Modeling of the Bread Baking Process Using Moving Boundary and Arbitrary-Lagrangian-Eulerian (ALE) Approaches

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Bread Baking - Overview

Flour, yeast, salt and water

Mixing → Molding → Prooving → Baking → Cooling

(i) Evaporation of water

(ii) Volume expansion

(iii) Gelatinization of starch

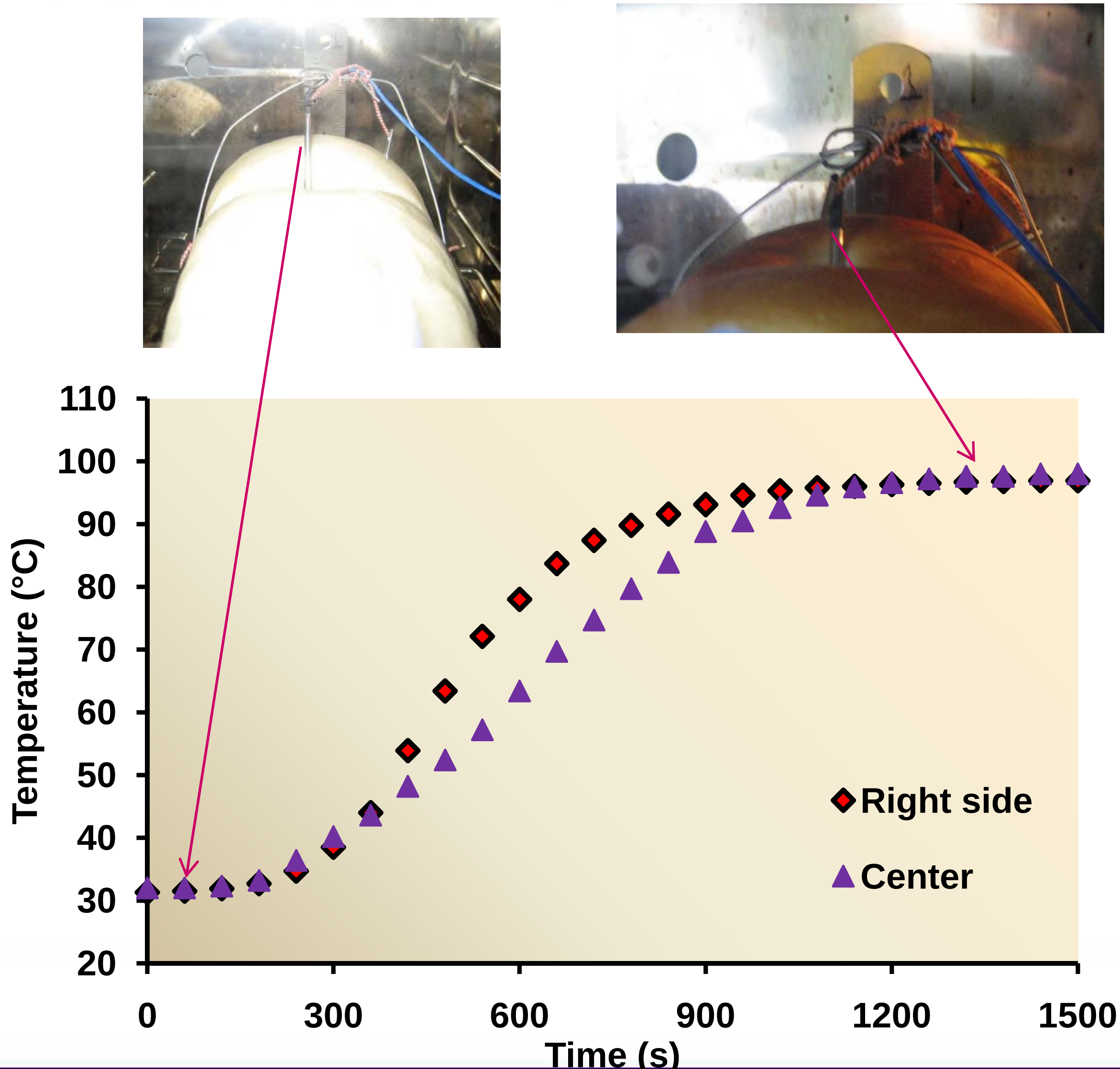
(iv) Denaturation of protein

(v) Crust formation

Bread

Experimental Measurement of Temperature

Pilot-Scale electric heating oven



Moving Boundary Problem (MBP) Formulation for Bread Baking

Bread can be modelled as a system containing three different regions:

Bread

1. **Crumb:** Wet inner zone, where temperature does not exceed 100°C and dehydration does not occur.

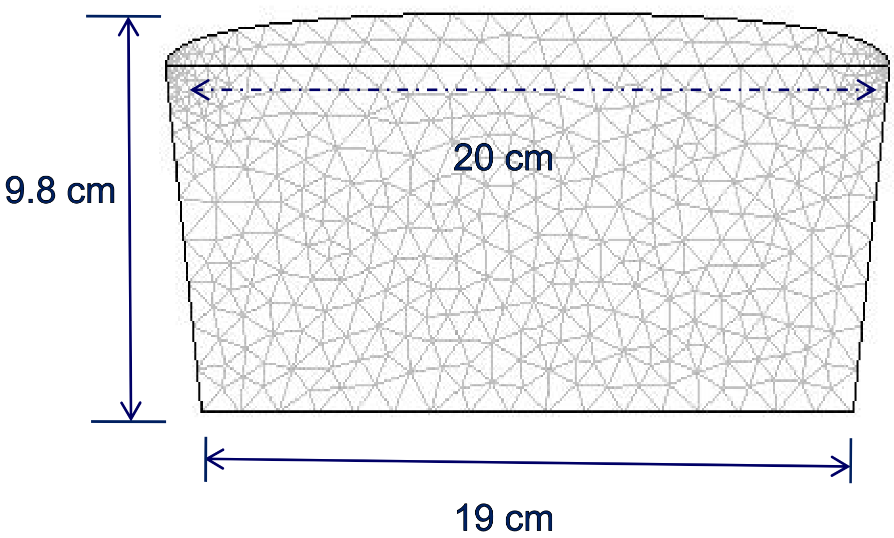
2. **Crust:** Dry outer zone, where temperature increases above 100°C and dehydration takes place.

3. **Evaporation front:** Between the crumb and crust, where temperature is 100°C and water evaporates.

