A simulation test bench for decay times in room acoustics

R. Magalotti[1], V. Cardinali
B&C Speakers S.p.A., Bagno a Ripoli (FI), Italy

Introduction

Reverberation time is the most common measurement used to assess how fast the energy of a sound field in a room decays, once the source has been turned off. It was introduced by W.C. Sabine at the end of the XIX century, and has been constantly used in room acoustics since then, in its original form or one of the variations proposed at various times.

Despite its widespread use, the model of reverberation is not universally valid: it relies on some necessary assumptions on the uniformity and diffuseness of the sound field. The consensus is that reverberation is a good model above the Schroeder frequency [1] only.

Below the Schroeder frequency, the behavior of the sound field is dominated by isolated modal resonances. Therefore, the decay of acoustic energy in that range is determined by the decay times of the modes.

The aim of this work is to investigate the relationship between the acoustical impedance of the room boundaries and the decay times of room modes, with the help of COMSOL® simulations.

The authors have set up in COMSOL® the simplest model in room acoustics: the rectangular room. Analytical solutions for the rectangular room are well known, and all modes can be classified in three families: axial, tangential and oblique.

Once finite impedances are assigned to the walls, modal decay times can be computed from the results of an eigenfrequency study.

Normal modes in a rectangular room

The simulated room is a rectangular parallepiped, with dimensions of 5.02 × 4.15 × 3.36 m and a resulting volume of 70 m^3. This choice of dimensions guarantees the absence of degenerate or quasi-degenerate modes below the (somewhat arbitrary) limit of 100 Hz.

An Eigenfrequency Study with the default Sound Hard Boundary condition on all walls yields the 14 modes listed in Table 1. The right column shows the resonance frequency, while the left column shows the "mode index": a set of three integers detailing, for each mode, the number of nodal planes perpendicular to each of the coordinate axes *x*, *y* and *z*.

Mode	Frequency (Hz)
[1,0,0]	34.2
[0,1,0]	41.4
[0,0,1]	51.1
[1,1,0]	53.7
[1,0,1]	61.5
[0,1,1]	65.7
[2,0,0]	68.4
[1,1,1]	74.1
[2,1,0]	79.9
[0,2,0]	82.7
[2,0,1]	85.3
[1,2,0]	89.5
[2,1,1]	94.8
[0,2,1]	97.2

Table 1: Mode index and resonance frequency for the first 14 modes of the room, classified as axial (green), tangential (red) and oblique (blue)

Mode index leads to a standard classification of modes in a rectangular room: those with one non-zero value only are called *axial modes*, with two non-zero values are *tangential modes* and with all three non-zero values are *oblique modes*. Figure 1 shows the shapes of some modal resonances and their classification.

Decay time and wall impedance

The basic Eigenfrequency Study in Pressure Acoustics relies on the default condition of Sound Hard Boundary for all walls, and yields real values for the resonance frequencies *f*. Since we want to investigate decay times, we need to assign finite impedances to the walls. When there is a loss of energy, the eigenfrequency values become complex, with the imaginary part of *f* representing the damping of the mode.

It is customary to define a decay time MT_{60}, analogous to reverberation time, as the time required for the energy of the mode to decay by 60 dB [3,4]. The link between the imaginary part of the eigenfrequency and the decay time is expressed by the formula

$$MT_{60} = \frac{60}{20\log_{10}(\mathrm{e}) \cdot 2\pi \cdot \mathrm{imag}(f)} \cong \frac{1.1}{\mathrm{imag}(f)}$$

