Modeling Energy Harvesting from Membrane Vibrations in COMSOL

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Abstract: This paper presents an ongoing effort, motivated by developing self-contained sensors for structural health monitoring of inflatable structures, to model the process of extracting useable electrical power from the mechanical vibrations of thin, prestressed membrane structures. The multiphysics package COMSOL is being used to estimate the time-domain response of a piezoelectric patch placed onto a thin-film membrane to induced vibrations. Rayleigh damping coefficients will be used to represent the energy dissipated from the system through the harvesting process. Currently, prestress and eigenmodes have been successfully modeled in COMSOL. In following work, the prestressed solution will be used in the time-domain analysis for a more realistic representation of the added stiffness in the system.

Keywords: energy harvesting, nonlinear vibration, membranes, piezoelectric, COMSOL.

1. Introduction

Though the piezoelectric effect was discovered by the Curie brothers over a century ago, energy harvesting via piezoelectrics has become of increasing importance today [1]. The demand for smaller, more autonomous methods of powering devices is growing, as is the demand for sustainable energy sources. Energy harvesting from vibration energy is attractive because it uses system energy that would otherwise be lost through ambient vibrations, and converts that mechanical energy to electric power for use elsewhere.

The focus of this research is to develop a model for an efficient method of extracting useable electrical power from the mechanical vibrations of thin, prestressed membrane structures. These structures may not be easily accessible, as in the cases of large space structures, vehicle tires, and high roofs similar to those at Denver International Airport (Figure 1).

Figure 1. Denver International Airport.

Energy harvesting from mechanical vibrations of these structures may provide enough power to operate both the sensors as well as wireless devices used to transmit data to a remote data collection device. These energy harvesting technologies are also applicable to powering the components of micro air vehicles (specifically from the wing-like membrane structures), which would aid their autonomy.

Various designs for energy harvesting transducers (i.e. piezoceramic, ionic polymers, PVDF) have already been documented for comparison, in terms of strengths and weaknesses. Since the goal is to transfer as much mechanical energy to power as possible, the transducer should support large strains and allow for higher electromechanical coupling at low frequencies.

A list of manufacturers for flexible piezoelectric materials was acquired, as regular piezoceramics would not support large strains. Smart Material's macro-fiber composite (MFC) product was both flexible and specifically developed for high coupling coefficients in transverse and bender configurations, so it was chosen for this work [2].

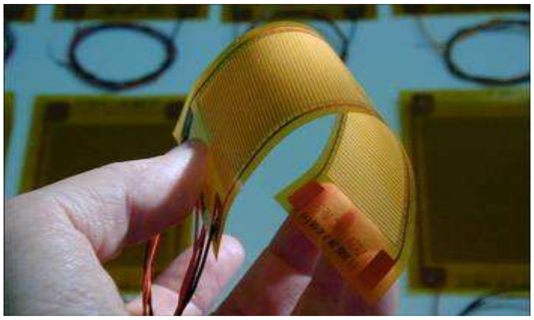

Figure 2. A macro-fiber composite patch.

The multi-physics package COMSOL was found to be the finite element program of choice, due to the ease in which the electrical and mechanical domains can be coupled during processing. A paper by Zurkinden documents the use of COMSOL in successfully modeling a wave energy converter with PVDFs, confirming that the program could be used for complete analyses for harvester systems [3].

2. Use of COMSOL Multiphysics

Before any trade studies or optimization techniques can be conducted, the simplest form of the stated problem had to be attempted. Below, insight into the modeling of the transverse nonlinear vibration of a prestressed membrane with a piezoelectric patch is documented.

2.1 Boundary Conditions and Background

Figure 3 shows the system for analysis. A 0.54m x 0.54m x .001m nylon membrane is stretched in-plane by 3cm on all four sides. This keeps the membrane under uniform and equal prestress (tensile stress) along the x and y directions. Then, a 0.1m x 0.1m x .001m MFC piezoelectric patch is placed on the membrane, centered at (0.3, 0.05, .0015) [m].

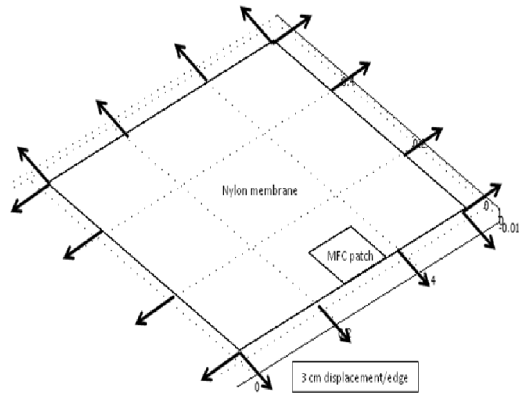

Figure 3. Model set up with initial prestress.

The constitutive equations (see Figure 4) governing this system show the coupled nature of the problem. Below, the matrix variables are defined as follows: S is the strain tensor, D is the electric displacement, s^E is the compliance matrix for constant electric field, d is the piezoelectric coupling matrix, where the subscript T denotes transpose, ε is the dielectric matrix, T is the stress tensor, and E is electric field [4].

$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s^E & d \\ d_T & \varepsilon^S \end{bmatrix} \begin{bmatrix} T \\ E \end{bmatrix}$$

Figure 4. Constitutive equation for piezoelectricity.

