Modeling Energy Harvesting From Membrane Vibrations in COMSOL

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Outline

- Importance of Energy Harvesting
- Prestressed Membrane Structures
- Challenges in Energy Harvesting
- Transducer Materials
- Estimate of Harvested Energy
- Optimal Prestress and Transducer Locations

Importance of Energy Harvesting

- Recycling energy vs. expending energy
- Useful for multiple applications
- Increases autonomy
- Reach inaccessible locations

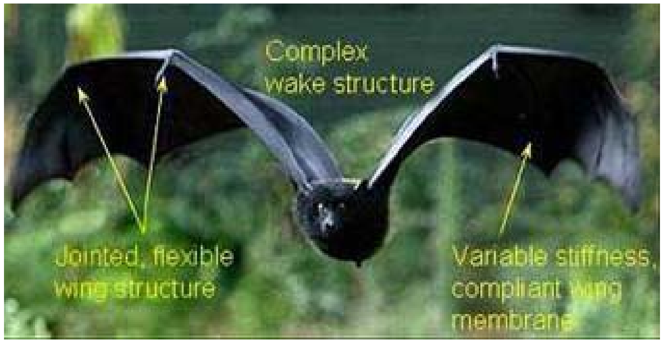
Why Membranes?

- Possibility of large amplitude vibrations

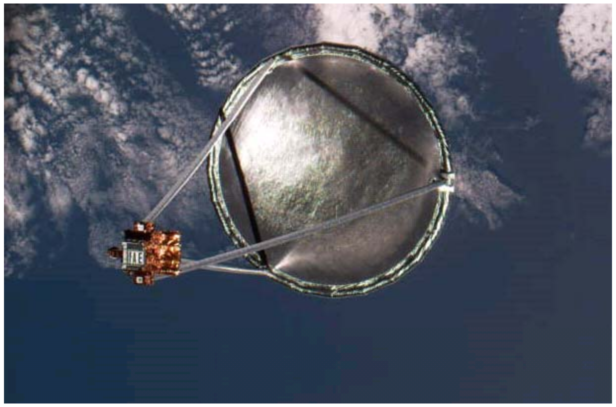
- Natural frequencies and mode shapes can be tuned by changing the prestress

Prestressed Membrane Structures

- Prestress is applied to ultra-lightweight membrane structures to keep them in desired shape and provide stiffness



Bat-Wing Micro Air Vehicles



Inflatable Space Antenna

Roof of Denver International Airport

Challenges in Energy Harvesting

- Maximize ability to support large strains
- Maximize power output
- Place transducers at points of high deformation

Transducer Materials

- Piezoceramics
  - Do not support large strains
  - Produce high voltages and useable power
- Electroactive polymers (i.e. ionic polymers)
  - Supports large strains
  - Do not produce useable amounts of power
- Flexible piezomaterials (PVDF, macro-fiber composites, etc.)
  - Designed to accommodate large strains while sustaining high piezoelectric constants

Transducer Materials

- Macro-fiber composites was found to have the highest piezoelectric coefficient for transverse stresses

- PVDF support highest strain limit, but would not produce higher output

Modeling Challenges

- Find relationships between membrane deformation and inputs
- Find relationships between electric field and deformation
- Finite element approach more suitable for nonlinear problem
- For MSC/NASTRAN use “thermal-piezo analogy”
- ANSYS/ABAQUS, can use piezoelectric elements directly

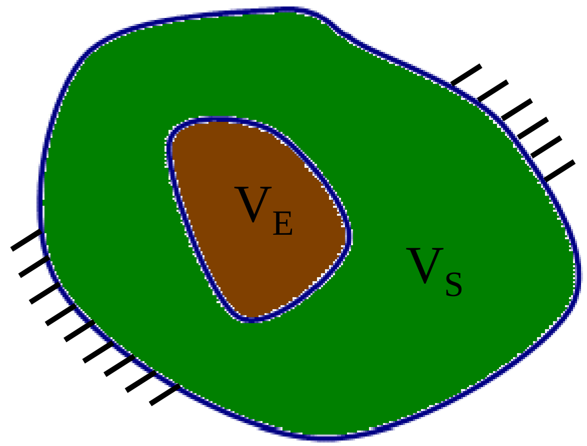
Governing Equation of a Membrane with Transducer