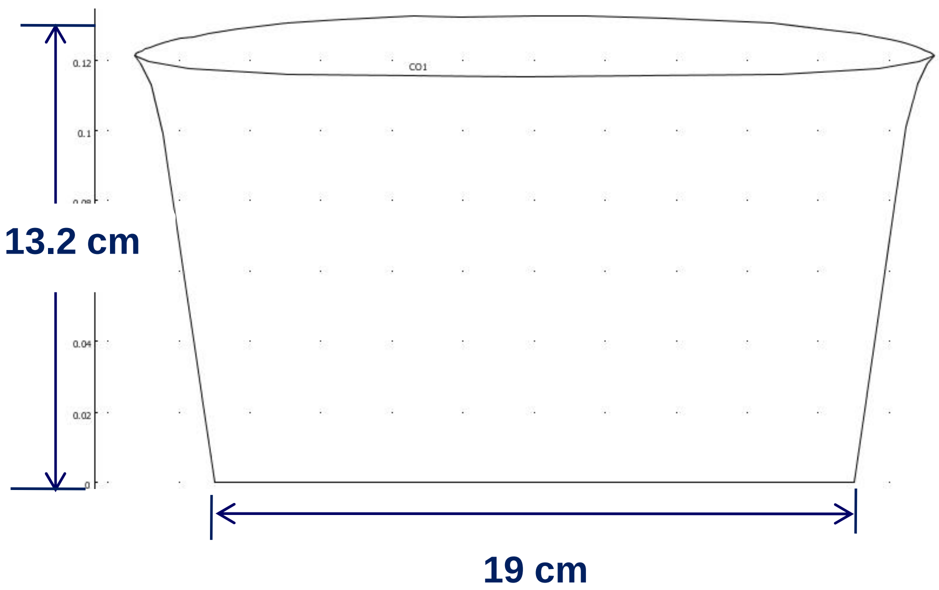
Model Frames

- Control volume is homogeneous.
- Energy from oven ambient to bread surface is transferred by conduction, convection and radiation.
- The Arbitrary-Lagrangian-Eulerian (ALE) method was applied to describe movement of mesh during volume expansion.
- Only liquid diffusion in the crumb and only vapour diffusion in the crust are assumed to occur.
- Water evaporates at 100°C.
- A smoothed Heaviside function replaces the delta function for defining equivalent thermophysical properties in the Moving Boundary Problem formulation.

Bread Geometry and Meshing



Geometry of bread before baking



Geometry of bread after baking

Model Development: Step 1- Governing Equations

Heat Balance Equation:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot \left(k \nabla T \right)$$

Boundary conditions:

The heat arriving to the bread surface by convection and radiation is balanced by conduction inside the bread

Hence,

$$-k\nabla T = h(T_s - T_\alpha) + \varepsilon\sigma\left(T_s^{\,4} - T_\alpha^{\,4}\right)$$

Step 1- Governing Equations

Mass Balance Equation:

$$\frac{\partial W}{\partial t} = \nabla \cdot \left(D \nabla W \right)$$

Boundary conditions:

The water migrating towards the bread surface is balanced by convective flux

$$-D\nabla W = k_g \left[\left(P_s(T_s) - P_\alpha(T_\infty) \right) \right]$$

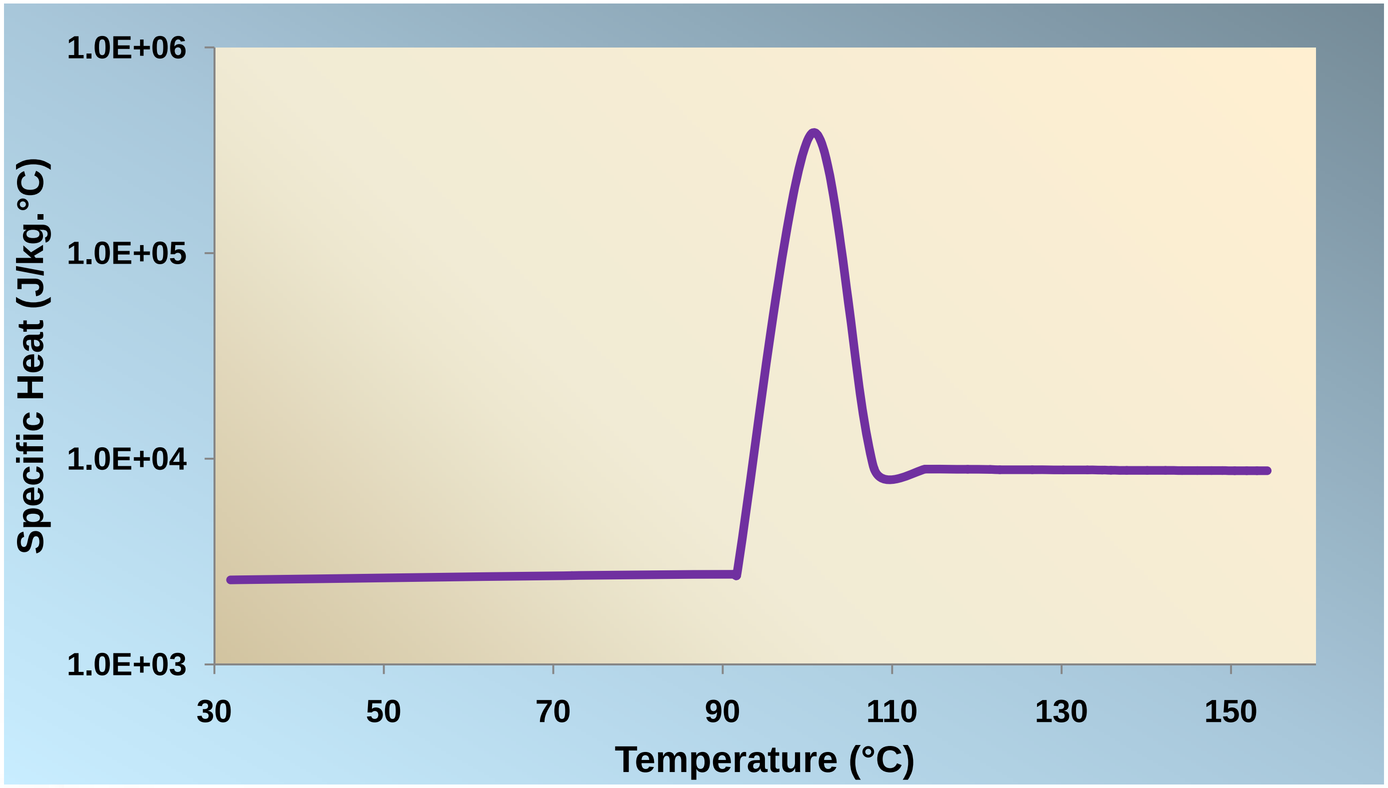
where,

$$P_s = a_w P_{sat}(T_s)$$

$$P_\alpha = RH \times P_{sat}(T_\alpha)$$

MBP Formulation: Specific Heat

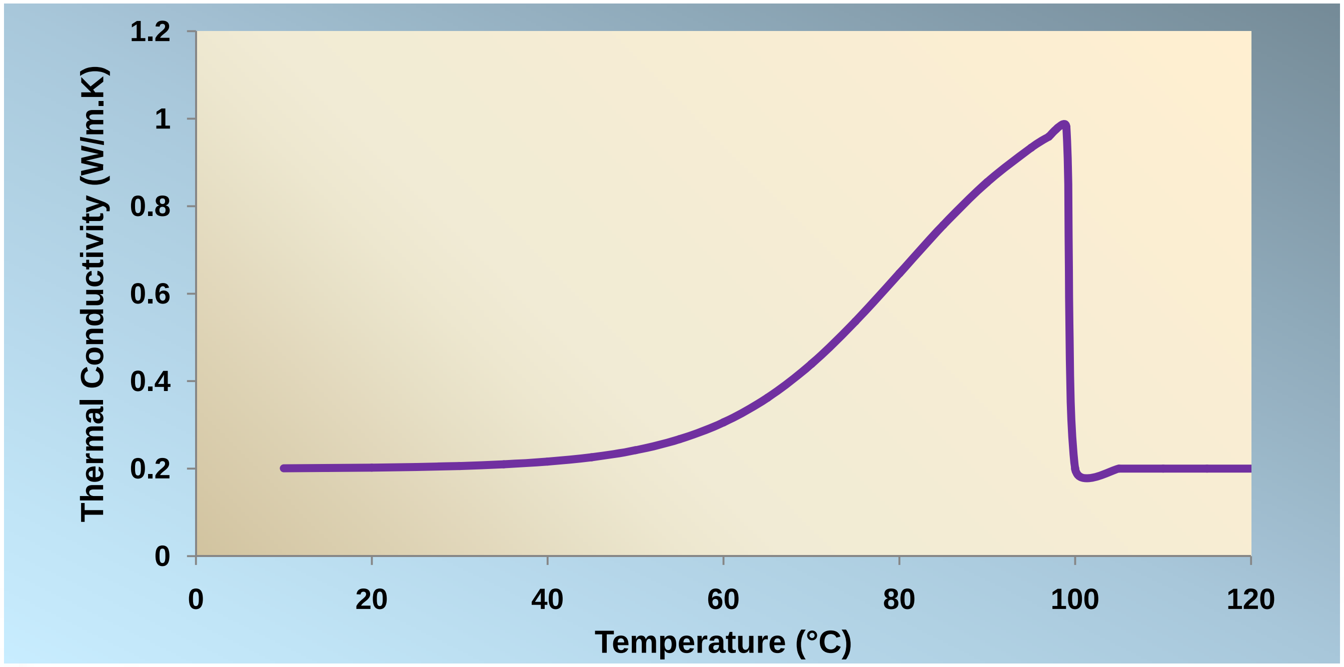
$$C_p = [(0.013T^2 - 0.217T + 4179)X_w + (5T + 1390)(1 - X_w)] + \lambda\delta(T - T_f, dT)$$



MBP Formulation: Thermal conductivity

$$k(T) = \frac{0.9}{1 + \exp(-0.1(T - 353.16))} + 0.2$$ If T ≤ 100 C

$$k(T) = 0.2$$ If T > 100 C



MBP Formulation: Density and Diffusivity

Assuming the dough porosity equal to 75%, an apparent density is defined as follows (Hamdami et al., 2004):

$$\begin{aligned}\rho(T) &= 180.61 \quad \text{if} \quad T \leq T_f - \Delta T \\ &\; 321.31 \quad \text{if} \quad T \geq T_f + \Delta T\end{aligned}$$

Mass Diffusivity

$$D(T) = \begin{cases} 1\times 10^{-10} & \text{If } T \leq T_V - dT \\ 6.4\times 10^{-8} & \text{If } T \geq T_V + dT \end{cases}$$

Step 2: MBP Formulation – Water activity

Change in water activity with respect to temperature and moisture content:

$$a_w(T,W) = \left[\left(\frac{100W}{\exp(-0.0056T + 5.5)} \right)^{\frac{-1}{0.38}} + 1 \right]^{-1}$$

In the present work, we use the Oswin model (Lind and Rask, 1991)

Setup for the volume expansion of bread

Arbitrary Lagrangian-Eulerian (ALE) is a formulation for defining a moving mesh where dependent variables represent the mesh displacement or mesh velocity.

The ALE is an intermediate method between the Lagrangian and the Eulerian Combines the best features of both and allow moving boundaries without the need for the mesh movement to follow the material.

$$\frac{\partial^2}{\partial X^2}\left(\frac{\partial x}{\partial t}\right)+\frac{\partial^2}{\partial Y^2}\left(\frac{\partial x}{\partial t}\right)=0$$

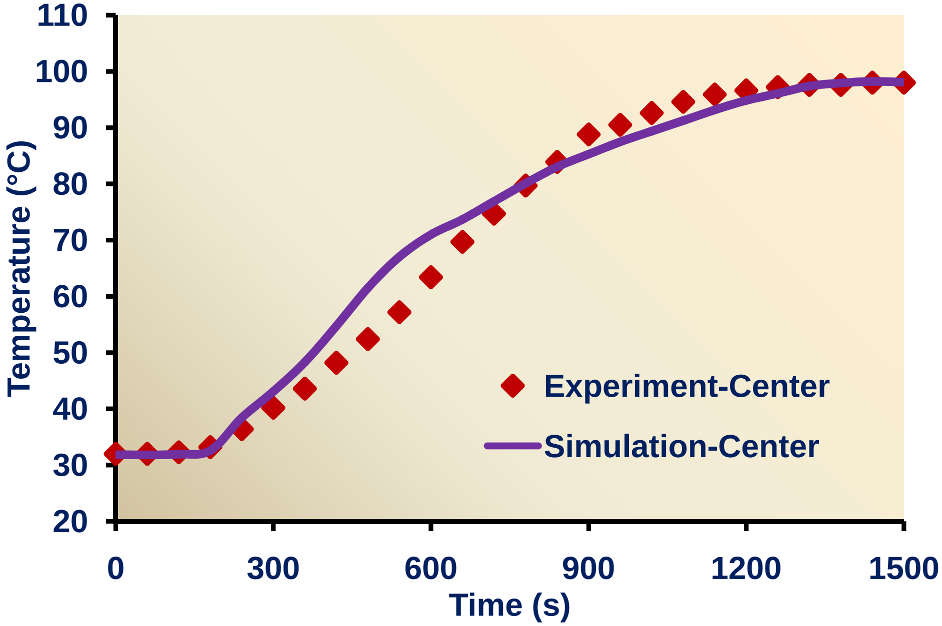
We propose a following model, where expansion rate is proportional to the temperature of bread

$$V_n = K*T$$

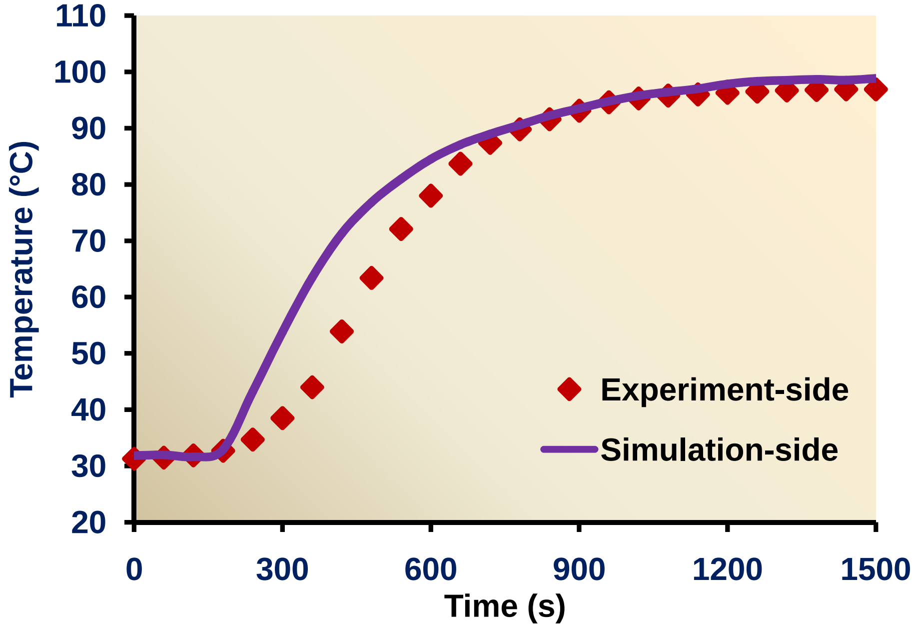
where, V_n **represents the mesh velocity,**
K = coefficient of proportionality = 3.5×10^{-7} $m^3/K\,s$.