[1] Corresponding author: rmagalotti@bcspeakers.com

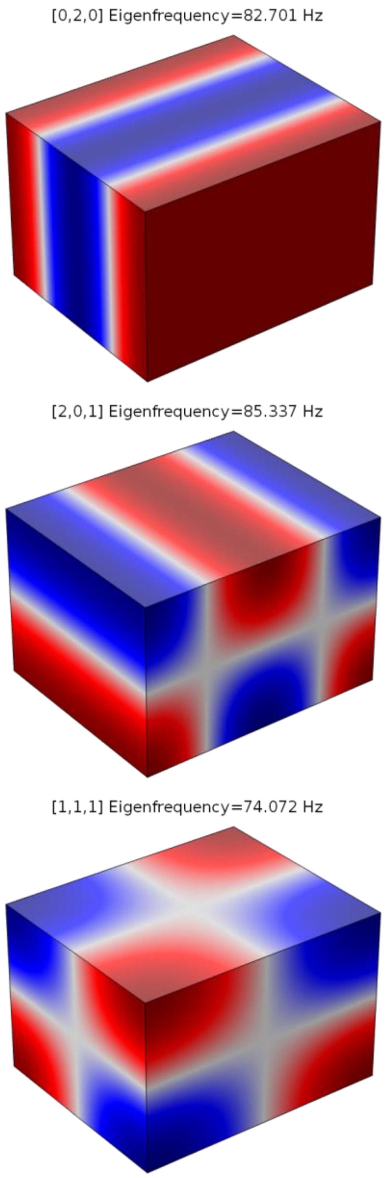

Figure 1. Examples of modal shapes for (top to bottom) axial, tangential and oblique modes.

We used simulations to verify the relationship between wall impedance and decay time. For example, following Morse and Ingard [2], it is possible to set all decay times of axial modes to the same value, when each pair of walls has an acoustical conductance proportional to the perpendicular dimension of the room.
Incorporating the previous formula, this translates to a purely real normalized acoustic impedance of

$$\zeta_i = \frac{Z_i}{\rho c} = \frac{4c\,\log_{10}(e)}{3} \cdot \frac{MT_{60}}{l_i} \cong 200 \cdot \frac{MT_{60}}{l_i}$$

where i is either x, y, or z and the coefficient 200 is in m/s.

This condition is easy to enforce in COMSOL®, as shown in Figure 2.

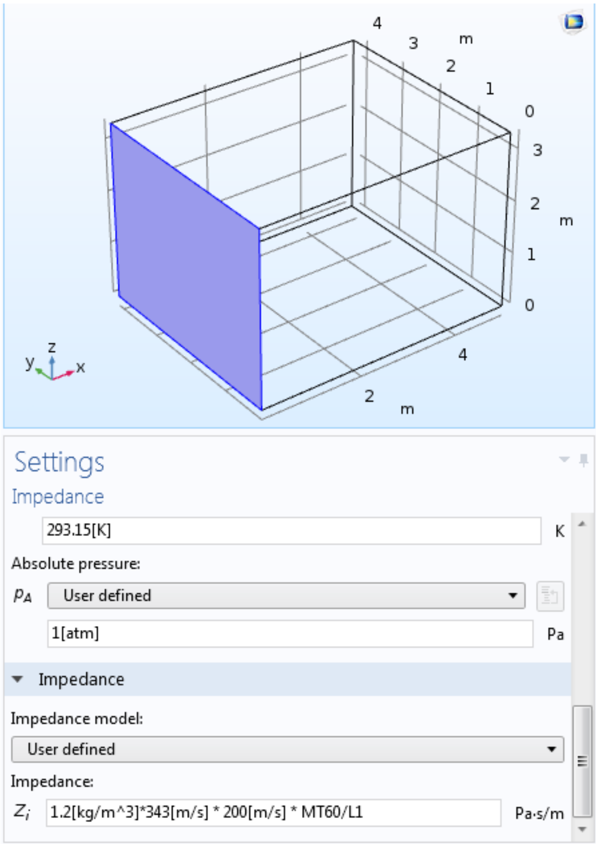

Figure 2. Setting the acoustic impedance of walls in COMSOL®

Simulation results for decay times

Figure 3 shows the simulation results for a study where all six walls in the room had an acoustic impedance computed with the previous formula, with a target MT_{60} for axial modes of three seconds. Axial modes actually have the required 3 s of decay time, while tangential modes have 2.4 s and oblique modes have 2 s.

An explanation of those ratios is found in Morse/Ingard [2]. Simplifying the third row of their formula (9.4.31), the modal wavenumber K_n can be written:

$$K_n \cong \eta_n - \frac{ik}{\eta_n}\left(\epsilon_{n_x}\frac{\beta_x}{l_x} + \epsilon_{n_y}\frac{\beta_y}{l_y} + \epsilon_{n_z}\frac{\beta_z}{l_z}\right)$$

Applying our formula for the impedance is equivalent to setting the values of acoustic conductance β proportional to lengths l. The formula becomes thus:

$$K_n \cong \eta_n - \frac{ik\,\beta}{\eta_n\,l}\left(\epsilon_{n_x} + \epsilon_{n_y} + \epsilon_{n_z}\right)$$

and shows that the total damping depends on the sum of ε_n in parenthesis. The value of the ε_n is 1 when the corresponding mode index is zero and 2 when it's non-zero, so the sum is 4 for axial modes, 5 for tangential modes and 6 for oblique modes; decay times are inversely proportional to damping.