- Governing equations are determined from the following condition:

$$\delta\Pi = 0$$

$$\Pi = \int_{t_1}^{t_2} (T - U + \int_A pwdA)dt$$

$$T = \frac{1}{2}\left[\int_{v_E} \rho^E v^2 dv + \int_{v_S} \rho^S v^2 dv\right] = \text{Kinetic Energy}$$



$$U = \frac{1}{2}\left[\int_{V_E} \sigma_{pr}{}^T \varepsilon dv_E + \int_{V_s} \sigma^T \varepsilon dv_s + \int_{V_E} \sigma^T \varepsilon dv_E\right] - \int_{V_E} D^T E dv_E = \text{Strain Energy}$$

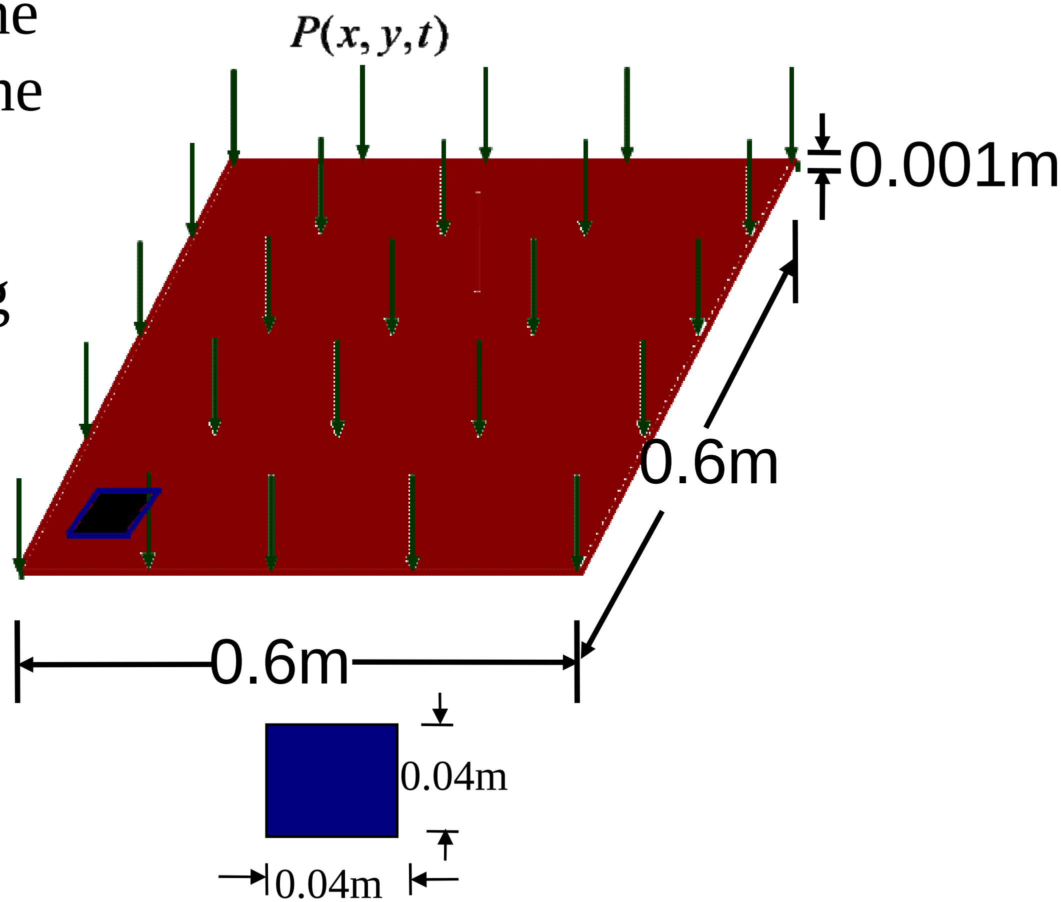
- Substitution of constitutive relations and strain-displacement relationship into Π yields governing differential equations (nonlinear coupled PDEs)