As this system involves the conversion of mechanical strain energy into electrical power, the piezo patch was modeled as a generator, while the harvesting process was modeled as a dissipater. From a circuit system standpoint, the sinusoidal form of the loading meant that the piezoelectric patch could be modeled as an AC voltage source in series with a capacitor, which represents the dielectric properties of the material. An equivalent circuit for the described example is shown in Figure 5.

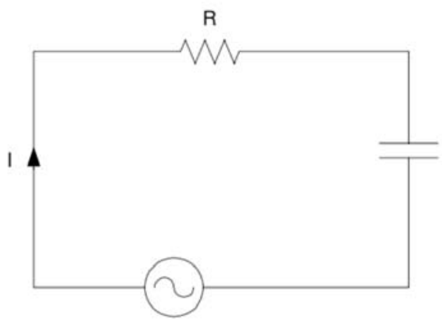

Figure 5. Equivalent circuit diagram.

2.2 Analysis Setup

COMSOL 3.5a was used to estimate the coupled mechanical and electrical responses of the MFC patch due to a sinusoidal pressure load acting in the out-of-plane direction on the bottom of the membrane. As vibration amplitudes are increased near resonance frequencies, the excitation frequency needed to be around the natural frequency of the first eigenmode. So in addition to a static analysis to set up the prestress in the membrane, an eigenfrequency analysis would need to be completed before approaching the transient analysis.

Within COMSOL, the model was setup using the Structural Mechanics → 3D Solid and Structural Mechanics → Piezo Solid physics settings. The piezo patch was only active in the Piezo Solid domain, while the membrane was active in the 3D Solid domain. Using the large deformation option, the displacement conditions were applied using the Prescribed Displacement tool. Then, a static analysis step was run to check the prestress values. Figure 6 shows the result of the static step, with a boundary plot of total displacement (blue is the zero value).

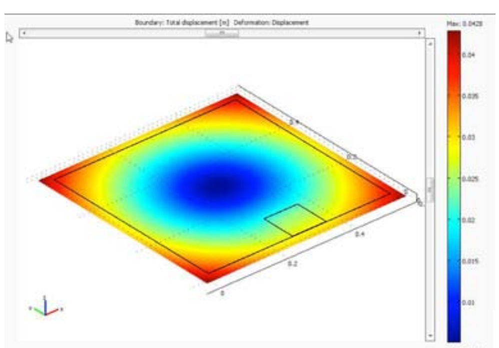

Figure 6. Total displacement in membrane after prestress.

To keep the prestressed state, the Solver Manager was used in an attempt to perform an eigenfrequency analysis using the stored solution. Thus far, the solver sequence has not been able to capture the deformed state, and so further work would require a better understanding of multi-step analysis procedures. However, an algorithm for computing the nonlinear response was developed, and is previewed below.

3. Future Work

When energy is harvested from a system, the energy required for sustaining the amplitude of vibration decreases, and so a damped behavior is observed. As COMSOL has no direct input for equivalent resistance in the piezoelectric circuit, the dissipative harvesting process will be estimated using a Rayleigh damping coefficient. To find the corresponding Rayleigh damping coefficient for a desired resistance, an analytical relationship between the energy or power harvested over a number of load cycles and the damping ratio of the system was derived. With this relation, an algorithm for replicating the harvesting system was formed. First, a eigenfrequency analysis is run to find the amplitude and phase angle for the voltage signal generated by the piezoelectric patch for frequencies near resonance of the first mode.

Then, a time-dependent analysis is run in COMSOL using an estimate of the Rayleigh damping coefficient, to extract an approximation of the power harvested (dissipated) over a number of cycles. Following that, the analytical equation for power over time is solved for the damping ratio based on the input resistance, the electromechanical properties of the system, the loading conditions, and the initial estimate of the power harvested. Using this damping ratio, solve for the Rayleigh damping coefficient, which is then entered into COMSOL. A time-dependent analysis is run again, and this process is iterated until a solution is converged upon.

4. Conclusions

Initial work for modeling the complete electromechanical solution of a piezoelectric harvesting system for vibrating prestressed membranes was presented. The prestress in the nylon membrane was successfully applied, but problems arise in keeping the deformed properties for further steps, i.e. eigenfrequency or time-domain analyses. Once this process is more concretely understood, then the dissipation in the vibrations of the membrane system can be accounted for using both COMSOL and analytical expressions for the power and energy harvested over time. This will provide a realistic approximation of the coupled harvesting process. Future work involves running the algorithm for a simple membrane vibration problem and comparing to analytical results using Adomian decomposition method.

5. References

1. V. A. Koptsik and I. S. Rez, Pierre Curie's works in the field of crystal physics (on the one-hundreth anniversary of the discovery of the piezoelectric effect), *Soviet Physics Uspekhi*, **Volume** 24 (1981).

2. A. J. Schoenecker, T. Daue, et al, Overview on Macro Fiber Composite Applications, *Proceedings of SPIE*, **Volume** 6170, (2006).

3. A. S. Zurkinden, F. Campanile, and L. Martinelli, Wave Energy Converter through Piezoelectric Polymers, *Proceedings of the COMSOL Users Conference 2007 Grenoble*, (2007)

4. O. J. Myers, M. Anjanappa and C. Freidhoff, Designing Piezoelectric Interdigitated Microactuators Using COMSOL, *Proceedings of the COMSOL Conference 2008 Boston*, (2008).

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