On The COMSOL Software Ability On Studying Transition Flows For Low Prandtl Number Fluids

H. Jamai[1] , A. Elamari[1], M. El Ganaoui[2], F. S. Oueslati[1], B.Pateyron [2] et H. Sammouda [3]
[1] LETTM, Departement of physics, F.S.T, Tunisia.
[2]SPCTS UMR 6638 CNRS, Faculte of Sciences, 87000 Limoges, France.
[3]ESST-H. Sousse, Tunisie
hanene_jamai@yahoo.fr

Abstract: COMSOL industrial software is offering an important alternative to home codes for modeling and simulation of complex problems with including coupled effects on Heat and Masse Transfers. The present work focuses on low Prandtl number fluid melts subject to symmetry breaking and transition to unsteady regimes. These configurations are for practical interest in crystal growth industry namely the Bridgman configuration is widely used to produce single crystals of compound semi- conductors and 2D enclosures provide a basic model for studying hydrodynamic regime occurring on such processes. A first step in this work is to study the effect of magnetic field on liquid metal flow and heat transfer occurring in an annular cavity.

Keywords: Natural convection, magneto-convection, molten metal (fluids on low Prandtl number), annular cavity

Nomenclature

$A = H/L$ Aspect ratio

$\vec{B}$ Diensionless magnetic vector (tesla)

$\vec{g}$ Gravitational vector acceleration (m/s^{-2})

H Cylinder height (m)

$\vec{j}$ Electric Vector density

$k = R_2/R_1$ Ratio of rays

P Dimonsionless pression

t time(s)

$L = R_2 - R_1$ Difference of rays (m)

T Dimensionless temperatur

r,z Dimensionless radial and axial coordinates

u,w Radial and axial velocity vector components

Grecs Letters

α Thermal diffusivity (m^2/s)

β Thermal expansion coefficient (1/k)

ρ Density (kg/m^3)

μ Dynamic viscosity, (kg/m.s)

σ Electric Conductivity (S/m)

ψ Electric potentielle Fonction

1. Introduction

Liquids metals, in the presence of a magnetic field (magneto-convection) have are the subject of a great number of researches. The interest of these flows lies in their presence in many natural and applied phenomena. In the same way, metallurgical industry, cooling engines in nuclear industry, crystalline growth for the industry of the semiconductors generate several questions for controlling the stability of these flows [1, 2, 3].

The appearance of the convection during crystal growth can produce inhomogenuities which lead to striations and which affect the quality of the obtained crystals [4]. In these cases, the application of magnetic field to the thermal convection appears of great importance for the control of the stability of these flows and the heat transfers while resulting.

Our study relates to the convective flow, of a fluid of low Prandtl number in an annular and vertical cavity

formed by two coaxial cylinders, differentially heated in the presence of a constant magnetic field [5, 6].

2. Mathematical Model

The system considered in this paper is shown schematically in fig.1

Boussinesq approximation is adopted, that the density of fluid is given by:

$$\rho = \rho_0 \left[1 - \beta_T \left(T - T_0\right)\right] \qquad (1)$$

The dimensionless equations which describe the problem, are respectively equations of, continuity, conservation of the momentum, the Ohm law and the energy conservation equation:

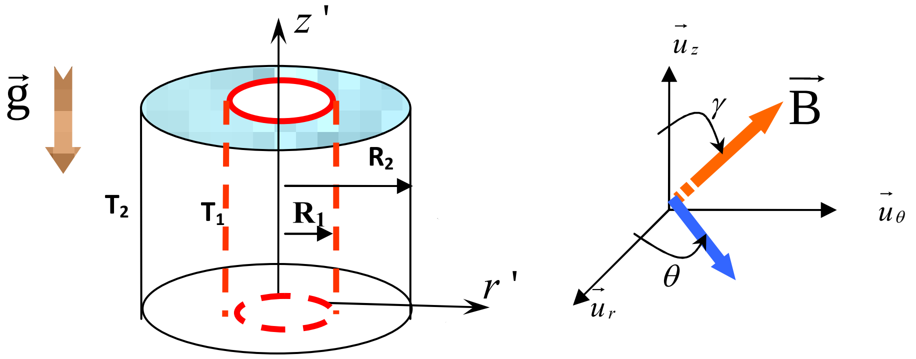


Figure 1. Geometrical configuration

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\left\{\frac{\partial u}{\partial t}+\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)\right\}=-\frac{\partial P}{\partial r}+\Pr\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)+\Pr.Ha^2\left(\vec{j}\wedge\vec{e}_B\right).\vec{u}_r \qquad (2)$$

$$\left\{\frac{\partial w}{\partial t}+\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)\right\}=-\frac{\partial P}{\partial z}+\Pr\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+\left(\Pr Ra\right)T+\Pr.Ha^2\left(\vec{j}\wedge\vec{e}_B\right).\vec{u}_z \qquad (3)$$

$$\vec{j}=(-\vec{\nabla}\phi+\vec{V}\wedge\vec{e}_B) \qquad (4)$$

$$\frac{\partial T}{\partial t}+\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \qquad (6)$$