Experimental Validation: Bread Temperature

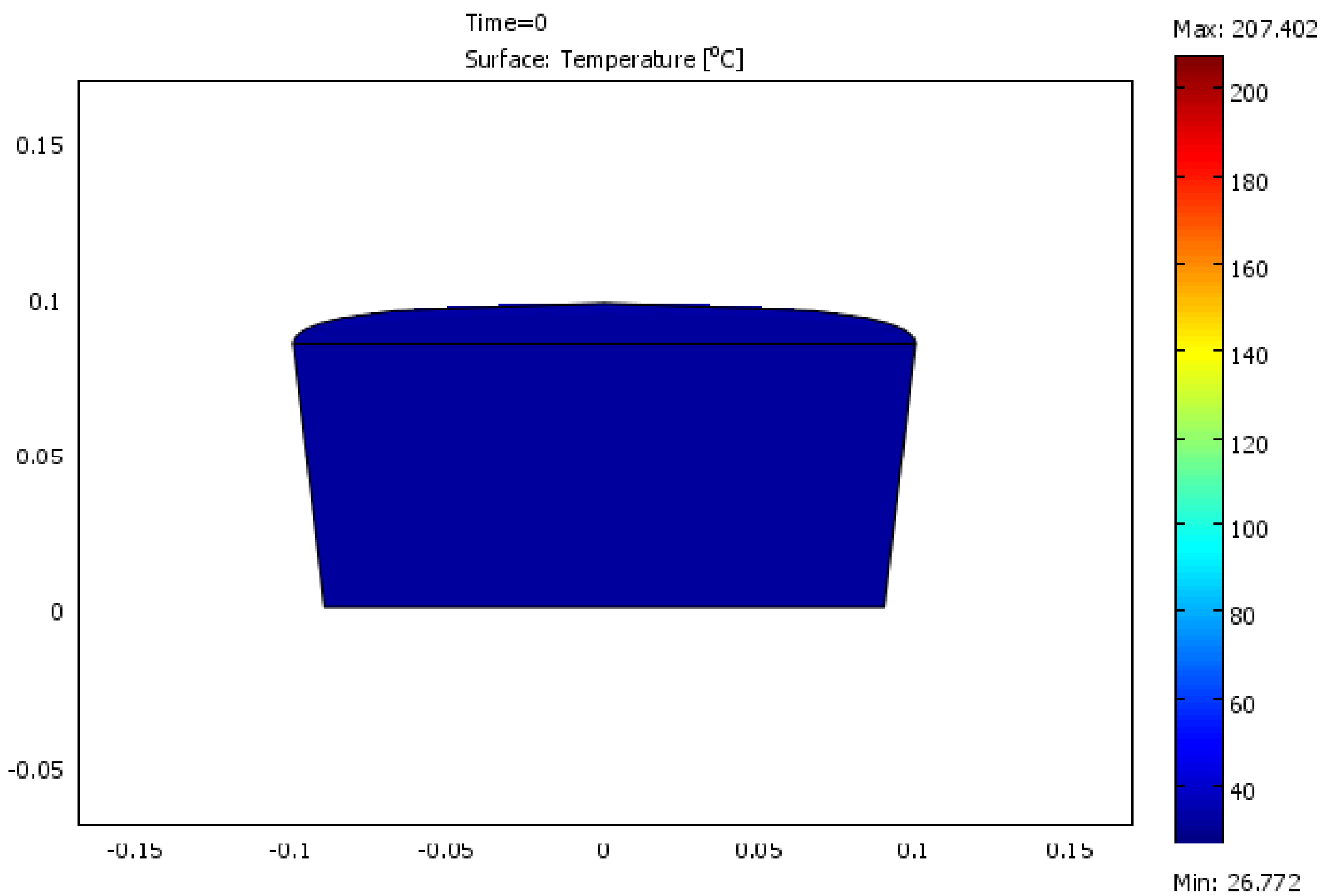
Centre of the bread



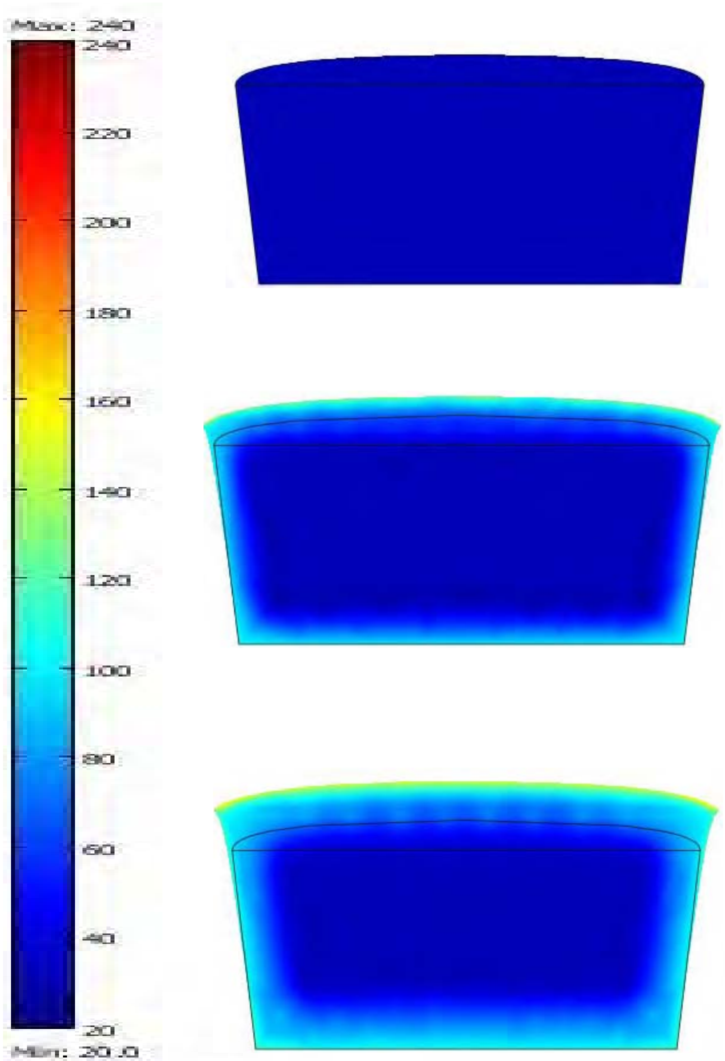
6 cm from centre to right side of the bread



