]$$

Using the last definition of $M_{ij}$, the mutual inductance for the following case is evaluated; loop_3 is excited using a loop potential of one volt and frequency 50 Hz, where the induced open circuit voltage is evaluated over loop_2.

However, to implement Faraday's law correctly based on the above definition for the mutual inductance $M_{ij}$, it should be clear that in this scenario loop_2 must be kept as an open circuit. For this purpose, the current in loop_2 should be forced to zero (i.e., simulating an open circuit situation) so that the flux linking loop_2 has no contribution from the induced current in this loop. This is achieved by firstly evaluating the total induced current in loop_2, then inserting the negative value of the induced current density in this loop as an external current, resulting in a zero total current in loop_2.

Using the previous comments, the obtained values are;

$V_{induced23}$ (loop_2) = $26.55\text{x}10^{-3}\angle$-95.19 V

$I_3$ (loop_3) = $1920.85\angle$-5.19 A

$$M_{23} = 4.39\text{x}10^{-8} \mathrm{H}$$

For the purpose of verification, the inverse situation is also investigated;

$V_{induced32}$ (loop_3) = $37.146\text{x}10^{-3}\angle$-94.75 V

$I_2$ (loop_2) = $2691.89.\angle$-4.75 A

$$M_{32} = 4.39\text{x}10^{-8} \mathrm{H}$$

From the above calculations, it is clearly verified that $M_{ij}=M_{ji}$, which is a well known property for the mutual inductance.

It is also possible to verify and demonstrate that the mutual inductance depends only on the geometrical configuration and the permeability of the medium. Consequently, the mutual inductance could be calculated using the basic definition given by;

$$M_{ij} = \frac{\phi_{ij}}{I_j} [\mathrm{H}]$$

The $M_{ij}$ expression given above could be evaluated independently of frequency, i.e.; under DC condition. As a demonstration, the simulation is performed at zero frequency, where the flux linking and the current are evaluated. The value of the mutual inductance is given by;

$M_{32}$= $4.39\text{x}10^{-8}$ [H]

Which is the same as that obtained using Faraday's law.

As a final verification, the obtained result for the mutual inductance is compared with approximate formula. For the case of $M_{23}$; the following formula may be used;

$$M_{23} = \frac{\mu_o \pi R_2^2}{2R_3} = 3.52\text{x}10^{-8} \mathrm{H}$$

Clearly, there is a considerable difference between approximate formula and that obtained based on simulation. This is due to the fact that the approximate formula is based on the assumption that the magnetic flux density is constant over the closed area of the closed loop. However, in most of actual cases, this assumption is not valid.

The variation of the magnetic flux density along the radial distance of loop_2 is evaluated and presented in Fig. 8. The calculation has been performed at zero frequency, with only loop_3 excited. It is clearly observed that the flux density is not constant over the radial distance from the symmetry axis to the center of wire; rather it has a large dynamic variation.

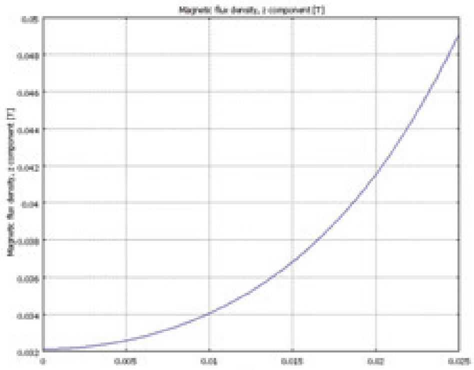

**Figure 8:** Variation of $B_z$ across the radial distance of loop_2

From the obtained results and previously mentioned comments, the benefits of using such developed platform for teaching purposes are obvious.

However, although the presented geometry involves only 2D concentric loops, where the normal magnetic flux density is clearly the z-component, another more general 3D geometry has been used. Figure 9 shows the case of a two parallel non concentric loops.

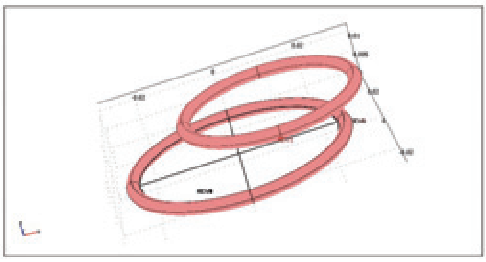

**Figure 9**: 3D configuration of two parallel non concentric loops

Different variations on the proposed 3D geometry are possible. Figure 10 shows the previous 3D structure, where the two loops are no longer parallel.

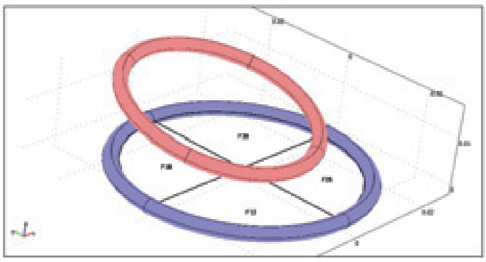

**Figure 10:** 3D configuration of two randomly oriented loops

It would be a challenging, but interesting, exercise for students to follow the previously described procedure and apply it on the configuration presented in Fig. 10. In such case, students are requested to use their engineering experience to determine the normal component of the magnetic flux density with respect to any selected loop. This is due to the fact that in such configuration, the normal component of the magnetic flux will not be simply the x- or the y- or the z-component, rather a combined formula is required. Similarly, the determination of the induced voltage in any loop requires line integration over the loop perimeter, which will not be a simple multiplication process as in the 2D case. In fact, the general 3D configuration shown in Figure 10 represents an advanced step to achieve a very useful teaching tool for induction-related issues.

## 4. Conclusions

An educational platform that could be used in teaching electromagnetic field concepts has been presented. The main advantage of using such technique for teaching is that it permits student to get access to all elementary electromagnetic field variables, which are always represented mathematically and thus remain distant from students understanding. In addition, students can use raw electromagnetic field variables to verify different correlated concepts based on both visualization and simulation results.

## 5. References


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