Thin Membrane Modelling for the Electrical Stimulation of Auditory Nerve

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Abstract: Modelling of $2-5$ μm thin membranes into a cochlea with a width of 2 cm is computationally. The paper is focused on two approximative methods used to overcome this problem and in addition a simple model challenging of a plate capacitor with a thin membrane of different thickness in-between is presented. The results of simulations with both thin layer approximation methods are compared with those for a membrane with real thickness.

Keywords: Cochlea implants, multiscale nature, electrostatic, inner boundary, Thin Low Permittivity Gap

1 Introduction

The main role of a cochlea implant is the electrical stimulation of the auditory nerve for aurally handicapped people with functioning auditory nerve. For certain atraumatic care, new implantation techniques are investigated considering alternative electrode types and locations [2]. A precise electrical model of the cochlea considering its thin insulating membranes shall be realized. The specific numerical challenge lies in the discretisation costs caused by the multiscale nature of the problem: There are ca. $2-5$ μm thin membranes to be modelled inside the cochlea of about 2 cm of diameter. In this paper, we present a comparison of two approximative methods for implementing these membranes into a cochlea model. The first method (*model A*) is to enlarge the thickness of membranes and consequently their permittivity [1]. Taking into account the model-inherent inaccuracy, the discretisation costs are still very high. In contrast, the second method (*model B*) avoids any discretisation of the membranes by modelling them as inner boundaries. To compare the above-described methods, a simple electrostatic model composed of a plate capacitor with membranes in-between implemented in the electrostatic mode of COMSOL Multiphysics® 3.5a is studied.

2 Modelling

2.1 Governing equations

All four Maxwell‘s equations are the basic equations for electrodynamics. Computing electromagnetic fields does not always require the solution of the complete system of differential equations. Yet, appropriate simplifications result in classifications of field problems. As cochlea implants have stimulation rates from 20 kHz to 80 kHz the electro-quasistationary approximation is maybe suitable. In the electro-quasistatic field problem the time derivative of the magnetic flux density ($\frac{\delta B}{\delta t} \approx 0$) neglected, but the time derivative of the displacement current ($\frac{\delta D}{\delta t} \neq 0$) is considered.

As for our initial studies we consider only low frequencies, the time dependency will be neglected and the problem turns into electrostatic one.

In electrostatic case, electric and magnetic fields are decoupled and magnetic fields are neglected. Then, only the system:

$$\begin{aligned} rot\mathbf{E} &= 0 \\ div\mathbf{D} &= \rho \end{aligned} \tag{1}$$

has to be solved. Here $\mathbf{E}$ is the electric field, $\mathbf{D}$ the electric displacement field and ρ the total charge density. With equations (1) and $\mathbf{E} = -grad\,\Phi$ Poisson‘s equation $\nabla^2\Phi = -\frac{\rho}{\epsilon}$, where ϵ is the permittivity and Φ is the potential, can be derived and has to be solved. The equation is solved by COMSOL Multiphysics® 3.5a, which is based on the Finite-Element-Method.

2.2 Model A

Numerical computation of very thin components in larger models causes high costs in discretisation and CPU time. In order to avoid this problem *model A* is used. Here, the very thin layers are assumed as a plate capacitor and a conductor of uniform cross section described by the following equations: $C = \frac{\epsilon A}{l}$, with the permittivity ϵ, the surface area A and the distance l of the capacitor plates, and $R = \frac{l}{\sigma A}$, with the length l of the conductor and the conductivity σ. As already mentioned above, the problem is studied in electrostatics with dielectric materials only and several transformations are conducted: Mainly the thin layers are assumed as plate capacitors, which allow enlargement of the thickness and simultaneously scaling up the permittivity. Thereby the assumed capacity does not change.

2.3 Model B

To avoid high discretisation costs at all, in COMSOL Multiphysics® 3.5a offers the possibility to define inner boundaries.The existing thin layers lie between the two components of the model and these two components are defined as a pair (Identity Pair) allowing to choose the boundary condition *Thin Low Permittivity Gap*. The following equations are adapted for this boundary condition:

$$(\mathbf{n} \bullet \mathbf{D})_1 = \epsilon_0 \epsilon_g (V1 - V2)/d \tag{2}$$

$$(\mathbf{n} \bullet \mathbf{D})_2 = \epsilon_0 \epsilon_g (V2 - V1)/d \tag{3}$$

Where D1 and D2 are the electric displacement fields and V1 and V2 are the electric potentials on the boundaries of component 1 and 2. V1 and V2 correspond to the charges of the plate capacitor and thus homogeneous fields are assumed in the substituted membrane. The gap has the permittivity ϵ_g and thickness d.