Transducer Materials

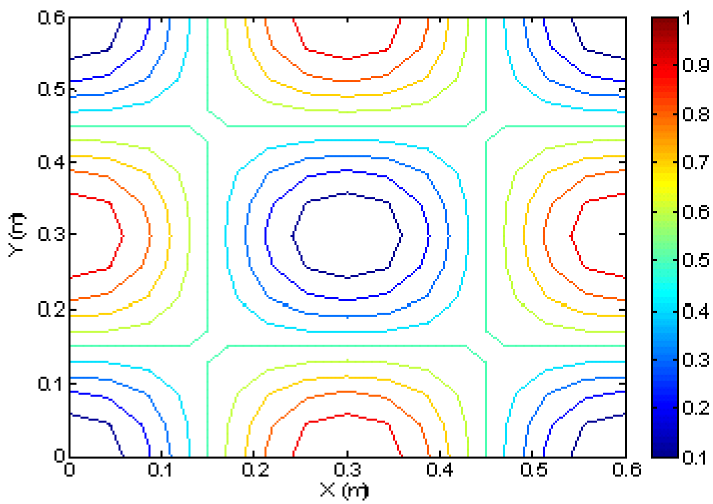
- Analyzed the response of the membrane to find out the electric filed generated in the transducer

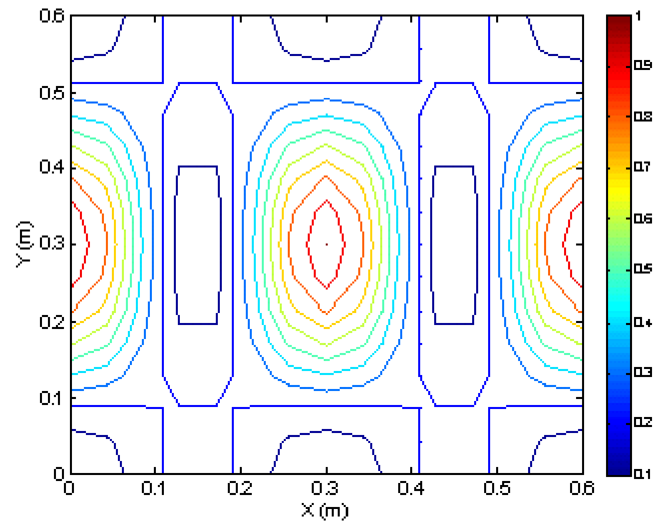
- The non linear governing differential equations were solved using the Adomian decomposition method

- The analysis was performed for different load cases for different transducer locations



Modeling Challenges



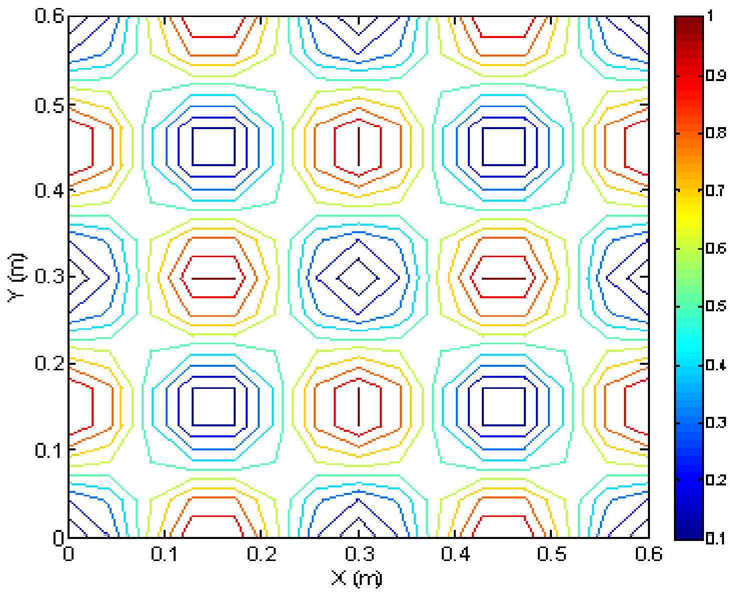


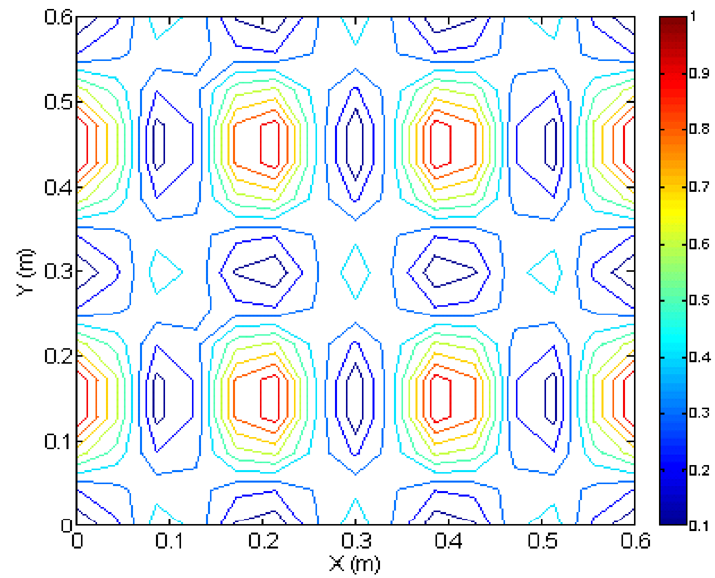
$$P_1(x,y,t) = P_0 Sin\left(\frac{\pi x}{L}\right) Sin\left(\frac{\pi y}{L}\right)$$