Volume Expansion with Temperature changes



Volume Expansion of Bread During Baking Process With Temperature Profile



Process time-0 sec

Bread height -9.8 cm

60 sec

10.5 cm

120 sec

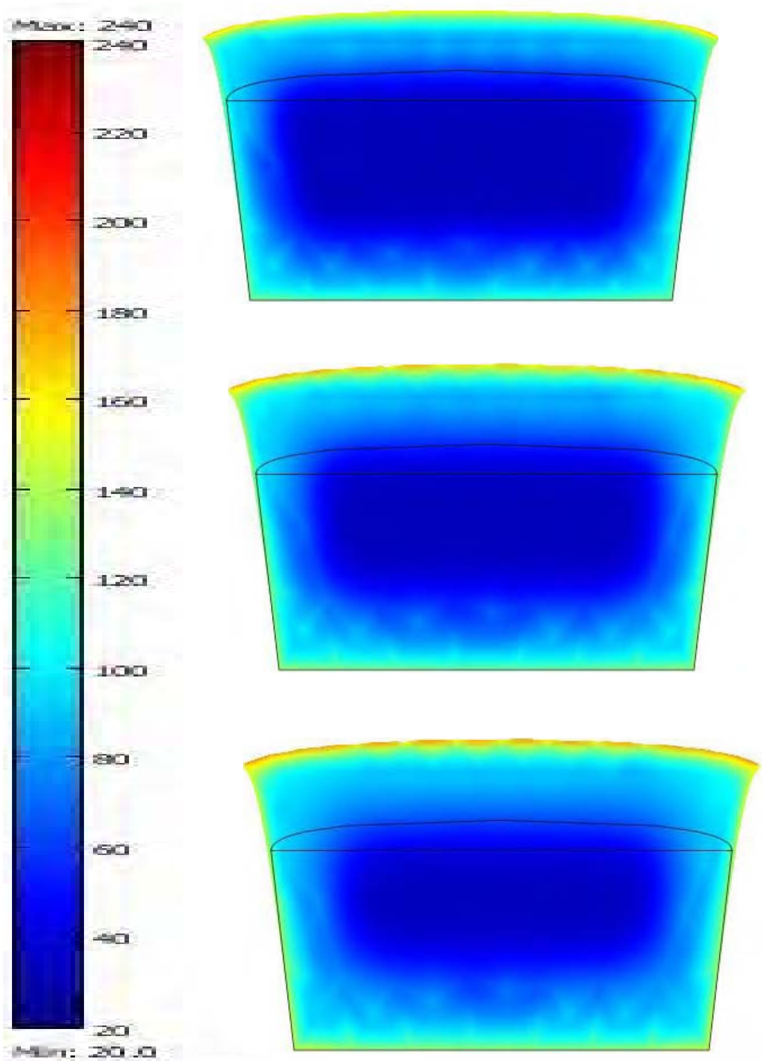
11.4 cm

9.8 cm

10.8 cm

11.8 cm