3 Numerical Model

As a numerical model, a simple plate capacitor is chosen with a thin membrane in-between. The shape of the modeled membrane is defined as a triangle in order to test the methods in domains with sharp edges as well. The actual thickness is $2\mu m$ and the actual permittivities based on the values in a real cochlea are $\epsilon_M = 2$ for the membrane and $\epsilon_S = 109$ for the surrounding material. Furthermore for the 2D case, the plate capacitor has the following size: $1.2\ mm \times 0.8\ mm$. The electrodes are positioned on the top and on the bottom of the capacitor and for them the potentials of +5 and $-5\ V$ are chosen (boundary condition). The computation is done in the application mode electrostatics in the AC/DC-module with the solver UMPFPACK.

For *model A*, the membrane is modeled as described above as an extra subdomain with a defined thickness of 0.05 *mm* (1) and an equivalent scaled ϵ of 50 and 20 and a scaling factor of $s_1 = 25$ and $s_2 = 10$. 2464 triangular elements are used and 0.1 *sec* of CPU-time are needed for s_1 and 5656 elements are used and 0.2 *sec* of CPU-time are needed for s_2. In both cases the subdomain with the membrane is discretised with 2 layers of elements.

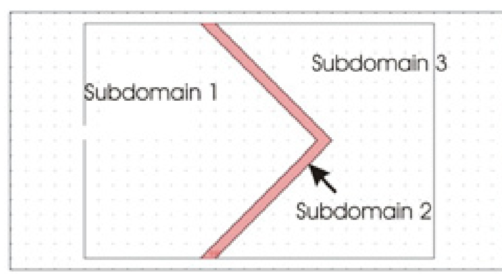

Figure 1: Arrangement of *model A* with the membrane as extra subdomain.

In *model B* the two subdomains around the thin layer are combined and the boundary is modeled as "Thin Low Permittivity Gap" (2). Here, 1728 grid elements are used and 0.1 *sec* CPU-time is needed.

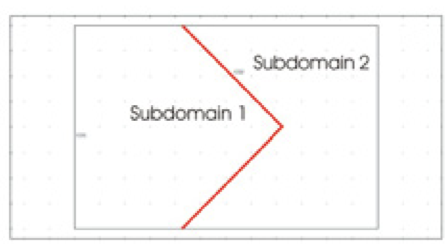

Figure 2: Arrangement of *model B* with the membrane as a boundary.

For comparison, a realistic membrane ($2 \mu m$ thick) is modeled (*model C*). This example is discretised with 133504 elements. A CPU-time of 5.7 *sec* is needed.

4 Results and Discussion

First results for both methods are shown on fig. 6. Less visible field distribution inside the membrane is caused by color ranging for comparing the far field distribution of the membrane between *model A* and *model B*. There are less differences, but to see detailed differences near the membrane, the computed field distribution is measured along the above side of the membrane with a distance of 0.01 *mm* from the membrane .

Next, these results are compared with the results of *model C*. In fig. 3 the relative deviation between model A ($s = 25$) and *model C* and between *model B* and *model C* are shown. The figure shows that *model B* seems to be the better approach, because the deviation is lower than the deviation of *model A*. At the end of the measuring line, at the sharp corner, high differences are visible which might be caused by discretisation differences near the sharp corner. For comparing the far field of the membrane for *model A* and *model B* a second measuring line is defined from the corner in the middle of the membrane to the right outer boundary of the whole model. Fig. 4 shows the relative deviation between *model A* and *model C* and between *model B* and *model C*. Obviously, the difference between both models is below 0.3 % for $x \geq 0.95$ *mm*.

The used scaling factor in *model A* is rather high. To compare both methods also for a lower scaling factor *model A* is also calculated with a scaling factor of $s = 10$ (fig. 5). It is now visible that the results from *model A* are much closer to the results of model C.