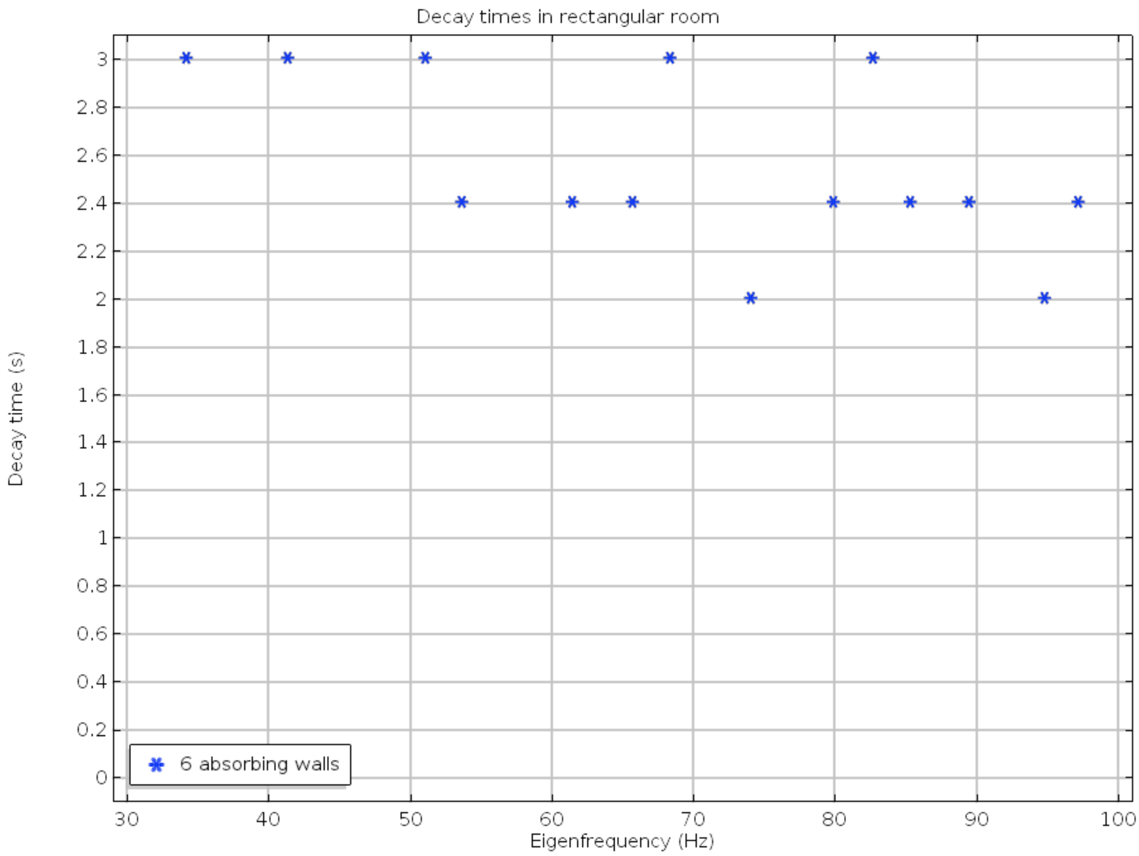

Figure 3. Simulated decay times of the first 14 modes in the rectangular room

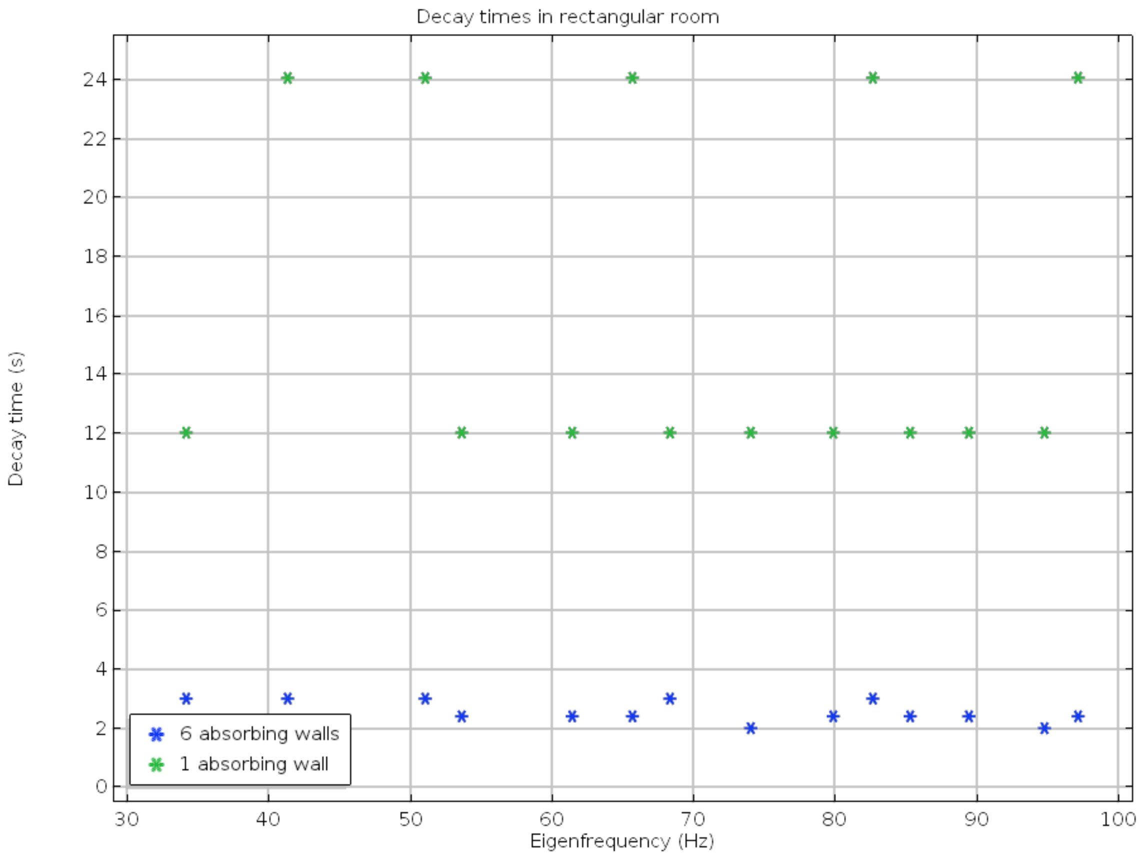

Figure 4. Comparison of simulated decay times with 6 vs. 1 absorbing walls

Another study was performed, applying the same impedance to one wall normal to *x* only (as in Figure 2) and reverting the other five walls to the default Sound Hard Boundary condition. Figure 4 shows the results.
In this case, there are two decay times only: 12 s and 24 s. The critical factor is the first integer (related to *x*) in the mode index. When that number is zero, the effective absorption of the *x* wall is halved and therefore the decay time is doubled.

Conclusions and further work

The paper has shown how decay times of modes in a room can be computed from an Eigenfrequency Study in COMSOL®. Such a simulation is a good test bench to verify the predictions and limits of models in room acoustics. Moreover, this kind of analysis has important consequences on the experimental side: once the relationship is known, measurements of decay times can be used to assess the impedance of boundaries in real rooms.
The authors plan to extend the strategy to more complex and realistic room shapes, in order to build detailed numerical models of the acoustical behavior of real environments. The ultimate goal is using simulation as a support tool to evaluate techniques of sound absorption in the low frequency range.

References

1. M.R. Schroeder, **The “Schroeder frequency” revisited**, *J. Acoust. Soc. Am.*, **Vol. 99**, pp. 3240-3241 (May 1996)
2. P.M. Morse, K.U. Ingard, ***Theoretical Acoustics***, pp. 554-576, Princeton University Press, Princeton, NJ, USA (1986)
3. A. Mäkivirta, P. Antsalo, M. Karjalainen, V. Välimäki, **Low-Frequency Modal Equalization Of Loudspeaker-Room Responses**, *111th Convention of the Audio Engineering Society, Convention Paper 5480*, New York, NY, USA, 2001 September 21-24
4. E. Rivet, S. Karkar, H Lissek, T.N. Thorsen, V. Adam, **Experimental assessment of low-frequency electroacoustic absorbers for modal equalization in actual listening rooms**, *140th Convention of the Audio Engineering Society, Convention Paper 9505*, Paris (France), 2016 June 4-6