With $\vec{e}_B$ is the unit vector in the direction of magnetic field $\vec{B}$

$Ra=g\,\beta\,\Delta T\,l^3/\nu\,\alpha$, $Ha=l.B.\sqrt{\sigma/\mu}$, and $\Pr=\upsilon/\alpha$ are the Rayleigh, Hartmann and Prandtl numbers, which are the dimensionless numbers governing the problem.

The boundary conditions

The boundary conditions adopted for the resolution of the problem are:

Hydrodynamic Condition: zero velocity on the walls. u=w=0

Thermal Condition:

$\{r=R_1: T=T_1,\ r=R_2: T=T_2,\ z=0,1: \partial T/\partial z=0\}$

Electrical conditions: electrical insulation on the walls $\vec{j}.\vec{n}=0$

The governing equations along with the boundary conditions are solved numerically using the COMSOL code which based on the finite element method, allowing a coupling between the dynamic, thermal and electric problems. To test and assess grid independence of the numerical scheme, various meshes are examined. Very fine no uniform grids close to the walls are adopted.

We assume that the solution is converged when the error is less than 10^{-6}.

3. The effect of an axial magnetic field

In this part, the magnetic field is supposed according to the z direction and we neglect the electric term in the force of Laplace. We present on Figure 2, the Streamlines, the isotherms and velocity profiles for Ra = 10^4 and the aspect ratio A = 1 for various values of the Hartmann number. In the absence of magnetic field (Ha = 0, Fig. 2(a)), the flow exhibits a simple circulating pattern rising along the hot wall and descending along the cold wall of the cavity. It is interesting to note that as the strength of the magnetic field increases (Ha = 40), the central streamlines are elongated horizontally and the temperature stratification in the core diminishes (Fig. 2(b)). As Hartmann number is further increased (Ha = 100), the isotherms are almost parallel and are nearly conduction like (Fig. 2(c)) and this is due to the suppression of convection by the magnetic field. Also it can be seen that the flow field becomes bicellular as strength of the magnetic field is increased.

It can also be seen from the figure3 (a and b) that the flow oscillation in shallow cavity is suppressed more effectively by the axial magnetic field than the radial magnetic field.

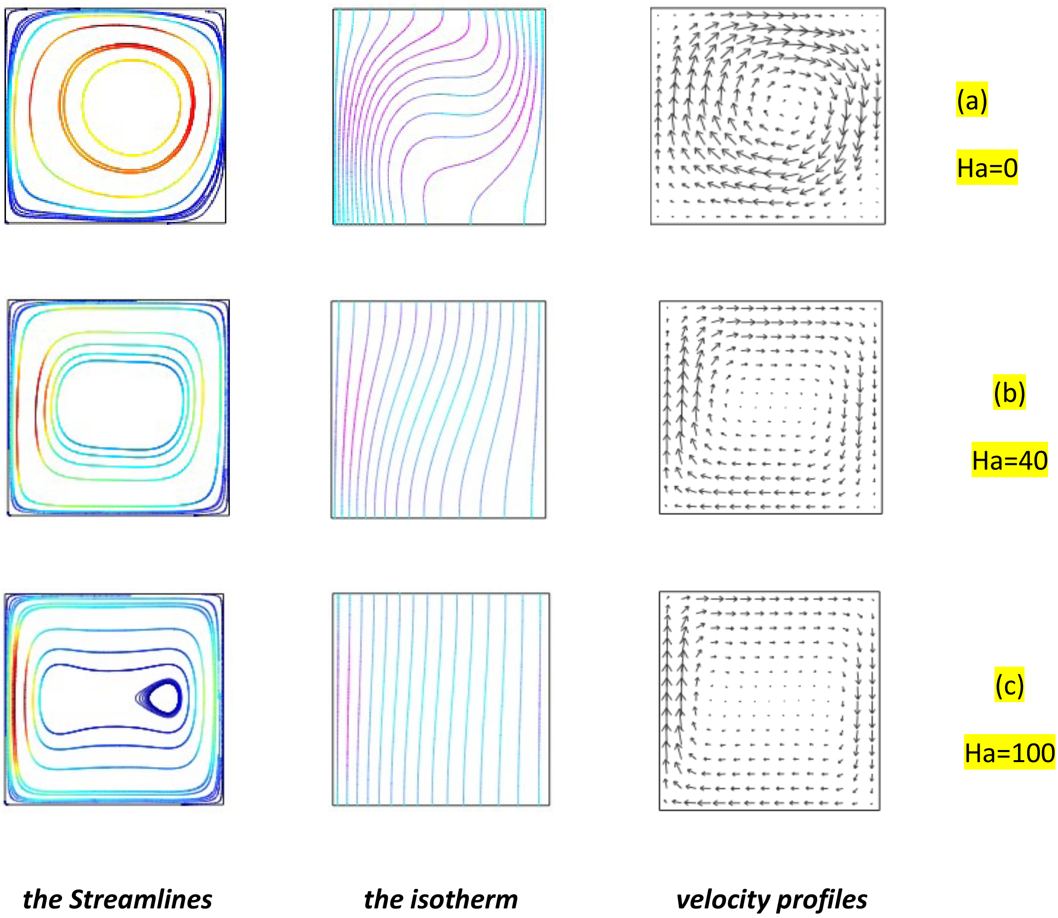
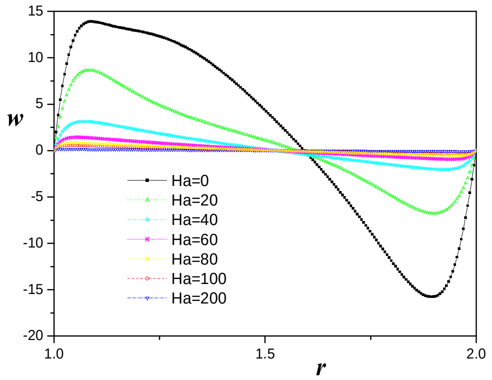