Volume Expansion of Bread During Baking Process With Temperature Profile



180 sec

12.36 cm

12.4 cm

240 sec

13.15 cm

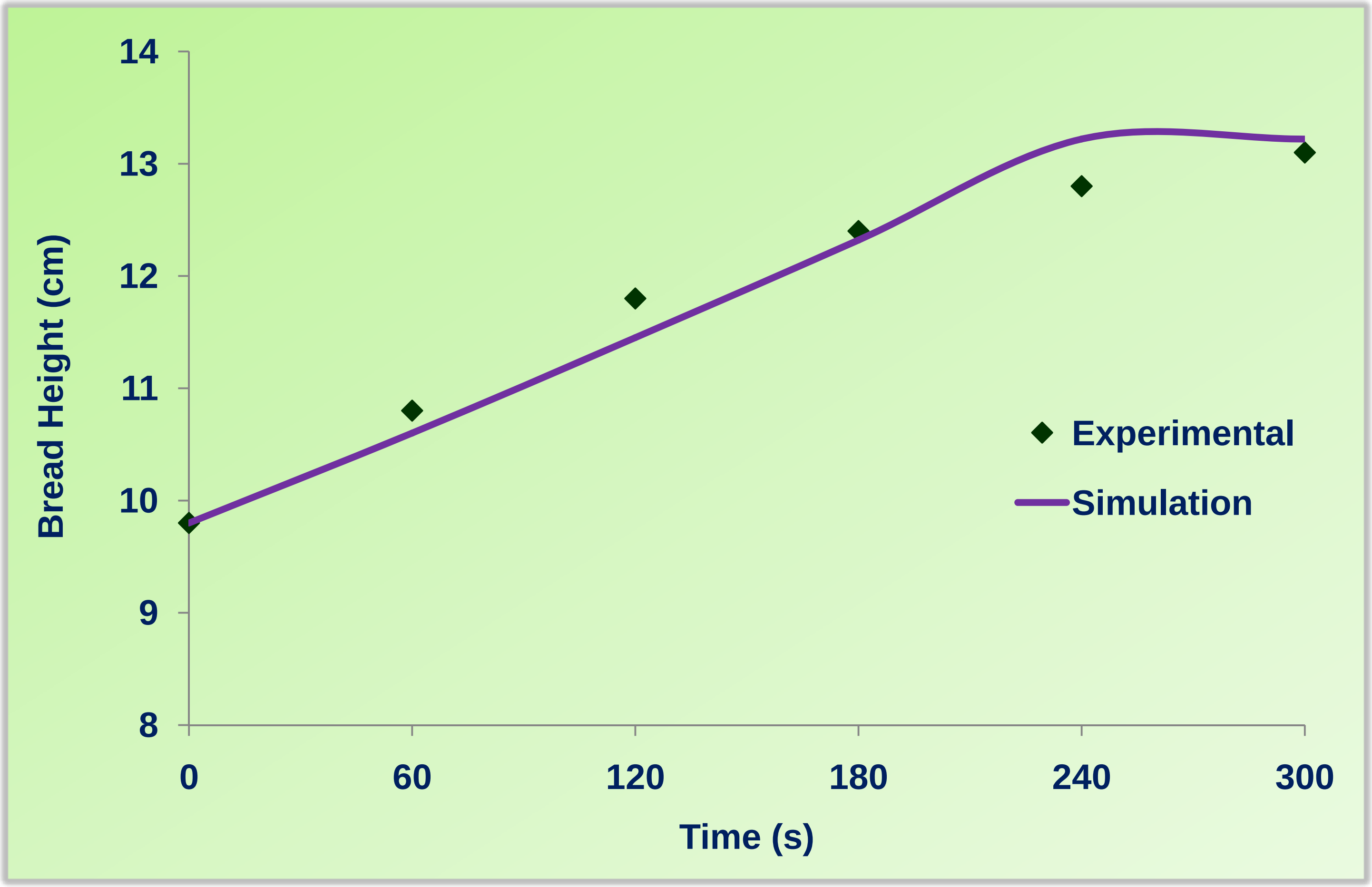
12.8 cm

300 sec

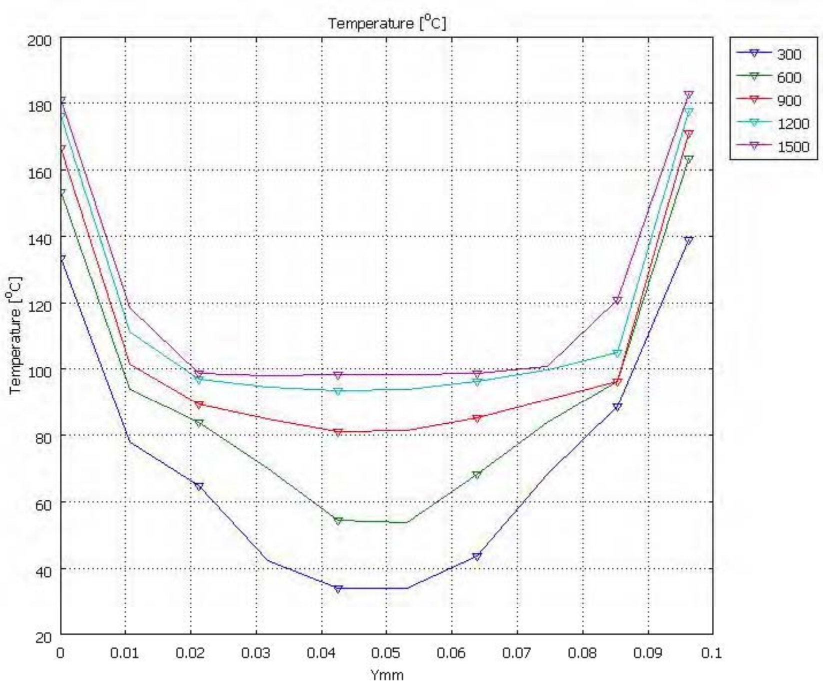
13.15 cm

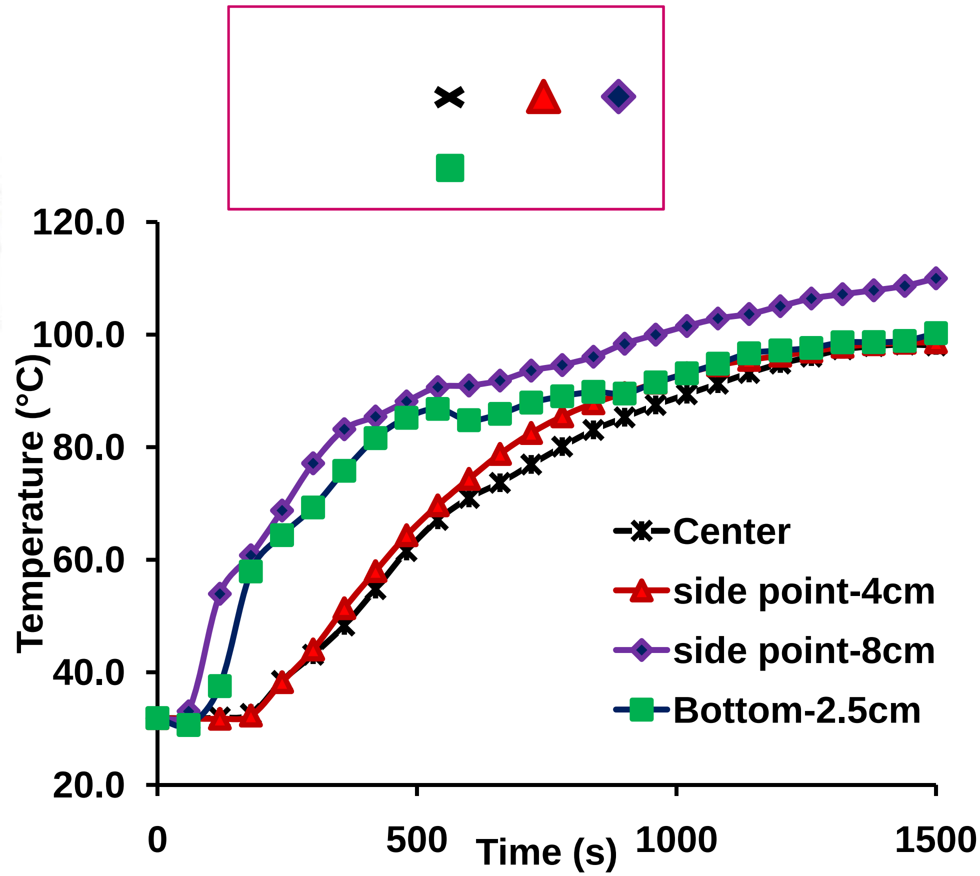
13.1 cm