We can conclude that *model B* can be used instead of *model A* for complex structures. Otherwise, if the structure allows for a smaller scaling factor, it is better to use *model A*.

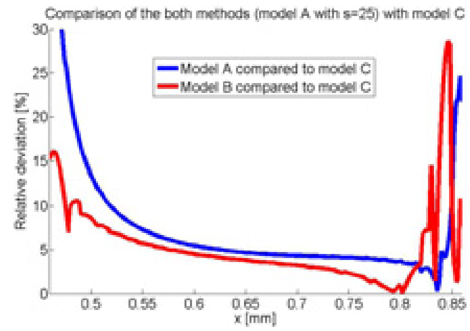

Figure 3: Comparison of both methods, using a scaling factor of 25 in model A: Relative deviation of electric field shows high differences especially at sharp corner.

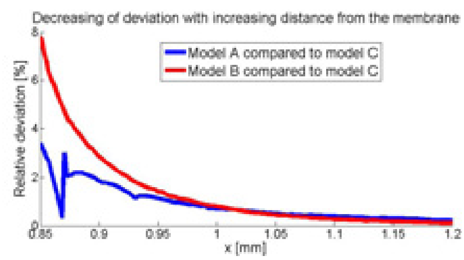

Figure 4: Decreasing of deviation with increasing distance from the membrane on the sharp corner.

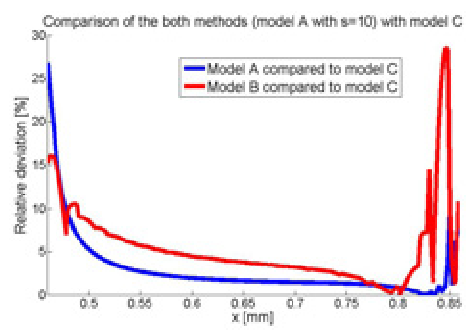

Figure 5: Comparison of both methods, using a scaling factor of 10 in model A: Model A seems to be the better approach.

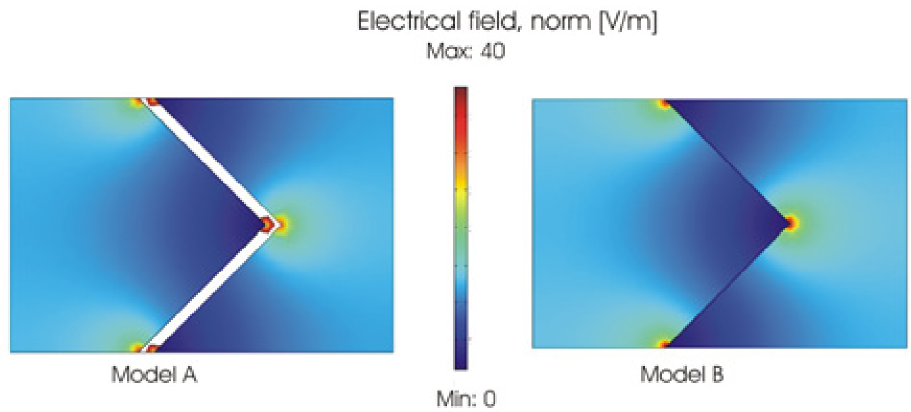

Figure 6: Electric field distribution of both models.

5 Conclusion

A precise electrical model of the cochlea is necessary in order to predict the electric field distribution in a cochlea stimulated by cochlea implants. The very thin membranes inside the cochlea afford for a fine discretisation. To avoid high discretisation costs a method (*model A*) of enlarging the thickness of the membranes and synchronously enlarging their material properties is used. This method is compared with a second method (*model B*) which comprises modelling the membranes as inner boundaries. With COMSOL Multiphysics® 3.5a it is possible to define the membranes as *Thin Low Permittivity Gap*.

Results obtained from the electrostationary approximation show that *model B* can be used to reduce CPU-time and discretisation costs. Higher accuracy depends on appropriate choice of the scaling factor of *model A* which depends on the complexity of the model. But as described above, the electrostatic case is a less accurate assumption. Therefore, it will be necessary to perform deeper investigations with electro-quasistationary approximation to achieve more profound knowledge of the field distribution near the thin membranes.

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