Figure 2 : dynamics, thermals Fields and velocity from various values of Hartmann,

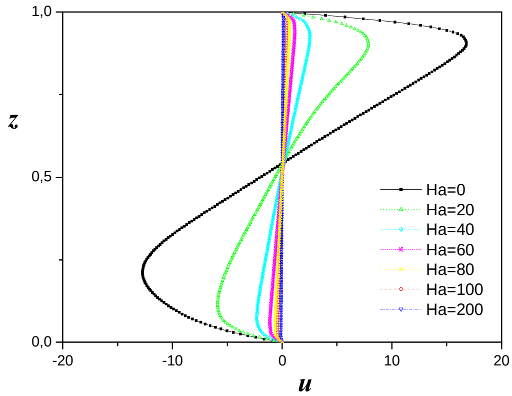
A = 1, k=2, Ra = 10^4 and $\theta = \pi/2$.

This behavior is consistent with the fact that the magnetic field suppresses the flow more effectively

when the magnetic field is imposed perpendicular to the direction of the flow.



-a-



- b-

Figure 3: Vertical (a) and horizontal (b) velocity at midheight for, A = 1, k=2, Ra = 10^4 and $\theta = \pi/2$.

4. Heat Transfer

Quantitative heat transfer results are presented in terms of average Nusselt number. The effect of magnetic field on the heat transfer rate is shown in Fig. 4. As expected, for a given value of Ra it can be seen that Nu is a decreasing function of Ha. This is due to the fact that with the increase in Hartmann number the convection is progressively reduced by the magnetic drag, resulting in a lower heat transfer.

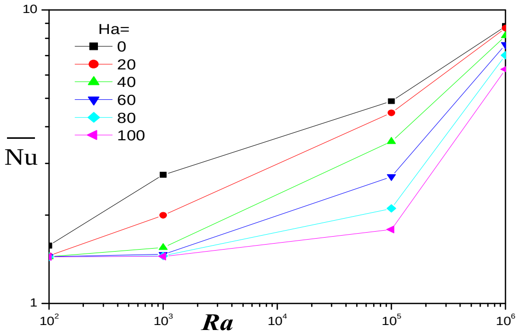


Figure 4: Local heat transfer rate for deferent values of Ha and Ra

5. Conclusion

The numerical results indicate that the magnetic field suppresses the convective flow and eliminates the flow oscillations.

Quantitative results are presented in terms of the local and average Nusselt number. The heat transfer rate increases with radii ratio and decreases with the Hartmann numbers. The direction of magnetic field plays an important role in suppressing the convective flows. The magnetic field is more effective when it is perpendicular to the direction of the primary flow. This phenomenon has a serious implication on the design of magnetic systems for stabilizing or weakening the convective effects. The numerical results of the present study are in good agreement with the existing numerical studies, in the absence of magnetic field.

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