Experimental Validation of Volume Expansion

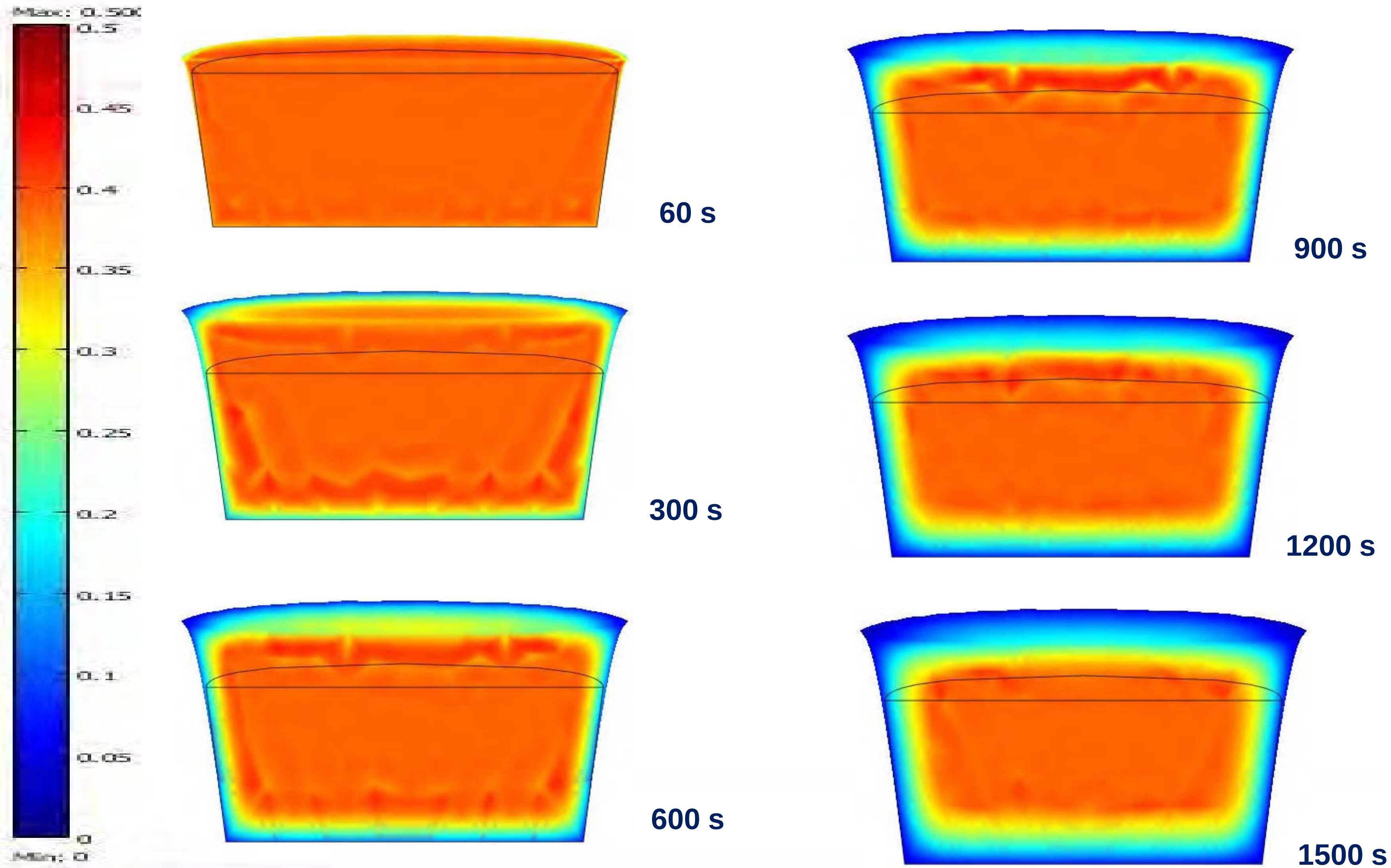


Temperature Profile of Bread during Baking Process

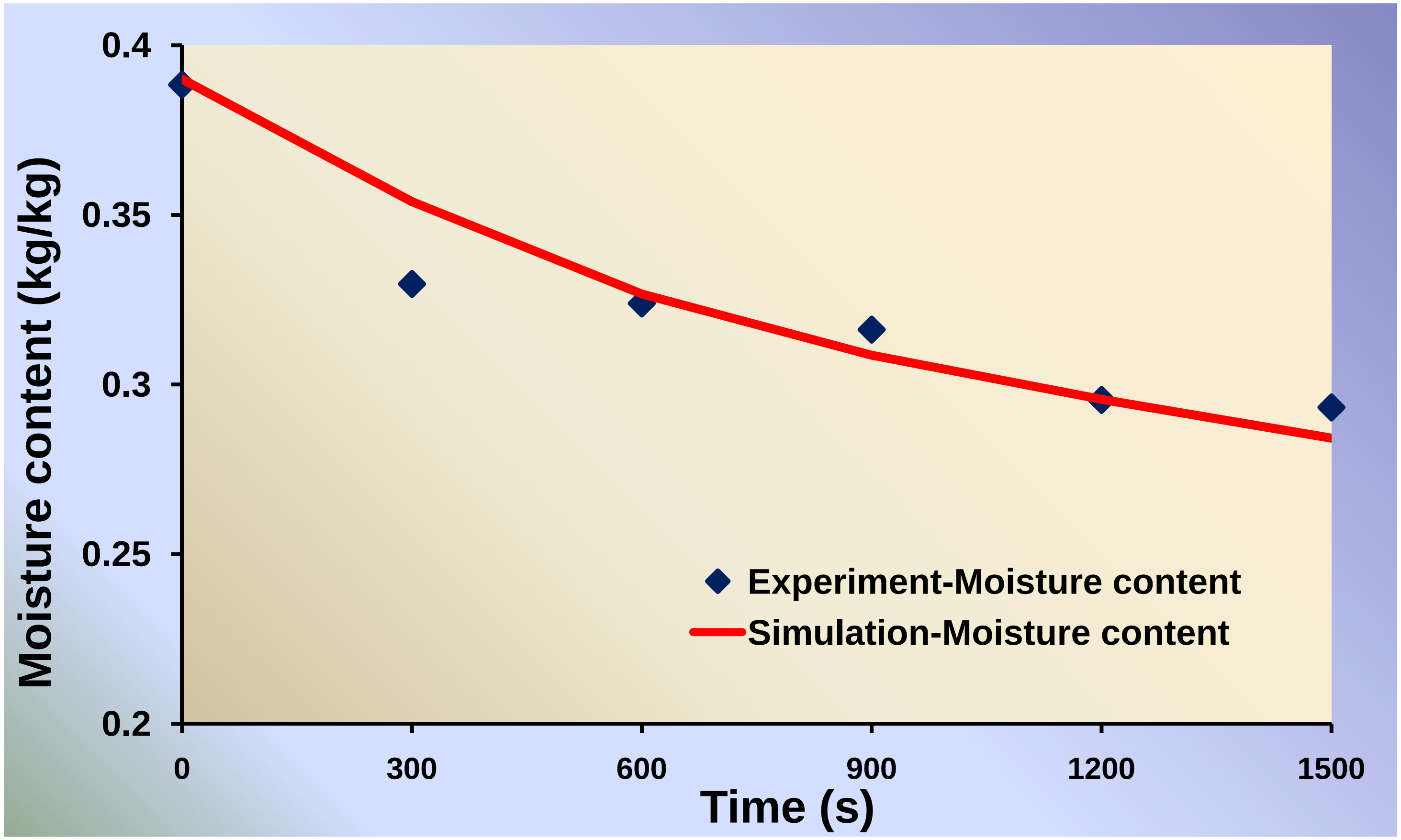




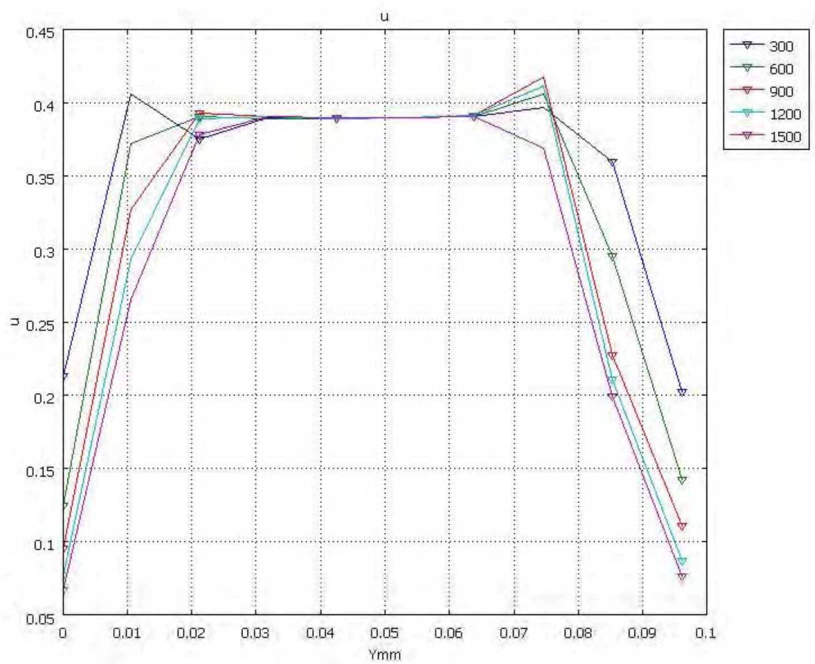
Moisture Content of Bread During Baking Process