$$P_2(x,y,t) = P_0 Sin\left(\frac{2\pi x}{L}\right) Sin\left(\frac{\pi y}{L}\right)$$

Contour Map Showing the Electric Field That Can Be Harvested at Each Location

Governing Equation of a Membrane with Transducer





$$P_3(x,y,t) = P_0 Sin\left(\frac{2\pi x}{L}\right) Sin\left(\frac{2\pi y}{L}\right)$$

$$P_4(x,y,t) = P_0 Sin\left(\frac{3\pi x}{L}\right) Sin\left(\frac{2\pi y}{L}\right)$$

Contour Map Showing the Electric Field That Can Be Harvested at Each Location

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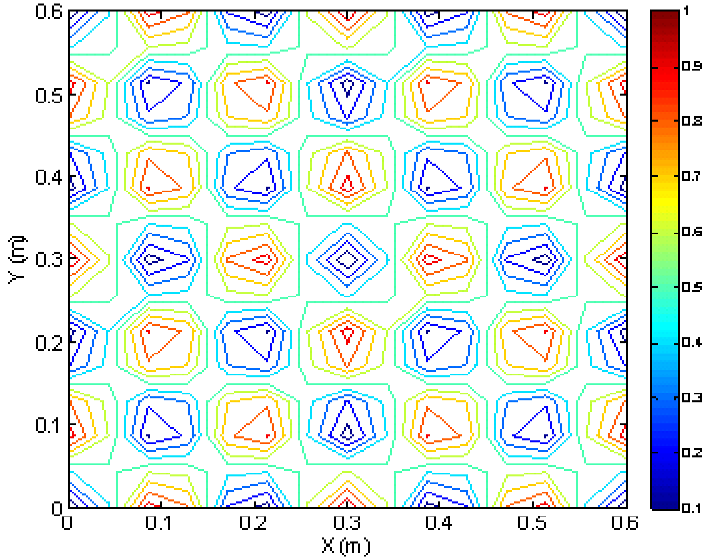
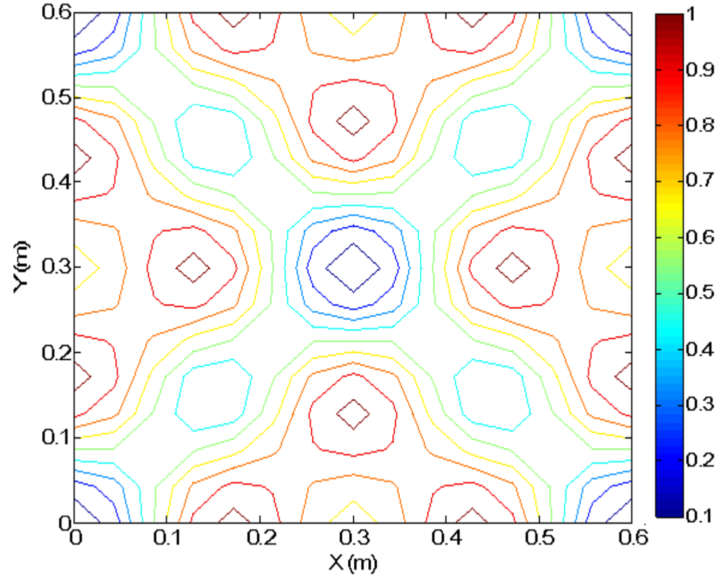
Solution of the Equations





$$P_5(x,y,t) = P_0 Sin\left(\frac{3\pi x}{L}\right) Sin\left(\frac{2\pi y}{L}\right)$$

$$P_6(x,y,t) = P_0 Sin\left(\frac{3\pi x}{L}\right) Sin\left(\frac{2\pi y}{L}\right)$$