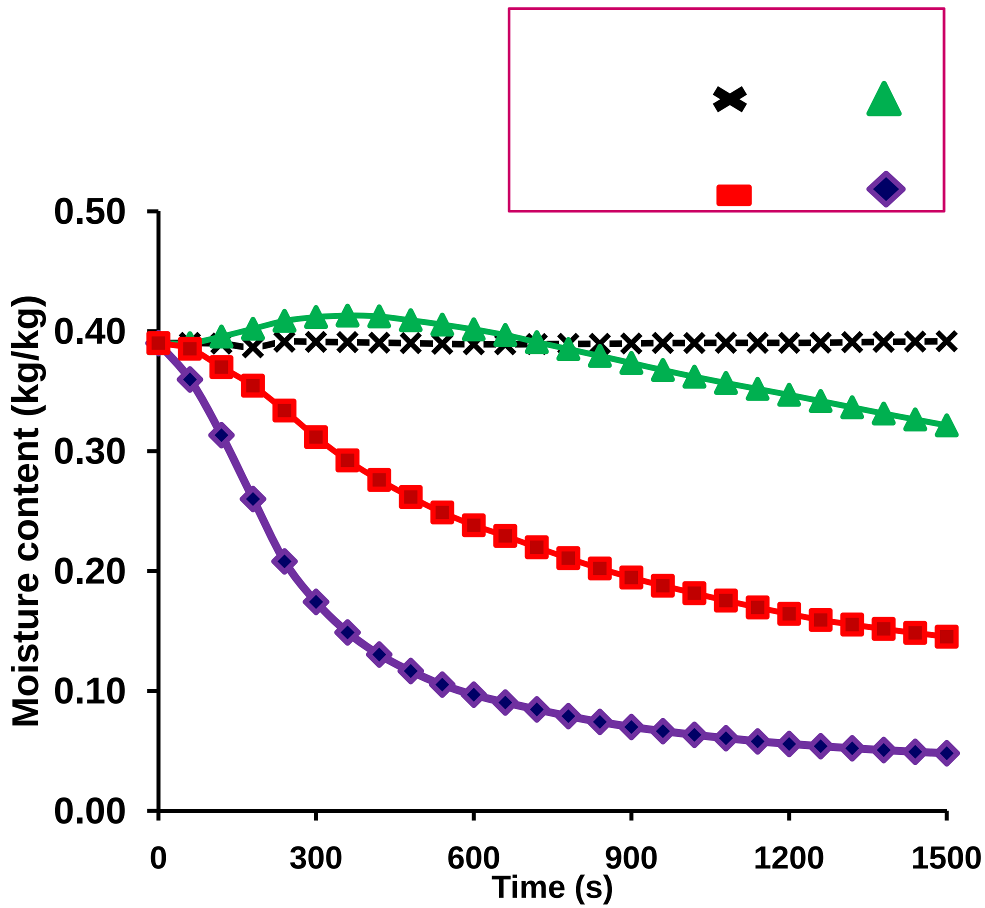


Experimental Validation : Moisture Content



Moisture Profile of Bread during Baking Process





Simulation of Starch Gelatinization

The Starch gelatinization mechanism during bread baking follows first order kinetics (Zanoni *et. al*., 1995)

$$1-\alpha = \exp(-kt)$$

α - degree of gelatinization, k - reaction rate constant, t – time (s)

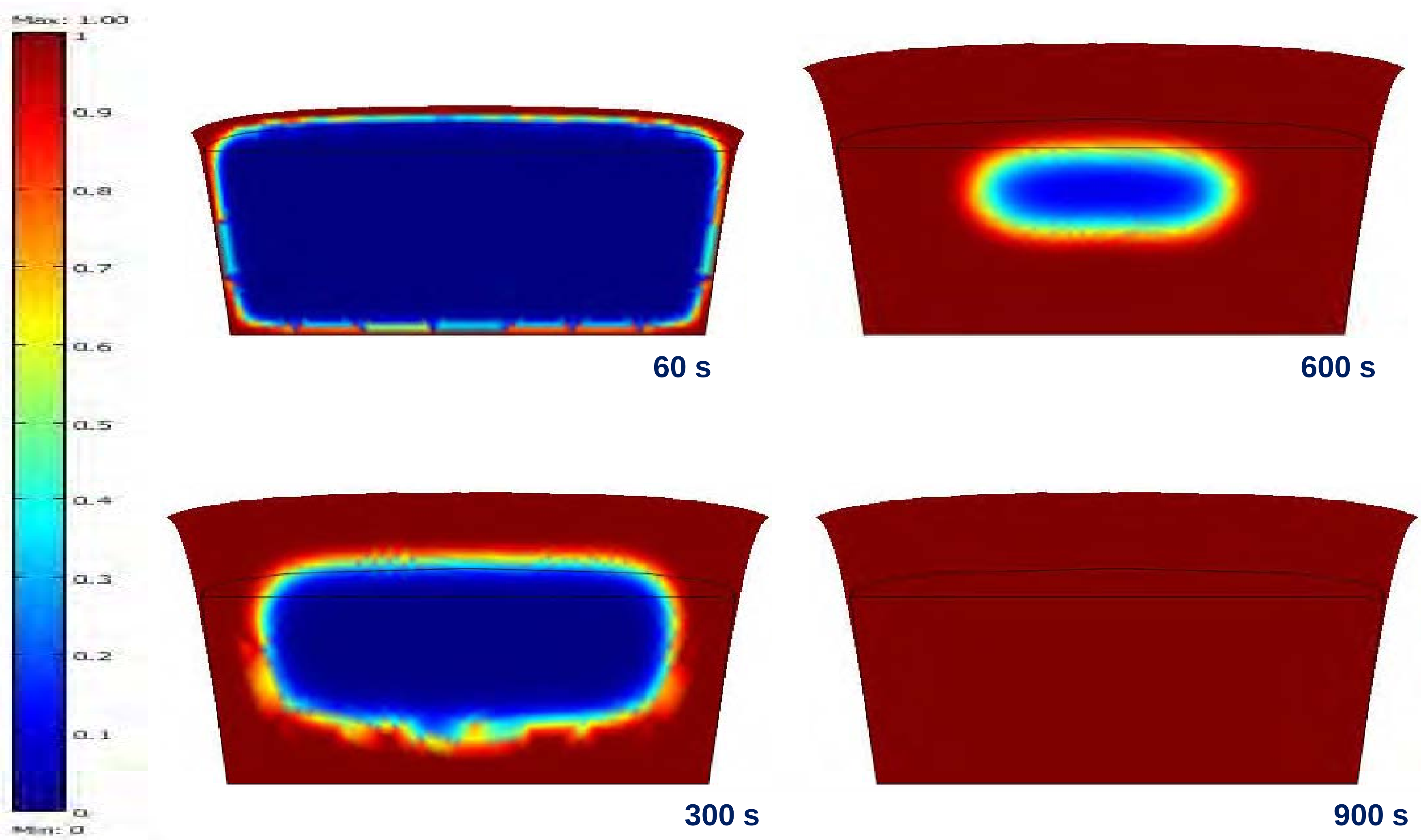
$$k = k_o \exp\left(-\frac{E_a}{RT}\right)$$

k can be calculated using *Arrhenius equation*.

k_0 - 2.8 x 10^{18} s-1

E_a - 138 kJ/mol (based on DSC experiments)

Starch Gelatinization Counters During Bread Baking Process



Summary

- A two dimensional model was developed for the bread baking process taking into account of heat and mass transfer processes as well as volume expansion.
- Phase change and evaporation-condensation mechanism were incorporated by defining thermo-physical properties of bread as a function of temperature and moisture content.
- Simulation results were validated with experimental measurements of bread temperature and moisture content.
- Occurrence of evaporation-condensation and high value of latent heat of phase change keeps crumb temperature below 100°C.
- Linear increase in volume was observed for the first four minutes and volume remains constant for remaining entire baking process. Baking process was completed in 25 minutes.

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Thank You..

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