Contour Map Showing the Electric Field That Can Be Harvested at Each Location

Challenges in COMSOL

- Keys to modeling prestressed membranes are understanding the following:
- Large deformations must be accounted for
- Membranes have no bending stiffness, while COMSOL only has shell and solid elements, unlike ABAQUS
- Prestress increases system stiffness, and thus alters eigenfrequencies

Challenges in COMSOL

- Alternating between COMSOL 3.5 and 4.0 for computational analysis
- Modeling prestress in membrane was difficult to adapt for COMSOL
- Keeping the prestress and having it interact with the piezoelectric patch more difficult
- Simple problem of analyzing the natural frequency of a plain square membrane successful

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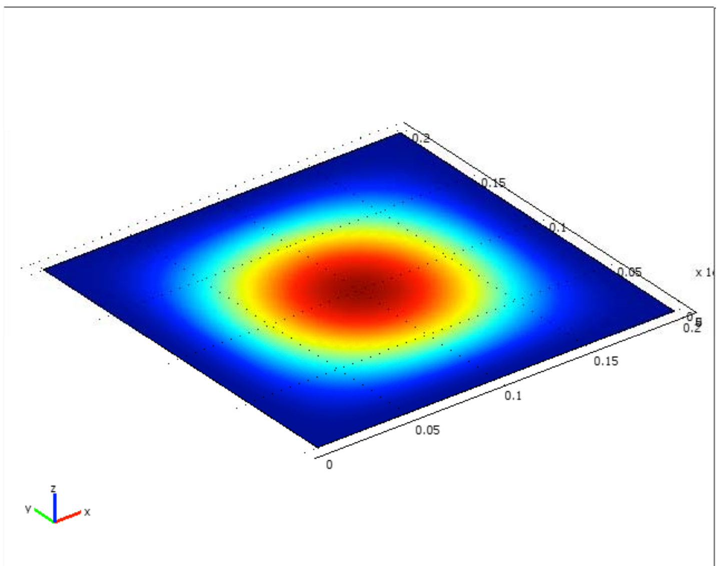
Simple Membrane Comparison

- Below are the results from a comparison made between literature[1] and COMSOL for a simple membrane
- Prestress applied to .2 x .2 x .0001 m Kapton membrane in static step, then eigenfrequency analysis performed (1st mode shown)

[1] S. Kukathasan and S. Pellegrino, Vibration of Prestressed Membrane Reflectors, ESA Contractor Report



Prestress (N/m)	COMSOL Frequency	Analytical Frequency	ABAQUS Frequency
10	34.46 Hz	39.78 Hz	39.66 Hz
20	51.28 Hz	56.25 Hz	56.08 Hz
30	63.89 Hz	68.9 Hz	68.69 Hz
40	75.25 Hz	79.56 Hz	79.31 Hz
50	87.87 Hz	88.95 Hz	88.67 Hz

Conclusions

- Energy harvesting from membranes requires heterogeneous materials and multi-physics
- Many methods/software require sequential analysis and additional intermediate processing
- Success has been obtained using analytical methods (Adomian) + FEM tools
- Barriers still exist in implementing the solution in multi-physics software, e.g. COMSOL

Overview of the Adomian Decomposition Method

- Consider the differential equation

$$LY + NY + RY = g(t)$$

$$\downarrow$$

$$Y = Y_0 - L^{-1}NY - L^{-1}RY$$

where $Y_0 = Y(0) + tY'(0) + L^{-1}g(t)$

L linear operator

N Non linear operator

R Remaining linear part

- The general solution is assumed to be of the form

$$Y = \sum_{n=0}^{\infty} Y_n \qquad \text{where } Y_n = -L^{-1}NY_{n-1} - L^{-1}RY_{n-1}$$

$$Y_0 = Y(0) + tY'(0) + L^{-1}g(t) \qquad A_0 = f(Y_0)$$

$$Y_1 = -L^{-1}RY_0 - L^{-1}NA_0 \qquad A_1 = Y_1 \frac{d}{dY_0} f(Y_0)$$

Limitations

- Let us write $S_n = \sum_{i=0}^{n} Y_n$
- $\lim_{n \to \infty} S_n = Y$
- The solution Y converges if

$$\exists 0 \le \alpha < 1, \ \ \| Y_{k+1} \| < \alpha \| Y_k \|, \ \ \forall k \in NU\{0\}$$